Random tournaments: who plays with whom and how many times?

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Abstract. The weighted feedback arc set problem on tournaments (WFAS-T) is defined by a weighted tournament graph whose nodes represents the teams/items, and the goal is to find a ranking over the teams that minimizes the sum of the weights of the feedback arcs. We consider the probabilistic version of WFAS-T, in which the weights of the directed edges between every pair of teams sum up to one. A WFAS-T with this probabilistic constraint naturally determines a distribution over the tournament graphs. In this study, we investigate an online learning problem where the learner can observe tournament graphs drawn from this distribution. The goal of the learner is to approximate the solution ranking of the underlying probabilistic WFAS-T problem. We also investigate the partial information case, known from the multi-armed bandit problem, where the learner is allowed to select single edges and observe the corresponding value. Since the probabilistic WFAS-T problem is in general NP-hard [1], our learning algorithm relies on some recent approximation results for WFAS-T [2, 3]. We also consider some interesting special cases where the learner is able to estimate the exact solution, for example where the weights of the tournament graph satisfy the Bradley-Terry assumption [4].

Keywords: weighted feedback arc set problem, tournaments, online learning, bandit feedback

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