# **Comparison of Fuzzy Membership Functions for Value of Information Determination**

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#### Abstract

Network-centric military operations are redefining information overload as military commanders and staffs are inundated with vast amounts of information. Recent research has developed a fuzzy-based system to assign a Value of Information (VoI) determination for individual pieces of information. This paper presents an investigation on the effect of using triangular and trapezoidal fuzzy membership functions within the system.

#### Introduction

Today's military operations utilize information from a myriad of sources that provide overwhelming amounts of data. A primary challenge of decision makers at all levels is to identify the most important information with respect to the mission at hand, and often do so within a limited amount of time. The process of assigning a Value of Information (VoI) determination to a piece of information has historically been a multi-step, human-intensive exercise requiring intelligence collectors and analysts to make judgments within differing operational situations. Recently, a fuzzy associative memory architecture was used to develop a system to calculate VoI in complex military environments based on the information's content, source reliability, latency, and the specific mission context under consideration (Hanratty, Hammell, and Heilman 2011; Hammell, Hanaratty, and Heilman 2012). Military intelligence analysts were used as subject matter experts to provide the fuzzy association rules from which the system was constructed, and preliminary results from the system have been demonstrated and "validated" in principal and context (Hanratty et al. 2012; Hanratty et al. 2013). Efforts are continuing towards a more formal validation of the system and to empirically evaluate the effects of the system on intelligence analyst performance (Newcomb and Hammell 2012; Newcomb and Hammell 2013).

This paper presents an investigation on the effect of using two different membership functions within the fuzzy-based system and a comparative analysis of the differences between them. The paper is organized as follows: the next section presents background information on VoI as well as the design of the original fuzzy system. This is followed by a section that discusses the experimental framework used for this work, and then a section describing the experiments and results. The paper concludes with a section that provides conclusions and future work.

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## Value of Information (VoI)

In order to turn large amounts of disparate information into useful knowledge, it is vital to have some way to judge the importance of individual pieces of information; the Value of Information (VoI) metric is used to do this. Ranking the "value" of information is a formidable task involving not only the sheer amount and diversity of information, but also the idea that the value of a piece of information will likely be influenced by the specific mission context to which it will be applied.

Before going further, it is useful to briefly address what is meant by information "value", and differentiate it from what could be meant by information "quality". One viewpoint is that "quality" refers to the *fitness* of data with respect to the inherent attributes of the data (accuracy, precision, timeliness, freshness, resolution, etc.) while "value" addresses the *utility* of the data within a specific application context (Bisdikian et al. 2009). The definition used in this paper comes from that provided by (Wilkins, Lee, and Berry 2003). Wilkins considers the practical importance of the information to the receiver, suggesting that information with value supports the receiver's ability to make informed decisions.

#### **VoI Determination**

U.S. military doctrinal guidance for determining VoI is vague at best (US Army 2006; NATO 1997) and does not address integrating mission context into the decision. The guidance provides two tables for judging the "reliability" and "content" of a piece of data, with each characteristic broken into six categories. *Reliability* relates to the information source, and is ranked from A to F (reliable, usually reliable, fairly reliable, not usually reliable, unreliable, and cannot judge). Information content is ranked from 1 to 6 (confirmed, probably true, possibly true, doubtfully true, improbable, and cannot judge).

Doctrinal guidance does not provide any process for combining these determinations into a VoI metric. Additionally, it is obvious that combining only these two assessments of a piece of information would fall far short of representing all the critical aspects for a useful VoI determination.

Two other potential data characteristics include *mission context* and *timeliness*. Timeliness relates to how long ago the piece of information was collected, while mission context is set by the operational tempo of the military operation underway. The operational tempo relates to the decision cycle for the mission; that is, the time that can or will be used to plan, prepare, and execute the mission. Fast tempo operations may have a decision cycle measured in minutes to hours, while slower tempo operations may be measured in months or longer.

## VoI System

While it is likely that numerous characteristics could be applicable to determining VoI, the aspects of source reliability, information content, timeliness, and mission context were used as the starting point to develop an automated VoI system.

A Fuzzy Associative Memory (FAM) model was chosen to construct the prototype fuzzy system. A FAM is a *k*dimensional table where each dimension corresponds to one of the input universes of the rules. The *i*th dimension of the table is indexed by the fuzzy sets that compromise the decomposition of the *i*th input domain. Fuzzy if-then rules are represented within the FAM. For the prototype system, three inputs are used to make the VoI decision: source reliability, information content, and timeliness (how mission context contributes to the determination will be explained shortly).

