GeospatialRules: A Datalog$^+$ RuleML Rulebase for Geospatial Reasoning

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Abstract. Representing and reasoning with qualitative geospatial relationships among regions is an important task in many geospatial applications. In this paper, we present a Datalog$^+$ rulebase, GeospatialRules, which can be used for this task. The rulebase is built on top of the Region Connection Calculus (RCC). It includes rules, facts, and queries. The rules in GeospatialRules consist of a set of rules that are equivalent to the Datalog$^+$ fragment of the RCC axioms and additional rules which express part of the RCC knowledge that are not captured by the Datalog$^+$ fragment. The XML version of the rulebase complies to the Deliberation RuleML 1.01 standard, so that it allows the use of RuleML-compatible implementations for geospatial reasoning.

1 Introduction

In geographical information systems, knowledge representation and querying over topological relations between geospatial units play an important role. To fulfill the need for qualitative representation and reasoning with these relations, the Region Connection Calculus (RCC) [1, 2] has been developed, which defines a set of relations between regions through first-order logic.

In order to allow the use of RuleML-compatible implementations for geospatial reasoning, we developed a rulebase, GeospatialRules, based on the logic formalisms of RCC relations. The rulebase includes rules, facts, and queries. The rules consist of a set of rules equivalent to the Datalog$^+$ fragment of RCC definitions, as well as extra rules which express part of the RCC knowledge that are not captured by the Datalog$^+$ fragment. The rulebase complies to the recently developed Deliberation RuleML 1.01 subfamily for the Datalog$^+$ language.

The remaining paper is organized as follows: Section 2 gives the background of Datalog$^+$ and RCC. Section 3 formally develops the RCC rulebase. Section 4 concludes the paper and gives an outlook on future work. Appendix A provides a complete listing of GeospatialRules in RuleML/XML.

2 Background

In this section, we will give the background of the relevant technologies, starting with a brief introduction of the Datalog$^+$ language, and then continuing with an overview of the Region Connection Calculus.

2.1 Datalog$^+$

Datalog$^+$ is an extension of Datalog with existential quantifiers, equality, and falsity in rule conclusions. This extension addresses the modeling needs of many applications, e.g. of knowledge-based data access [3]. A family of decidable sub-languages of Datalog$^+$, Datalog$^\pm$, has been proposed for ontological querying [4]. A reasoning engine, Nyaya [5], has been developed to answer queries in Datalog$^\pm$. The new release of the Deliberation RuleML 1.01[1] standard introduces Relax NG and XSD schemas for the Datalog$^+$ language, leaving Datalog$^\pm$ restrictions to a future release.

2.2 Region Connection Calculus

The Region Connection Calculus (RCC) is an axiomization of spatial relations between regions in first-order logic [1,2]. The theory assumes one primitive binary relation $\mathsf{C}(x, y)$, which represents "x connects with y". The relation C is reflexive and symmetric, captured by the axioms

$$
\begin{array}{ll}
(1) & \forall x : \mathsf{C}(x, x) \\
(2) & \forall x, y : \mathsf{C}(y, x) \rightarrow \mathsf{C}(x, y)
\end{array}
$$

On top of C, several binary spatial relations can be defined. The meanings and first-order definitions of these relations are shown in Table 1. All top-level free variables are implicitly universally quantified. Among these relations, P,PP,TPP, and NTPP are asymmetric and their inverses can be defined. The notation $\Phi\mathsf{i}$ is used to represent the inverses for $\Phi \in \{\mathsf{P}, \mathsf{PP}, \mathsf{TPP}, \mathsf{NTPP}\}$, e.g. PPi is the inverse of PP. The first-order definition for the inverses are $\Phi\mathsf{i}(x, y) \equiv_{\text{def}} \Phi(y, x)$.

Table 1. Definitions of RCC Relations

Relation Definition	Meaning
$\mathsf{C}(x, y)$ (primitive)	x connects with y
(3) $\mathsf{DC}(x, y) \equiv_{\text{def}} \neg\mathsf{C}(x, y)$	x is disconnected from y
(4) $\mathsf{P}(x, y) \equiv_{\text{def}} \forall z[\mathsf{C}(z, x) \rightarrow \mathsf{C}(z, y)]$	x is part of y
(5) $\mathsf{O}(x, y) \equiv_{\text{def}} \exists z[\mathsf{P}(z, x) \wedge \mathsf{P}(z, y)]$	x overlaps with y
(6) $\mathsf{DR}(x, y) \equiv_{\text{def}} \neg\mathsf{O}(x, y)$	x is discrete from y
(7) $\mathsf{EC}(x, y) \equiv_{\text{def}} \mathsf{C}(x, y) \wedge \mathsf{DR}(x, y)$	x is externally connected to y
(8) $\mathsf{PO}(x, y) \equiv_{\text{def}} \mathsf{O}(x, y) \wedge \neg\mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x)$	x partially overlaps with y
(9) $\mathsf{EQ}(x, y) \equiv_{\text{def}} \mathsf{P}(x, y) \wedge \mathsf{P}(y, x)$	x is equal to y
(10) $\mathsf{PP}(x, y) \equiv_{\text{def}} \mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x)$	x is a proper part of y
(11) $\mathsf{TPP}(x, y) \equiv_{\text{def}} \mathsf{PP}(x, y) \wedge \exists z[\mathsf{EC}(z, x) \wedge \mathsf{EC}(z, y)]$	x is a tangential proper part of y
(12) $\mathsf{NTPP}(x, y) \equiv_{\text{def}} \mathsf{PP}(x, y) \wedge \neg\exists z[\mathsf{EC}(z, x) \wedge \mathsf{EC}(z, y)]$	x is a nontangential proper part of y

[1] http://deliberation.ruleml.org/1.01

Among these relations, the set $\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{EQ}, \mathsf{TPP}, \mathsf{NTPP}, \mathsf{PPi}, \mathsf{NTPPi}\}$ are provably jointly exhaustive and pairwise disjoint [1]. This set of eight relations is usually referred to as RCC8. Figure 1 [2] gives a graphical view of all RCC8 relations.

