Decidable Contextualized DLs with Rigid Roles

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Description logics (DLs) of context can be employed to represent and reason about contextualized knowledge, which naturally occurs in practice [5,4,7,9,8]. Consider, for instance, the rôles played by a person in different contexts. The person Bob, who works for the company Siemens, plays the rôle of an employee of Siemens in the work context, whereas he might play the rôle of a customer of Siemens in the context of private life. Here, access restrictions to the data of Siemens might critically depend on Bob's rôle. Moreover, DLs capable of representing contexts are vital to integrate distributed knowledge as argued in [5,4].

DLs are well-suited to describe contexts as formal objects with formal properties that are organized in relational structures, which are fundamental requirements for modeling contexts [11,12]. However, classical DLs lack expressive power to formalize that some individuals satisfy certain concepts and relate to other individuals depending on a specific context. Therefore, often two-dimensional DLs are employed [7,9,8]: One DL $\mathcal{L}_M$ (the *meta* or *outer logic*) is used to represent the contexts and their relationships to each other. $\mathcal{L}_M$ is combined with a DL $\mathcal{L}_O$ (the *object* or *inner logic*) that captures the relational structure within each context. Moreover, while some pieces of information depend on the context, other pieces of information are shared throughout all contexts. For instance, a person's name is typically independent of the actual context. To be able to express that, some concepts and roles a designated to be *rigid*, i.e. they are required to be interpreted the same in all contexts. Unfortunately, if rigid roles are admitted, reasoning in contextualized DLs is usually undecidable [7].

We propose and investigate a family of two-dimensional DLs $\mathcal{L}_M[\![\mathcal{L}_O]\!]$ that meet the above requirements, but are a restricted form of the one defined in [7] in the sense that we limit the interaction of $\mathcal{L}_M$ and $\mathcal{L}_O$. More precisely, in our family of contextualized DLs the outer DL can refer to the internal structure of each context, but not vice versa. This represents contexts in a top-down perspective. Interestingly, reasoning in $\mathcal{L}_M[\![\mathcal{L}_O]\!]$ stays decidable with such a restriction, even in the presence of rigid roles. In some sense our family of contextualized DLs are similar to temporalized DLs investigated in [2,3,10].

For providing better intuition on how our formalism works, we examine the above mentioned example a bit further. Consider the following axioms:

$$\top \sqsubseteq [\![\exists\, \mathit{worksFor}.\{\mathit{Siemens}\} \sqsubseteq \exists\, \mathit{hasAccessRights}.\{\mathit{Siemens}\}]\!] \tag{1}$$

$$\textsc{Work} \sqsubseteq [\![\mathit{worksFor}(\mathit{Bob}, \mathit{Siemens})]\!] \tag{2}$$

$$\top \sqsubseteq [\![\exists\, \mathit{isCustomerOf}.\top \sqsubseteq \mathit{HasMoney}]\!] \tag{3}$$

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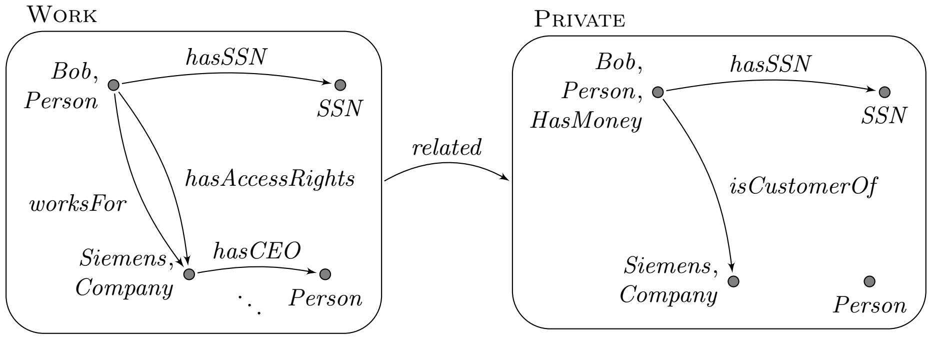