The overall architecture of the fuzzy system is shown in Fig. 1. Instead of using one 3-dimensional FAM, two 2-dimensional FAMs were used. The reasoning behind this decision was presented in detail in (Hammell, Hanratty, and Heilman 2012) but essentially it provided a simpler knowledge elicitation process, decreased the total number of fuzzy rules, and provided a potential for the output of the first FAM to be useful on its own.

As seen in Fig. 1, two inputs feed into the *Applicability* FAM: source reliability (SR) and information content (IC); the output of this FAM is termed the information applicability decision. Likewise, two inputs feed into the *VoI* FAM: one of these (information applicability) is the output of the first FAM; the other input is the information timeliness rating. The output of the second FAM, and the overall system output, is the VoI metric.

The fuzzy rules represented in the FAMs capture the relationships between the input and output domains. Since both FAMs have two inputs and one output, all the fuzzy rules in the system will be of the form "If *X* is *A* and *Y* is *B*, then *Z* is *C*", where *A* and *B* are fuzzy sets over the input domains and *C* is a fuzzy set over the output domain. For example, an actual rule in the Applicability FAM might be: "if *Source Reliability* is *Usually Reliable* and *Information Content* is *Probably True*, then *Information Applicability* is



Figure 1. Vol System Architecture

*Highly Applicable.*" Knowledge elicitation from military intelligence Subject Matter Experts (SMEs) was used to construct the fuzzy rules (Hanratty et al. 2012).

Within the Applicability FAM, the two input domains (source reliability and information content) are divided into five fuzzy sets following the guidance provided in (US Army 2006). The omission of the "cannot judge" category from both of the input domains is explained in (Hammell, Hanratty, and Heilman 2012). The "information applicability" output domain was decomposed into nine fuzzy sets (ranging from *not applicable* to *extremely applicable*) while the VoI output domain utilized eleven fuzzy sets (ranging from *not valuable* to *extremely valuable*).

Up to this point, the contribution of *mission context* has not been apparent. To account for differing mission tempos, three separate *Vol* FAMs were derived to represent three different tempos. Missions were characterized as either 'tactical' (high-tempo), 'operational' (moderatetempo), or 'strategic' (slow-tempo). The system selects the correct *Vol* FAM based on the indicated mission context, thereby utilizing the appropriate fuzzy rule base to produce the Vol determination.

More detailed descriptions of the FAMs, the fuzzy rules bases, the domain decompositions, and other implementation aspects of the prototype system can be found in (Hanratty et al. 2013). The series of surveys and interviews with SMEs that were used to integrate cognitive requirements, collect functional requirements, and elicit the fuzzy rules is presented in (Hanratty et al. 2012).

The VoI system has been demonstrated to the SMEs and its output has met SME expectations (Newcomb and Hammell 2012). Note that there is no current system against which the results can be compared. As such, the system has not been tested comprehensively due to the human-centric, context-based nature of the problem and usage of the system. Formal validation of the VoI system requires a comprehensive experiment which is currently under development separately.

# **Experimental Framework**

A major factor in the design of any fuzzy system relates to the decomposition of the input and output domains into fuzzy sets. The "shape" of the fuzzy sets defines the membership functions for the system. While there are numerous shapes for fuzzy sets (triangular, trapezoidal, Gaussian, bell, and the like), triangular membership functions were used in the initial VoI system. To further facilitate computational efficiency, it was also required that the triangles were isosceles with bases of the same width; this <u>triangular</u> <u>d</u>ecomposition with <u>e</u>venly spaced midpoints has been referred to as a TPE system (Sudkamp and Hammell 1994). Fig. 3(a) shows the TPE decomposition of a domain ranging from 1 to 5; Fig. 3(b), 3(c), and 3(d) illustrate isosceles triangular decompositions of the same range that do *not* adhere to the restriction of having bases of the same width. Triangular decompositions, with and without bases of the same width, are included in our experimental framework.

In addition to using triangular membership functions, trapezoidal decompositions are another approach we would like to explore. Similar to the triangles, isosceles trapezoids both with and without bases of the same width are considered. Fig. 5(a) shows the decomposition of a domain ranging from 1 to 5 using isosceles trapezoids with bases of the same width; Fig. 5(b), 5(c), and 5(d) depict similar decompositions using isosceles trapezoids without the requirement for equally sized bases.