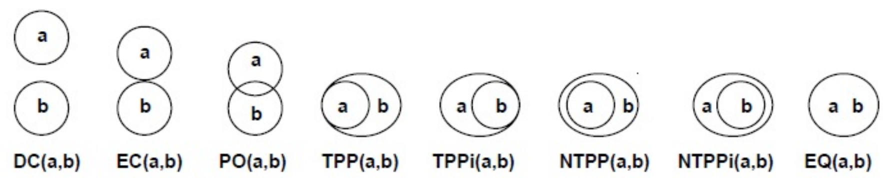

Fig. 1. RCC8 relations.

The first-order definitions of Table 1 with inverses imply a lattice of all defined relations, as shown in Figure 2 [2].

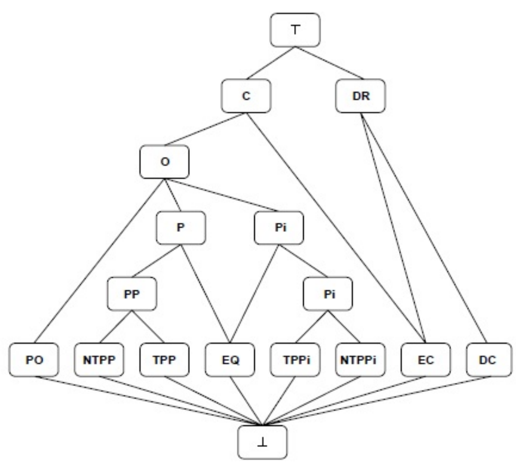

Fig. 2. Subsumption Lattice of RCC8 relations.

To assist with consistency checking and query answering in RCC, composition tables are used [2]. The RCC8 composition table can be seen as a set of first-order rules of the form $\forall x, y, z : R_1(x, y) \wedge R_2(y, z) \rightarrow \bigvee_{i=1}^{n} S_i(x, z)$, where R_1, R_2, and $S_1, \ldots, S_n$ are RCC8 relations. The table enables a quick retrieval of all possible RCC8 relations $S_1, \ldots, S_n$ between regions x and z given $R_1(x, y)$ and $R_2(y, z)$. A Datalog$^+$ subset of these rules, i.e. the ones with $i = 1$, are included in GeospatialRules.

3 Development of GeospatialRules

The GeospatialRules rulebase contains rules, facts and queries. The rules consist of two parts of knowledge: (1) the Datalog$^+$ fragment of the first-order RCC definitions; (2) rules which express part of the RCC knowledge that are not captured by (1). We will explain all the parts in the following paragraphs. The rulebase will be shown here in the presentation syntax of PSOA RuleML [6] and converted to RuleML 1.01/XML for Appendix A.

In GeospatialRules, we use RCC-prefixed CamelCase for the relation names to improve readability. The mapping between RCC relations and Geospatial-Rules relations is shown in Table 2:

Table 2. RCC Relations in GeospatialRules

RCC Relation	GeospatialRules Relation
C	`RCCConnected`
DC	`RCCDisconnected`
P	`RCCPartOf`
O	`RCCOverlapped`
DR	`RCCDiscrete`
EC	`RCCExternallyConnected`
PO	`RCCPartiallyOverlapped`
EQ	`RCCEqual`
PP	`RCCProperPartOf`
TPP	`RCCTangentialProperPartOf`
NTPP	`RCCNonTangentialProperPartOf`
Pi	`RCCInversePartOf`
PPi	`RCCInverseProperPartOf`
TPPi	`RCCInverseTangentialProperPartOf`
NTPPi	`RCCInverseNonTangentialProperPartOf`

The first part of rules in GeospatialRules is the Datalog$^+$ fragment of the axioms (1)–(12) and the definitions of inverse RCC relations shown in Section 2.2. Axioms (1) and (2) are kept unchanged in GeospatialRules:

```
Forall ?X ( RCCConnected(?X ?X) )
Forall ?X ?Y ( RCCConnected(?Y ?X) :- RCCConnected(?X ?Y) )
```

Axiom (4) is equivalent to

$$(\mathsf{P}(x,y) \to \forall z[\mathsf{C}(z,x) \to \mathsf{C}(z,y)]) \land (\forall z[\mathsf{C}(z,x) \to \mathsf{C}(z,y)] \to \mathsf{P}(x,y))$$

The second conjunct is non-Horn, hence cannot be expressed in Datalog$^+$. The first conjunct can be transformed to the equivalent Datalog rule

$$\mathsf{C}(z,x) \land \mathsf{P}(x,y) \to \mathsf{C}(z,y),$$

which is written in GeospatialRules as

```
Forall ?X ?Y ?Z (
  RCCConnected(?Z ?Y) :- And(RCCConnected(?Z ?X) RCCPartOf(?X ?Y))
)
```