Fig. 1. Model of Axioms 1–7

$$
\begin{aligned}
[\![(\exists\, \mathit{worksFor}.\top)(\mathit{Bob})]\!] &\sqsubseteq \exists\, \mathit{related}.(\textsc{Private} \sqcap [\![\mathit{HasMoney}(\mathit{Bob})]\!]) && (4)\\
\textsc{Private} &\sqsubseteq [\![\mathit{isCustomerOf}(\mathit{Bob}, \mathit{Siemens})]\!] && (5)\\
\textsc{Private} \sqcap \textsc{Work} &\sqsubseteq \bot && (6)\\
\neg\textsc{Work} &\sqsubseteq [\![\exists\, \mathit{worksFor}.\top \sqsubseteq \bot]\!] && (7)
\end{aligned}
$$

Axiom 1 states that it holds true in all contexts that somebody who works for Siemens also has access rights to certain data. Axiom 2 states that Bob is an employee of Siemens in any work context. Axioms 3 and 4 say intuitively that if Bob has a job, he will earn money, which he can spend as a customer. Axiom 5 formalizes that Bob is a customer of Siemens in any private context. Moreover, Axiom 6 ensures that private and work contexts are disjoint. Finally, Axiom 7 states that the *worksFor* relation only exists in work contexts. A fundamental reasoning task is to decide whether a set of axioms is *consistent*. For our example, Figure 1 depicts a model. There, Bob's social security number is linked to him using a rigid role *hasSSN* since it does not change over the contexts.

Our family $\mathcal{L}_M[\![\mathcal{L}_O]\!]$ consists of combinations of two DLs, where we focus on the cases where $\mathcal{L}_M$ and $\mathcal{L}_O$ are $\mathcal{EL}$ or DLs between $\mathcal{ALC}$ and $\mathcal{SHOQ}$. Let $\mathsf{O_C}$, $\mathsf{O_R}$, and $\mathsf{O_I}$ be respectively sets of concept, role, and individual names for the object logic $\mathcal{L}_O$. Analogously, we define the sets $\mathsf{M_C}$, $\mathsf{M_R}$, and $\mathsf{M_I}$ for the meta logic $\mathcal{L}_M$. Let $\mathsf{O} = (\mathsf{O_C}, \mathsf{O_R}, \mathsf{O_I})$ and $\mathsf{M} = (\mathsf{M_C}, \mathsf{M_R}, \mathsf{M_I})$. Concepts, GCIs and assertions are defined over the respective signatures O and M as usual [1].

Definition 1. Concepts of the object logic (o-concepts) *are $\mathcal{L}_O$-concepts over* O*;* o-axioms *are $\mathcal{L}_O$-axioms over* O. Concepts of the meta logic (m-concepts) *are defined inductively: each $\mathcal{L}_M$-concept over* M *is an m-concept, and $[\![\alpha]\!]$ is an m-concept for an o-axiom α; and* m-axioms *are defined analogously.* Boolean $\mathcal{L}_M[\![\mathcal{L}_O]\!]$-knowledge bases ($\mathcal{L}_M[\![\mathcal{L}_O]\!]$-BKBs) *are Boolean combinations of m-axioms.*

Note that the syntax of the object level is precisely the one of $\mathcal{L}_O$, whereas the syntax of the meta level also allows to put $\mathcal{L}_O$-axioms in place of concept names. The semantics of $\mathcal{L}_M[\![\mathcal{L}_O]\!]$ is defined using *nested interpretations*, which consist of O-interpretations (usual DL interpretations for the names in O) for the

Table 1. Complexity results for consistency in $\mathcal{L}_M[\![\mathcal{L}_O]\!]$

$\mathcal{L}_M$ \ $\mathcal{L}_O$	no rigid names			only rigid concepts			rigid roles		
	$\mathcal{EL}$	$\mathcal{ALC}$	$\mathcal{SHOQ}$	$\mathcal{EL}$	$\mathcal{ALC}$	$\mathcal{SHOQ}$	$\mathcal{EL}$	$\mathcal{ALC}$	$\mathcal{SHOQ}$
$\mathcal{EL}$	const	Exp	Exp	const	NExp	NExp	const	2Exp	2Exp
$\mathcal{ALC}$	Exp	Exp	Exp	NExp	NExp	NExp	NExp	2Exp	2Exp
$\mathcal{SHOQ}$	Exp	Exp	Exp	NExp	NExp	NExp	NExp	2Exp	2Exp

specific contexts and the relational structure between them (M-interpretation), where all contexts have the same domain. Also, let $\mathsf{O_{Crig}} \subseteq \mathsf{O_C}$ denote the set of *rigid concepts*, and let $\mathsf{O_{Rrig}} \subseteq \mathsf{O_R}$ denote the set of *rigid roles*. Moreover, we assume that individuals of $\mathcal{L}_O$ are always interpreted the same in all contexts.