While we mentioned several forms of membership functions from which to choose, we selected trapezoidal and triangular fuzzy sets for two primary reasons. First, the membership degree calculations for both are linear, thereby facilitating high computational efficiency. This is significant since the purpose of the fuzzy VoI system is to help intelligence specialists find the most important information within a potentially large amount of data while frequently adhering to restrictive time constraints.

The second reason is that these two forms can help in the data acquisition process. As implied earlier, significant knowledge elicitation efforts using intelligence specialists as Subject Matter Experts (SMEs) were required to construct the initial fuzzy rules; likewise, any membership function optimization will be determined by the SMEs. The triangular and trapezoidal functions are more visually understandable and provide an environment more conductive to human-in-the-loop knowledge acquisition. Based on these two reasons, trapezoidal and triangular membership functions are often used (Zimmerman 1996).

To facilitate the analysis of various domain decompositions using the triangular and trapezoidal fuzzy sets, we compare them from different aspects and display the results visually. Three categories of experiments are presented in the next section. First, results from using "standard" triangular and trapezoidal decompositions are compared, where "standard" means the use of isosceles shapes with bases of the same width (Fig. 3(a) and 5(a)). Next, "standard" triangular fuzzy membership functions are compared with "customized" triangular fuzzy membership functions, where "customized" means that the restriction for bases of the same width is removed (Fig. 3(b), 3(c), and 3(d)). Finally, "standard" trapezoidal fuzzy decompositions (Fig. 5(b), 5(c), and 5(d)).

#### **Results**

This section provides the experimental results from comparing triangular and trapezoidal fuzzy set membership functions. Three subsections will be used to present the results. First, a comparison of the "standard" triangular and trapezoidal sets will be shown. Next, several "customized" triangular decompositions will be compared with the initial TPE fuzzy sets. Finally, several "customized" trapezoidal decompositions will be compared with the standard trapezoidal fuzzy sets.

#### Standard Triangular vs Standard Trapezoidal

Fig. 2 compares the FAM outputs for the standard (TPE) triangular fuzzy sets (a, c) and the standard trapezoidal fuzzy sets (b, d). Fig. 2a and 2b show the applicability FAM output for the two models; that is, the relationship between source reliability (x-axis) and information content (y-axis). The values of two inputs are from one to five, with the smaller value of one being "better" (better reliability/content) and five meaning "worse" (less reliability/content). The applicability output values vary from one to nine where the larger values represent better applicability; the colors vary from blue to red where the higher value is in blue (high applicability meaning reliable, probable information) and the lower value is in red (unreliable, improbable information).

Fig. 2c and 2d show the *value of information* (VoI) FAM output based on the two inputs of applicability and timeliness. Applicability is as mentioned above. Timeliness reflects the temporal age of the information, with values ranging from one to three: one means "recent" while three means "old". As with the applicability graphs, the VoI values are represented in the color shades within the graph. The numerical values for VoI range from zero to ten (blue meaning ten; red meaning zero) and the higher values represent higher VoI (more valuable information). The mission context is assumed to be "tactical".

Comparing results for the models, the output landscape of the triangular fuzzy models (a, c) looks smoother while the trapezoidal fuzzy models (b, d) produce some fairly well defined rectangles. To see why, note that when an input (in these standard models) has a membership value equal to 1 in a fuzzy set, the input belongs only to that fuzzy set (see Fig. 3(a) and 5(a)). For example, in the triangular fuzzy model, only the integer input values (1, 2, etc.) belong to just one fuzzy set; that is, there is only one input value in each triangular fuzzy set that will have a membership equal to one. For the trapezoidal fuzzy sets, however, there are several values in each set that have a membership equal to one and, thus, belong to only that fuzzy set. This creates areas within the color graphs that have the same calculated output values for applicability or



Figure 2. Applicability and VoI: Standard Triangular and Trapezoidal Fuzzy Sets

VoI, thereby producing the more pronounced rectangles. Note these rectangles are seen within the color graph and at the four corners of the graph.

#### **Standard Triangular vs Customized Triangular**

In the experiments shown below, due to space limitations, comparisons between the different models will be illustrated using the results of the *applicability* FAM only. Fig. 3 and 4 are used to compare the applicability values for the standard and customized triangular fuzzy models; Fig. 3 shows the fuzzy set shapes for all domains in the standard model (3(a)) and the customized models (3(b), 3(c), 3(d)), and Fig. 4 provides the associated color graphs.