The correctness of the transformation can be shown by the following first-order equivalences:

$$\begin{aligned} & \forall x, y[\mathsf{P}(x, y) \rightarrow \forall z[\mathsf{C}(z, x) \rightarrow \mathsf{C}(z, y)]] \\ \equiv\ & \forall x, y[\neg\mathsf{P}(x, y) \vee \forall z[\neg\mathsf{C}(z, x) \vee \mathsf{C}(z, y)]] \\ \equiv\ & \forall x, y, z[\neg\mathsf{P}(x, y) \vee \neg\mathsf{C}(z, x) \vee \mathsf{C}(z, y)] \\ \equiv\ & \forall x, y, z[\neg(\mathsf{P}(x, y) \wedge \mathsf{C}(z, x)) \vee \mathsf{C}(z, y)] \\ \equiv\ & \forall x, y, z[\mathsf{C}(z, x) \wedge \mathsf{P}(x, y) \rightarrow \mathsf{C}(z, y)] \end{aligned}$$

The axioms (3),(5)–(12), and the definitions of the inverse relations are all of the form $r \equiv \bigwedge_{i=1}^{m} p_i \wedge \bigwedge_{j=1}^{n} \neg q_j$, where p_i and q_j are relational atoms or existential formulas. Such a formula is equivalent to the conjunction of two formulas

$$(r \rightarrow \bigwedge_{i=1}^{m} p_i \wedge \bigwedge_{j=1}^{n} \neg q_j) \wedge (\bigwedge_{i=1}^{m} p_i \wedge \bigwedge_{j=1}^{n} \neg q_j \rightarrow r)$$

The first formula is further separated into $m + n$ rules $\{r \rightarrow p_i\}_{i=1}^{m}$ and $\{r \wedge q_j \rightarrow \bot\}_{j=1}^{n}$, where $\bot$ denotes falsity (represented by `Or()` in the presentation and XML syntaxes in a queryable manner). The second formula is in Datalog$^+$ only if there are no negations in the premise, i.e. for $n = 0$. In such a case the formula is of the form $\bigwedge_{i=1}^{m} p_i \rightarrow r$. For any p_i being an existential formula, we move all premise-side existentially quantified variables to top-level universally quantified variables. Following are the rules belonging to this part. The rules employing `Or()` conclusions are negative-constraint rules which can be used for checking the consistency of the rulebase by querying `Or()`.

```
Forall ?X ?Y (
  Or() :- And(RCCConnected(?X ?Y) RCCDisconnected(?X ?Y))
)
Forall ?X ?Y ?Z (
  RCCOverlapped(?X ?Y) :- And(RCCPartOf(?Z ?X) RCCPartOf(?Z ?Y))
)
Forall ?X ?Y (
  Exists ?Z (And(RCCPartOf(?Z ?X) RCCPartOf(?Z ?Y))) :-
    RCCOverlapped(?X ?Y)
)
```

The above existential rule is transformed from axiom (5) in Table 1. The rule says if a region `?X` overlaps with a region `?Y`, then there exists a region `?Z` that is part of both `?X` and `?Y`.

```
Forall ?X ?Y ( Or() :- And(RCCDiscrete(?X ?Y) RCCOverlapped(?X ?Y)) )
Forall ?X ?Y (
  RCCExternallyConnected(?X ?Y) :-
    And(RCCConnected(?X ?Y) RCCDiscrete(?X ?Y))
)
```

```
Forall ?X ?Y ( RCCConnected(?X ?Y) :- RCCExternallyConnected(?X ?Y) )
Forall ?X ?Y ( RCCDiscrete(?X ?Y) :- RCCExternallyConnected(?X ?Y) )
Forall ?X ?Y ( RCCEqual(?X ?Y) :- And(RCCPartOf(?X ?Y) RCCPartOf(?Y ?X)) )
Forall ?X ?Y ( RCCPartOf(?X ?Y) :- RCCEqual(?X ?Y) )
Forall ?X ?Y ( RCCPartOf(?Y ?X) :- RCCEqual(?X ?Y) )
Forall ?X ?Y ( RCCOverlapped(?X ?Y) :- RCCPartiallyOverlapped(?X ?Y) )
Forall ?X ?Y ( Or() :- And(RCCPartiallyOverlapped(?X ?Y) RCCPartOf(?X ?Y)) )
Forall ?X ?Y ( Or() :- And(RCCPartiallyOverlapped(?X ?Y) RCCPartOf(?Y ?X)) )
Forall ?X ?Y ( RCCPartOf(?X ?Y) :- RCCProperPartOf(?X ?Y) )
Forall ?X ?Y ( Or() :- And(RCCProperPartOf(?X ?Y) RCCPartOf(?Y ?X)) )
Forall ?X ?Y ?Z (
  RCCTangentialProperPartOf(?X ?Y) :-
    And(RCCProperPartOf(?X ?Y)
        RCCExternallyConnected(?Z ?X)
        RCCExternallyConnected(?Z ?Y))
)
Forall ?X ?Y (
  Exists ?Z (
    And (RCCExternallyConnected(?Z ?X) RCCExternallyConnected(?Z ?Y))
  ) :- RCCTangentialProperPartOf(?X ?Y)
)

Forall ?X ?Y ?Z (
  Or() :-
     And(RCCNonTangentialProperPartOf(?X ?Y)
         RCCExternallyConnected(?Z ?X)
         RCCExternallyConnected(?Z ?Y))
)
Forall ?X ?Y ( RCCProperPartOf(?X ?Y) :- RCCTangentialProperPartOf(?X ?Y) )
Forall ?X ?Y ( RCCProperPartOf(?X ?Y) :- RCCNonTangentialProperPartOf(?X ?Y) )
```