Definition 2. *We call a tuple $\mathcal{J} = (\mathbb{C}, \cdot^{\mathcal{J}}, \Delta, (\cdot^{\mathcal{I}_c})_{c\in\mathbb{C}})$ a* nested interpretation*, where $\mathbb{C}$ is a non-empty set (called* contexts*) and $(\mathbb{C}, \cdot^{\mathcal{J}})$ is an* M*-interpretation. Moreover, $\mathcal{I}_c := (\Delta, \cdot^{\mathcal{I}_c})$ is an* O*-interpretation for every $c \in \mathbb{C}$, such that it holds for all $c, c' \in \mathbb{C}$ that $x^{\mathcal{I}_c} = x^{\mathcal{I}_{c'}}$ for all $x \in \mathsf{O_I} \cup \mathsf{O_{Crig}} \cup \mathsf{O_{Rrig}}$.*

Definition 3. *Let $\mathcal{J} = (\mathbb{C}, \cdot^{\mathcal{J}}, \Delta, (\cdot^{\mathcal{I}_c})_{c\in\mathbb{C}})$ be a nested interpretation. The mapping $\cdot^{\mathcal{J}}$ is extended as follows: $[\![\alpha]\!]^{\mathcal{J}} := \{c \in \mathbb{C} \mid \mathcal{I}_c \models \alpha\}$. Moreover, $\mathcal{J}$ is a* model *of the m-axiom β if $(\mathbb{C}, \cdot^{\mathcal{J}})$ is a model of β. This is extended to $\mathcal{L}_M[\![\mathcal{L}_O]\!]$-BKBs inductively as usual. We write $\mathcal{J} \models \mathcal{B}$ if $\mathcal{J}$ is a model of the $\mathcal{L}_M[\![\mathcal{L}_O]\!]$-BKB $\mathcal{B}$. We call $\mathcal{B}$* consistent *if it has a model.*

The complexity of consistency in $\mathcal{L}_M[\![\mathcal{L}_O]\!]$ is listed in Table 1. The lower bounds are obtained using the ideas of [2,3] and hold already for the fragment $\mathcal{EL}[\![\mathcal{ALC}]\!]$, even if only *conjunctions* of m-axioms are considered instead of BKBs. Without rigid names, Exp-hardness follows from the complexity of $\mathcal{L}_O$. If rigid concept and role names are allowed, we reduce the word problem for exponentially space-bounded alternating Turing machines to obtain 2Exp-hardness. If only rigid concept names are allowed, a reduction of an exponentially bounded version of the domino problem yields NExp-hardness. For the upper bounds, we proceed similar to what was done for $\mathcal{ALC}$-LTL in [2,3] and reduce the consistency problem to two separate decision problems. First, we abstract our $\mathcal{L}_M[\![\mathcal{L}_O]\!]$-BKB $\mathcal{B}$ by replacing the m-concepts that consist of o-axioms by fresh concept names and test this abstraction $\mathcal{B}'$ for consistency. With this abstraction, we loose, however, the information on the o-axioms. In each model of $\mathcal{B}'$, the fresh concept names have some extensions, and are treated completely independently. The induced o-axioms, however, may not be independent. It could be that some o-axioms are inconsistent together. We check this in a separate step.

To conclude, we have proposed novel combinations of two DLs for representing contextual knowledge and analyzed their complexity. Interestingly, even in the presence of rigid roles the consistency problem is still decidable. For more details, see [6]. As future work apart from others, we envision that our decision procedures can be adapted to deal with temporalized context DLs such as $LTL[\![\mathcal{ALC}[\![\mathcal{ALC}]\!]]\!]$.

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