## Case 1

In the standard model, both domains (source reliability and information content) are decomposed following the TPE restrictions as illustrated in Fig. 3(a). The resulting color graph for the standard (TPE) model is shown in Fig. 4(a). In the customized model, both inputs shrink the third fuzzy set from the standard 2 to 4 width (on the x-axis) to the customized width of 2.5 to 3.5 as shown in Fig. 3(b); the corresponding color graph is Fig. 4(b).

Compared with the applicability distribution of the standard model, the color graph for the customized model is much less smooth. It is also clear that two very similar color belts cross in the middle of the graph (as outlined); the edges are 2.5 and 3.5 (both vertically and horizontally). The middle of the graph for the customized model has similar color values; however, the outer edge of the color



Figure 3. Standard and Customized Triangular Fuzzy Membership Functions



Figure 4. Applicability: Standard and Customized Triangular Fuzzy Sets

belt has smaller changes than that in standard model and the inner edge has larger changes which cause the visible boundaries. Also, four rectangles in a solid color around the center are observable (and outlined) in Fig. 4(b). The reason for the observed differences is that in the customized model, inputs between 2 to 2.5 and 3.5 to 4 only belong to one fuzzy set. This leads to smooth visualization and solid squares of the same color. On the other hand, input values between 2.5 to 3.5 belong to two fuzzy sets. The third fuzzy membership degree is changed faster (narrower triangle; slope is larger) than that in the standard model. As a result, this enhances the representation of boundaries.

#### Case 2

In this case, the third fuzzy set is assigned a wider range, encompassing the entire input domain, as shown in Fig. 3(c). Again, in the standard model both domains are decomposed following the TPE restrictions (3(a)).

The values in the customized color graph in Fig. 4(c) look smoother in the center with sharp variations occurring in red and blue at the corners; the red and blue corner values have a much smaller area than in the standard fuzzy triangular model. The reason is that the third fuzzy set in the customized model affects all the fuzzy membership degree calculations since it spans the entire input domain. For high value inputs, the third fuzzy set causes lower FAM values to join the calculation, resulting in a lower output value than that in the standard model.

The reverse occurs for the low value inputs; the middle fuzzy set contributes higher FAM values to the applicability result. Thus, the red and blue boundaries contract to the corners of the customized graph as compared to the standard triangular fuzzy model results.

#### Case 3

Considering that some users maybe prefer a wide range in the middle fuzzy sets (most IC and SR inputs would fall in the "middle") but smaller ranges at the edges (only extreme IC and SR inputs are considered "best" or "worst"), Fig. 3(d) shows a fuzzy set pattern to provide such a system. In this model, the two ends are made narrower (range from 1 to 1.5 and 4.5 to 5), which means only a small range of inputs belong to these sets. The middle set has a wide input scope, which is from 1.5 to 4.5. Meanwhile, the input ranges of other two fuzzy sets are reduced appropriately.

Fig. 4(d) shows the applicability distribution based on this customized model which is much more "blocky" than that of the standard TPE model shown in Fig. 4(a). Because the middle fuzzy set is extended and covers numerous inputs, the resulting output has a number of areas in the middle values. The contraction of the other fuzzy sets causes much of the graph area to show up in the orange and cyan colors, while only the corners have extreme high or low values corresponding to dark blue and dark red.

#### Standard Trapezoidal vs Customized Trapezoidal

Fig. 5 and 6 are used to compare the applicability values for the standard and customized trapezoidal fuzzy models; Fig. 5 shows the fuzzy set shapes for all domains in the standard model (5(a)) and the customized models (5(b), 5(c), 5(d)), and Fig. 6 provides the associated color graphs.

#### Case 1

In the standard model, both domains (SR and IC) are decomposed as illustrated in Fig. 5(a). The resulting color graph for the standard model is shown in Fig. 6(a). In the customized case, the middle fuzzy set is still an isosceles trapezoid but the width is smaller than the other sets, as depicted in Fig. 5(b). The left and right bottom points are 2.5 and 3.5; note the upper base is the same 2.75 to 3.25 as in the standard trapezoidal model. The corresponding color graph is shown in Fig. 6(b).