The following rules are transformed from the definitions of the inverses of non-symmetric RCC relations. For each relation $\Phi \in \{\mathsf{P}, \mathsf{PP}, \mathsf{TPP}, \mathsf{NTPP}\}$, there are two rules $\Phi(x, y) \rightarrow \Phi\mathsf{i}(y, x)$ and $\Phi\mathsf{i}(x, y) \rightarrow \Phi(y, x)$.

```
Forall ?X ?Y ( RCCInversePartOf(?Y ?X) :- RCCPartOf(?X ?Y) )
Forall ?X ?Y ( RCCPartOf(?Y ?X) :- RCCInversePartOf(?X ?Y) )
Forall ?X ?Y ( RCCInverseProperPartOf(?Y ?X) :- RCCProperPartOf(?X ?Y) )
Forall ?X ?Y ( RCCProperPartOf(?Y ?X) :- RCCInverseProperPartOf(?X ?Y) )
Forall ?X ?Y (
  RCCInverseTangentialProperPartOf(?Y ?X) :- RCCTangentialProperPartOf(?X ?Y)
)
Forall ?X ?Y (
  RCCTangentialProperPartOf(?Y ?X) :- RCCInverseTangentialProperPartOf(?X ?Y)
)
Forall ?X ?Y (
  RCCInverseNonTangentialProperPartOf(?Y ?X) :- RCCNonTangentialProperPartOf(?X ?Y)
)
Forall ?X ?Y (
  RCCNonTangentialProperPartOf(?Y ?X) :- RCCInverseNonTangentialProperPartOf(?X ?Y)
)
```

The second part of rules in GeospatialRules contain rules that express part of the RCC knowledge that are not captured by the extracted Datalog$^+$ fragment. We first add all subsumptions in Figure 2 that are not implied by those extracted rules:

```
Forall ?X ?Y ( RCCConnected(?X ?Y) :- RCCOverlapped(?X ?Y))
Forall ?X ?Y ( RCCOverlapped(?X ?Y) :- RCCPartOf(?X ?Y))
Forall ?X ?Y ( RCCDiscrete(?X ?Y) :- RCCDisconnected(?X ?Y) )
```

Besides these subsumption rules, we also add rules for inferring new geospatial relations among entities from the relations explicitly expressed as facts in the rulebase. These rules are the Datalog$^+$ subset of the first-order form of the RCC8 composition table.

```
Forall ?X ?Y ?Z (
  RCCDisconnected(?X ?Z) :-
    And(RCCPartOf(?X ?Y) RCCDisconnected(?Y ?Z))
)
Forall ?X ?Y ?Z (
  RCCDiscrete(?X ?Z) :-
    And(RCCTangentialProperPartOf(?X ?Y) RCCExternallyConnected(?Y ?Z))
)
Forall ?X ?Y ?Z (
  RCCProperPartOf(?X ?Z) :-
    And(RCCTangentialProperPartOf(?X ?Y) RCCTangentialProperPartOf(?Y ?Z))
)
Forall ?X ?Y ?Z (
  RCCNonTangentialProperPartOf(?X ?Z) :-
    And(RCCPartOf(?X ?Y) RCCNonTangentialProperPartOf(?Y ?Z))
)
Forall ?X ?Y ?Z (
  RCCDisconnected(?X ?Z) :-
    And(RCCNonTangentialProperPartOf(?X ?Y) RCCExternallyConnected(?Y ?Z))
)
Forall ?X ?Y ?Z (
  RCCNonTangentialProperPartOf(?X ?Z) :-
    And(RCCNonTangentialProperPartOf(?X ?Y)
        RCCTangentialProperPartOf(?Y ?Z))
)
Forall ?X ?Y ?Z (
  RCCProperPartOf(?X ?Z) :-
    And(RCCProperPartOf(?X ?Y) RCCProperPartOf(?Y ?Z))
)
```

The rules in the second part compensate for a portion of information that is lost when the Datalog$^+$ fragment is extracted. These rules do not recover all the lost knowledge since some of it cannot be expressed in Datalog$^+$. However, they ensure that all subsumptions between the RCC relations are preserved in our rulebase. Moreover, the rules from the RCC8 composition table allow the users to make inferences that are usually needed in geospatial applications, which we will show in the next paragraph.