As in Case 1 of the triangular fuzzy model, the color graph for this customized trapezoidal model illustrates two similar color belts crossing in the middle of Fig. 6(b). Because the middle fuzzy set is narrower and more inputs belong to only the second or fourth fuzzy set, the edges corresponding to the middle SR and IC input values are smaller and more pronounced than those of the standard trapezoidal model in Fig. 6a. Also, the neighboring rectangles of solid color are larger than that in the standard model. Note that the areas associated with the four corners are similar in both the standard and customized color graphs.

#### Case 2

Considering the opposite setup with the middle fuzzy set as shown in Fig. 5(c), this case sets the middle fuzzy set to cover a wider input range, from 1 to 5. However, the upper base is still fixed from 2.75 to 3.25 and all other sets are the same as in the standard trapezoidal model.

Fig. 6(c) illustrates the associated color graph. The result of the customized trapezoidal model reveals a similar trend as that of the corresponding triangular model; more areas in the middle values can be observed and sharp variation happens in the corners as compared to the standard trapezoidal model in Fig. 6(a). Nevertheless, the graph still presents the basic features of the trapezoidal fuzzy model some rectangles in similar colors exist in the color graph which are not as obvious in the triangular fuzzy model.

#### Case 3

Based on the same scenario as with Case 3 for the triangular fuzzy sets, this customized trapezoidal model sets up a wide middle fuzzy set and narrower side sets as shown in Fig. 5(d). In this setup, only very high or low value inputs are regarded as extreme conditions.



Figure 5. Standard and Customized Trapezoidal Fuzzy Membership Functions

Fig. 6(d) shows the applicability distribution based on the customized model. Compared with the result of the standard trapezoidal fuzzy model in Fig. 6(a), similar results occur as with Case 3 for the triangular model. The areas in the middle values are larger than those in the standard trapezoidal model (Fig. 6(a)) and only small sections of dark red and blue in the corners represent the extreme high and low applicability values. Moreover, the result of this customized model retains the features of a trapezoidal fuzzy model which produces larger areas in the graph of solid colors. However, one difference is that color boundaries between the rectangles are much narrower in the customized model. This makes the boundaries more pronounced and provides well-defined solid color rectangles.

#### **Conclusion and Future Work**

This paper presents two approaches for codifying the contextual underpinnings (framework) and cognitive interpretation for capturing VoI utilizing source reliability, information content and latency based on triangular and trapezoidal fuzzy membership functions. While both approaches for capturing VoI are intuitively simple to comprehend and computationally easy to calculate, differences are observed.

The first major difference observed is that when using the triangular approach the results of the color graphs were



Figure 6. Applicability: Standard and Customized Trapezoidal Fuzzy Sets

strikingly different than those of the trapezoidal approach. Using the triangular fuzzy model produced graphs that were infinitely smoother in their transition between calculated values. The trapezoid models, on the other hand, produced plots that appeared "blockier", lending to larger areas of continual homogeneous values.

A second major difference observed is the increased flexibility for representing membership functions afforded by the trapezoidal representation. Using the trapezoids allowed an 'interval of values' that maximized the individual membership functions (top of the trapezoids) as compared to the triangle representation that permitted only one. The introduction of the trapezoid dramatically increases the ability of the user to capture representations over the more simplistic triangular shape.

With this understanding, one might mistakenly chose one approach over the other, thinking on one hand the trapezoidal approach is inferior because of the "blocky effect" or on the other hand superior because of the added flexibility. The fact is both approaches have their own strengths and weaknesses. For example, depending on the context of the situation, the blocky effect might provide a better representation of the military function being modeled. An example of this effect can be seen when comparing a logistic battle function against that of a tactical combat battle function. For the logistics operations the fidelity of the information required for moving equipment can be significantly less critical than that required when conducting a combat cordon and search operation; as such, the logistical representation of VoI may very well be represented with larger areas of homogeneous values (blocking effect).

Ultimately the goal of this research is targeted to improve the higher-level information fusion process (Hall, Hall, and Tate 2001) - effectively interleaving the human computer interaction (HCI) with the lower-level fusion process. To accomplish this goal further refinement of the VoI approach is necessary and includes the following activities: 1) vetting the VoI approaches with subject matter experts to provide direct feedback on applicability, 2) exercising the VoI construct within a task network model to assess the potential impact, and 3) conducting human-in-the-loop experiments to measure how cognitively aligned interfaces improve task performance.

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