The remaining of GeospatialRules consists of example facts and queries. The facts in the KB describe geospatial relationships among certain regions in North America, as well as the Pacific and the Atlantic oceans:

```
RCCProperPartOf(USA NorthAmerica)
RCCProperPartOf(California USA)
RCCProperPartOf(Virginia USA)
RCCExternallyConnected(Pacific California)
RCCExternallyConnected(Atlantic Virginia)
RCCExternallyConnected(Pacific USA)
RCCExternallyConnected(Atlantic USA)
State(California)
State(Virginia)
Ocean(Pacific)
Ocean(Atlantic)
```

Based on the above rules and facts, the following example queries can be asked:

1. `RCCProperPartOf(California NorthAmerica)`
   The ground query asks whether California is a proper part of North America. This query atom can be inferred by applying the rule

   ```
   Forall ?X ?Y ?Z (
     RCCProperPartOf(?X ?Z) :-
       And(RCCProperPartOf(?X ?Y) RCCProperPartOf(?Y ?Z))
   )
   ```

   to the facts `RCCProperPartOf(California USA)` and `RCCProperPartOf(USA NorthAmerica)`.
2. `And(State(?St) RCCTangentialProperPartOf(?St USA))`
   The query asks which states in the rulebase are tangential proper parts of the USA. From the geospatial relations among `Pacific`, `California`, and `USA`, and the definition rule of `RCCTangentialProperPartOf`:

   ```
   Forall ?X ?Y ?Z (
     RCCTangentialProperPartOf(?X ?Y) :-
       And(RCCProperPartOf(?X ?Y)
           RCCExternallyConnected(?Z ?X)
           RCCExternallyConnected(?Z ?Y))
   )
   ```

   the atom `RCCTangentialProperPartOf(California USA)` can be derived. Hence `California` is an answer of the query. Similarly, another answer, `Virginia`, can also be derived.
3. ```
   And( State(?St)
        Ocean(?O)
        RCCProperPartOf(?St NorthAmerica)
        RCCConnected(?St ?O))
   ```
   ```

This query asks for any state `?St` which is a proper part of North America and connects with an ocean `?O`. From the first query, we know that `California` is a proper part of North America. And since `RCCConnected` is a super-relation of `RCCExternallyConnected`, `RCCConnected(California Pacific)` can also be inferred. Therefore, one answer of the query is `?St=California, ?O=Pacific`. Another answer, `?St=Virginia,?O=Atlantic`, can be obtained in a similar way.

The full RuleML/XML version of the rulebase,[2] which complies to the Deliberation RuleML 1.01 specification, is included in Appendix A. It uses an abbreviated encoding allowed by the specification, which omits some edge tags and default attribute values to make the document more compact and easier to read. The compact version can be expanded to a normalized version using the Deliberation RuleML Normalizer XSLT stylesheet for Disjunctive Hornlog+ with Naf and Neg.[3] To increase readability and comprehensibility of the rulebase, we use proper indentations and add verbal definitions to each of the RCC relation as comments.

4 Conclusions and Future Work

In this paper, we describe a Datalog$^+$ rulebase, GeospatialRules, which can be used in geospatial applications for reasoning with geospatial relations. The rulebase is built on top of RCC relations. It contains rules, facts, and queries, where the rules consist of: (1) a Datalog$^+$ fragment of the first-order RCC relation definitions; (2) additional rules expressing part of the RCC knowledge that are not captured in (1), which ensure all subsumptions between RCC relations are preserved. The rulebase enables the use of RuleML-compatible implementations for qualitative geospatial reasoning.

In Semantic Web applications, graph representations are widely used for modeling knowledge and answering queries. In these representations, an object consists of a globally unique Object IDentifier (OID) typed by a class and described by an unordered collection of n attribute-value slots, where the value can again identify an object. Future versions of the rulebase can represent regions by OIDs with slots describing its connections with other regions, using graph versions of RCC relations as slot names, e.g. defined in the GeoSPARQL standard [7], in order to allow graph querying over the rulebase. The transformation from the relational to graph versions of RCC relations can be expressed in PSOA RuleML [6], which combines the graph and relational paradigms of modeling.

Further evaluation of the efficiency of reasoning in GeospatialRules is also an important research topic for the wide use of the rulebase. More facts and test queries will need to be developed for the evaluation. The rulebase and queries

[2] The file can be validated at and downloaded from `http://deliberation.ruleml.org/1.01/exa/RulebaseCompetition2014/GeospatialRulesRCC.ruleml`

[3] `http://wiki.ruleml.org/index.php/Specification_of_Deliberation_RuleML_1.01#Appendix_4:_Normalizing_or_Upgrading_RuleML_Files_with_XSLT`

will be posed to existing implementations of RuleML, e.g. PSOATransRun [8], to test the efficiency of query answering.

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A GeospatialRules in RuleML/XML

```
<?xml version="1.0" encoding="UTF-8"?>
<?xml-model href="http://deliberation.ruleml.org/1.01/relaxng/datalogplus_min_relaxed.rnc"?>
<!--
The rulebase is built on top of the definitions of the Region Connection Calculus (RCC).
It contains rules, facts, and queries. The rules consist of:
1) the Datalog+ fragment of the first-order definition of RCC relations;
2) additional rules that express part of the RCC knowledge that are not captured by 1).
-->
<RuleML xmlns="http://ruleml.org/spec">
  <!-- RCCConnected is a premitive relation in RCC. In RCC, two regions X and Y
       are connected if their topological closures share at least one point. -->
  <Assert>
    <!-- Reflexivity of RCCConnected -->
    <Forall>
      <Var>X</Var>
      <Atom>
        <Rel>RCCConnected</Rel>
        <Var>X</Var>
```

```
      <Var>X</Var>
    </Atom>
  </Forall>
</Assert>
<Assert>
  <!-- Symmetry of RCCConnected -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCConnected</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCConnected</Rel>
          <Var>Y</Var>
          <Var>X</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, a region X is part of a region Y if and only if all regions
     that connect to X also connect to Y.
-->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCConnected</Rel>
            <Var>Z</Var>
            <Var>X</Var>
          </Atom>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCConnected</Rel>
          <Var>Z</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, the RCCDisconnected relation is the negation of RCCConnected. The
     following rule expresses the Datalog+ fragment of this definition.
 -->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
```

```
      <if>
        <And>
          <Atom>
            <Rel>RCCConnected</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCDisconnected</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Or/>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, two region X and Y are overlapped if and only if there exists
     a region Z such that Z is part of X and Z is part of Y.
-->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>Z</Var>
            <Var>X</Var>
          </Atom>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>Z</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCOverlapped</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCOverlapped</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Exists>
          <Var>Z</Var>
```

```
        <And>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>Z</Var>
            <Var>X</Var>
          </Atom>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>Z</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </Exists>
    </then>
  </Implies>
 </Forall>
</Assert>
<!-- In RCC, RCCDiscrete is the negation of RCCOverlapped. -->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCDiscrete</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCOverlapped</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Or/>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, two region X and Y are externally connected if and only if X is connected
     with Y and X is discrete from Y. -->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCConnected</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCDiscrete</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCExternallyConnected</Rel>
          <Var>X</Var>
```

```
        <Var>Y</Var>
      </Atom>
    </then>
  </Implies>
 </Forall>
</Assert>
<Assert>
 <!-- RCCExternallyConnected is a subrelation of RCCConnected -->
 <Forall>
  <Var>X</Var>
  <Var>Y</Var>
  <Implies>
   <if>
    <Atom>
     <Rel>RCCExternallyConnected</Rel>
     <Var>X</Var>
     <Var>Y</Var>
    </Atom>
   </if>
   <then>
    <Atom>
     <Rel>RCCConnected</Rel>
     <Var>X</Var>
     <Var>Y</Var>
    </Atom>
   </then>
  </Implies>
 </Forall>
</Assert>
<Assert>
 <!-- RCCExternallyConnected is a subrelation of RCCDiscrete -->
 <Forall>
  <Var>X</Var>
  <Var>Y</Var>
  <Implies>
   <if>
    <Atom>
     <Rel>RCCExternallyConnected</Rel>
     <Var>X</Var>
     <Var>Y</Var>
    </Atom>
   </if>
   <then>
    <Atom>
     <Rel>RCCDiscrete</Rel>
     <Var>X</Var>
     <Var>Y</Var>
    </Atom>
   </then>
  </Implies>
 </Forall>
</Assert>
<!-- In RCC, two regions X and Y are equal if X is part of Y and Y is part of X. -->
<Assert>
 <Forall>
  <Var>X</Var>
  <Var>Y</Var>
  <Implies>
   <if>
    <And>
     <Atom>
      <Rel>RCCPartOf</Rel>
      <Var>X</Var>
      <Var>Y</Var>
     </Atom>
     <Atom>
      <Rel>RCCPartOf</Rel>
      <Var>Y</Var>
```

```
          <Var>X</Var>
        </Atom>
      </And>
    </if>
    <then>
      <Atom>
        <Rel>RCCEqual</Rel>
        <Var>X</Var>
        <Var>Y</Var>
      </Atom>
    </then>
  </Implies>
</Forall>
</Assert>
<Assert>
  <!-- RCCEqual is a subrelation of RCCPartOf -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCEqual</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCEqual</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCPartOf</Rel>
          <Var>Y</Var>
          <Var>X</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, two region X and Y are partially overlapped if and only if X and Y
     are overlapped, X is not part of Y, and Y is not part of Y. -->
<Assert>
  <!-- RCCPartiallyOverlapped is a subrelation of RCCOverlapped -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
```

```
        <Atom>
          <Rel>RCCPartiallyOverlapped</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCOverlapped</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCPartiallyOverlapped</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Or/>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCPartiallyOverlapped</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>Y</Var>
            <Var>X</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Or/>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, a region X is a proper part of a region Y if and only if
     X is part of Y and Y is not part of X. -->
```

```
<Assert>
  <!-- RCCProperPartOf is a subrelation of RCCPartOf -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>Y</Var>
            <Var>X</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Or/>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, a region X is a tangential proper part of a region Y if and only if
     X is a proper part of Y and there exists a region Z such that Z is
 externally connected to both X and Y. -->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCExternallyConnected</Rel>
            <Var>Z</Var>
            <Var>X</Var>
```

```
          </Atom>
          <Atom>
            <Rel>RCCExternallyConnected</Rel>
            <Var>Z</Var>
            <Var>Y</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Exists>
          <Var>Z</Var>
          <And>
            <Atom>
              <Rel>RCCExternallyConnected</Rel>
              <Var>Z</Var>
              <Var>X</Var>
            </Atom>
            <Atom>
              <Rel>RCCExternallyConnected</Rel>
              <Var>Z</Var>
              <Var>Y</Var>
            </Atom>
          </And>
        </Exists>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- In RCC, a region X is a nontangential proper part of a region Y if and only if
     X is a proper part of Y and there does not exist a region Z such that Z is
     externally connected to both X and Y.
-->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCNonTangentialProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
```

```
          <Rel>RCCExternallyConnected</Rel>
          <Var>Z</Var>
          <Var>X</Var>
        </Atom>
        <Atom>
          <Rel>RCCExternallyConnected</Rel>
          <Var>Z</Var>
          <Var>Y</Var>
        </Atom>
      </And>
    </if>
    <then>
      <Or/>
    </then>
  </Implies>
 </Forall>
</Assert>
<Assert>
 <!-- RCCTangentialProperPartOf is a subrelation of RCCProperPartOf -->
 <Forall>
  <Var>X</Var>
  <Var>Y</Var>
  <Implies>
    <if>
      <Atom>
        <Rel>RCCTangentialProperPartOf</Rel>
        <Var>X</Var>
        <Var>Y</Var>
      </Atom>
    </if>
    <then>
      <Atom>
        <Rel>RCCProperPartOf</Rel>
        <Var>X</Var>
        <Var>Y</Var>
      </Atom>
    </then>
  </Implies>
 </Forall>
</Assert>
<Assert>
 <!-- RCCNonTangentialProperPartOf is a subrelation of RCCProperPartOf
  -->
 <Forall>
  <Var>X</Var>
  <Var>Y</Var>
  <Implies>
    <if>
      <Atom>
        <Rel>RCCNonTangentialProperPartOf</Rel>
        <Var>X</Var>
        <Var>Y</Var>
      </Atom>
    </if>
    <then>
      <Atom>
        <Rel>RCCProperPartOf</Rel>
        <Var>X</Var>
        <Var>Y</Var>
      </Atom>
    </then>
  </Implies>
 </Forall>
</Assert>
<!-- Definitions of the inverses of non-symmetric RCC relations: RCCPartOf,
     RCCProperPartOf, RCCTangentialProperPartOf, and RCCNonTangentialProperPartOf.
-->
<Assert>
```

```
    <!-- RCCInversePartOf and RCCPartOf -->
    <Forall>
      <Var>X</Var>
      <Var>Y</Var>
      <Implies>
        <if>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </if>
        <then>
          <Atom>
            <Rel>RCCInversePartOf</Rel>
            <Var>Y</Var>
            <Var>X</Var>
          </Atom>
        </then>
      </Implies>
    </Forall>
  </Assert>
  <Assert>
    <!-- RCCInversePartOf and RCCPartOf -->
    <Forall>
      <Var>X</Var>
      <Var>Y</Var>
      <Implies>
        <if>
          <Atom>
            <Rel>RCCInversePartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </if>
        <then>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>Y</Var>
            <Var>X</Var>
          </Atom>
        </then>
      </Implies>
    </Forall>
  </Assert>
  <Assert>
    <!-- RCCInverseProperPartOf and RCCProperPartOf -->
    <Forall>
      <Var>X</Var>
      <Var>Y</Var>
      <Implies>
        <if>
          <Atom>
            <Rel>RCCProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
        </if>
        <then>
          <Atom>
            <Rel>RCCInverseProperPartOf</Rel>
            <Var>Y</Var>
            <Var>X</Var>
          </Atom>
        </then>
      </Implies>
    </Forall>
  </Assert>
```

```
<Assert>
  <!-- RCCInverseProperPartOf and RCCProperPartOf -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCInverseProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCProperPartOf</Rel>
          <Var>Y</Var>
          <Var>X</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <!-- RCCTangentialProperPartOf and RCCInverseTangentialProperPartOf -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCInverseTangentialProperPartOf</Rel>
          <Var>Y</Var>
          <Var>X</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <!-- RCCTangentialProperPartOf and RCCInverseTangentialProperPartOf -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCInverseTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCTangentialProperPartOf</Rel>
          <Var>Y</Var>
          <Var>X</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
```

```
</Assert>
<Assert>
  <!-- RCCNonTangentialProperPartOf and RCCInverseNonTangentialProperPartOf -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCNonTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCInverseNonTangentialProperPartOf</Rel>
          <Var>Y</Var>
          <Var>X</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <!-- RCCNonTangentialProperPartOf and RCCInverseNonTangentialProperPartOf -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCInverseNonTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCNonTangentialProperPartOf</Rel>
          <Var>Y</Var>
          <Var>X</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- Additional rules expressing subsumptions between RCC relations that are not implied by
     the Datalog+ fragment.
-->
<Assert>
  <!-- RCCOverlapped is a subrelation of RCCConnected -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCOverlapped</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCConnected</Rel>
          <Var>X</Var>
          <Var>Y</Var>
```

```
      </Atom>
    </then>
  </Implies>
</Forall>
</Assert>
<Assert>
  <!-- RCCPartOf is a subrelation of RCCOverlapped -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCPartOf</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCOverlapped</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <!-- RCCDisconnected is a subrelation of RCCDiscrete -->
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Implies>
      <if>
        <Atom>
          <Rel>RCCDisconnected</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </if>
      <then>
        <Atom>
          <Rel>RCCDiscrete</Rel>
          <Var>X</Var>
          <Var>Y</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<!-- Additional rules extracted from the RCC8 composition table. -->
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCDisconnected</Rel>
            <Var>Y</Var>
```

```
            <Var>Z</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCDisconnected</Rel>
          <Var>X</Var>
          <Var>Z</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCTangentialProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCExternallyConnected</Rel>
            <Var>Y</Var>
            <Var>Z</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCDiscrete</Rel>
          <Var>X</Var>
          <Var>Z</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCTangentialProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCTangentialProperPartOf</Rel>
            <Var>Y</Var>
            <Var>Z</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCProperPartOf</Rel>
          <Var>X</Var>
```

```
          <Var>Z</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCNonTangentialProperPartOf</Rel>
            <Var>Y</Var>
            <Var>Z</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCNonTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Z</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCNonTangentialProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCExternallyConnected</Rel>
            <Var>Y</Var>
            <Var>Z</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCDisconnected</Rel>
          <Var>X</Var>
          <Var>Z</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
```

```
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCNonTangentialProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCTangentialProperPartOf</Rel>
            <Var>Y</Var>
            <Var>Z</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCNonTangentialProperPartOf</Rel>
          <Var>X</Var>
          <Var>Z</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <Forall>
    <Var>X</Var>
    <Var>Y</Var>
    <Var>Z</Var>
    <Implies>
      <if>
        <And>
          <Atom>
            <Rel>RCCProperPartOf</Rel>
            <Var>X</Var>
            <Var>Y</Var>
          </Atom>
          <Atom>
            <Rel>RCCProperPartOf</Rel>
            <Var>Y</Var>
            <Var>Z</Var>
          </Atom>
        </And>
      </if>
      <then>
        <Atom>
          <Rel>RCCProperPartOf</Rel>
          <Var>X</Var>
          <Var>Z</Var>
        </Atom>
      </then>
    </Implies>
  </Forall>
</Assert>
<Assert>
  <!-- Facts of the rulebase -->
  <Atom>
    <Rel>RCCProperPartOf</Rel>
    <Ind>USA</Ind>
    <Ind>NorthAmerica</Ind>
  </Atom>
  <Atom>
    <Rel>RCCProperPartOf</Rel>
    <Ind>California</Ind>
```

```
    <Ind>USA</Ind>
  </Atom>
  <Atom>
    <Rel>RCCProperPartOf</Rel>
    <Ind>Virginia</Ind>
    <Ind>USA</Ind>
  </Atom>
  <Atom>
    <Rel>RCCExternallyConnected</Rel>
    <Ind>PacificOcean</Ind>
    <Ind>California</Ind>
  </Atom>
  <Atom>
    <Rel>RCCExternallyConnected</Rel>
    <Ind>AtlanticOcean</Ind>
    <Ind>Virginia</Ind>
  </Atom>
  <Atom>
    <Rel>RCCExternallyConnected</Rel>
    <Ind>PacificOcean</Ind>
    <Ind>USA</Ind>
  </Atom>
  <Atom>
    <Rel>RCCExternallyConnected</Rel>
    <Ind>AtlanticOcean</Ind>
    <Ind>USA</Ind>
  </Atom>
  <Atom>
    <Rel>State</Rel>
    <Ind>California</Ind>
  </Atom>
  <Atom>
    <Rel>State</Rel>
    <Ind>Virginia</Ind>
  </Atom>
  <Atom>
    <Rel>Ocean</Rel>
    <Ind>PacificOcean</Ind>
  </Atom>
  <Atom>
    <Rel>Ocean</Rel>
    <Ind>AtlanticOcean</Ind>
  </Atom>
</Assert>
<Query>
  <!--
      Is California a proper part of North America?
      Answer: True

      The query atom is obtained by applying the rule
      Forall ?X ?Y ?Z (
        RCCProperPartOf(?X ?Z) :- And(RCCProperPartOf(?X ?Y) RCCProperPartOf(?Y ?Z))
      )
      to the facts RCCProperPartOf(California USA) and RCCProperPartOf(California USA).
    -->
  <Atom>
    <Rel>RCCProperPartOf</Rel>
    <Ind>California</Ind>
    <Ind>NorthAmerica</Ind>
  </Atom>
</Query>
<Query>
  <!--
      Which states are tangential proper part of the USA?
      Answers:
      St: California
      St: Virginia
```

```
        The answers are obtained by applying the definition rule of RCCTangentialProperPartOf
        Forall ?X ?Y ?Z (
           RCCTangentialProperPartOf(?X ?Y) :-
               And(RCCProperPartOf(?X ?Y)
                   RCCExternallyConnected(?Z ?X)
                   RCCExternallyConnected(?Z ?Y))
        )
        to the facts of relations RCCProperPartOf and RCCExternallyConnected.
      -->
    <And>
      <Atom>
        <Rel>State</Rel>
        <Var>St</Var>
      </Atom>
      <Atom>
        <Rel>RCCTangentialProperPartOf</Rel>
        <Var>St</Var>
        <Ind>USA</Ind>
      </Atom>
    </And>
  </Query>
  <Query>
    <!--
        Which states in North America connect to an ocean?
        Answers:
        St: California O: PacificOcean
        St: Virginia O: AtlanticOcean

        The answers are obtained by applying the rules
        Forall ?X ?Y ?Z (
          RCCProperPartOf(?X ?Z) :- And(RCCProperPartOf(?X ?Y) RCCProperPartOf(?Y ?Z))
        )
        and
        Forall ?X ?Y ( RCCConnected(?X ?Y) :- RCCExternallyConnected(?X ?Y) )
        to the facts.
      -->
    <And>
      <Atom>
        <Rel>State</Rel>
        <Var>St</Var>
      </Atom>
      <Atom>
        <Rel>Ocean</Rel>
        <Var>O</Var>
      </Atom>
      <Atom>
        <Rel>RCCProperPartOf</Rel>
        <Var>St</Var>
        <Ind>NorthAmerica</Ind>
      </Atom>
      <Atom>
        <Rel>RCCConnected</Rel>
        <Var>St</Var>
        <Ind>O</Ind>
      </Atom>
    </And>
  </Query>
</RuleML>
```