Reasoning in a Rational Extension of $\mathcal{SROEL}(\sqcap, \times)$ (Extended Abstract)

Laura Giordano and Daniele Theseider Dupré

DISIT - Università del Piemonte Orientale, Alessandria, Italy
laura.giordano@uniupo.it, dtd@di.unipmn.it

The need for extending Description Logics (DLs) with nonmonotonic features has led, in the last decade, to the development of many extensions of DLs, obtained by combining them with the most well-known formalisms for nonmonomtnic reasoning [24, 2, 9, 11, 13, 16, 15, 18, 6, 4, 8, 23, 19, 7, 3] to deal with defeasible reasoning and inheritance, to allow for prototypical properties of concepts and to combine DLs with nonmonotonic rule-based languages [11, 10, 19, 17].

In this work we study a preferential extension of the logic $\mathcal{SROEL}(\sqcap, \times)$, introduced by Krötzsch [21], which is a low-complexity description logic of the $\mathcal{EL}$ family [1] that includes local reflexivity, conjunction of roles and concept products and is at the basis of OWL 2 EL. Our extension is based on Kraus, Lehmann and Magidor (KLM) preferential semantics [20], and, specifically, on ranked models [22]. We call the logic $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ and define notions of rational and minimal entailment for it.

The semantics of ranked interpretations for DLs was first studied in [6], where a rational extension of $\mathcal{ALC}$ is developed allowing for defeasible concept inclusions of the form $C \tilde{\sqsubseteq} D$. In this work, following [14, 16], we extend the language of $\mathcal{SROEL}(\sqcap, \times)$ with typicality concepts $\mathbf{T}(C)$, whose instances are intended to be the typical C elements. Defeasible inclusions $\mathbf{T}(C) \sqsubseteq D$ mean that "the typical C elements are Ds". Here, however, as in [5, 12], typicality concepts can freely occur in concept inclusions. The language is then more general than in [16], where minimal ranked models have been shown to provide a semantic characterization to rational closure for $\mathcal{ALC}$, which extends the rational closure by Lehmann and Magidor [22]. Alternative constructions of rational closure for $\mathcal{ALC}$ have been proposed in [8, 7]. All such constructions regard languages only containing strict or defeasible inclusions (i.e., in the language with typicality concepts, $\mathbf{T}(C)$ may only occur in inclusions $\mathbf{T}(C) \sqsubseteq D$ and in assertions).

In this work we define a Datalog translation for $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ which builds on the materialization calculus in [21], and, for typicality reasoning, is based on properties of ranked models, showing that instance checking under rational entailment is polynomial. While this result has the consequence that the Rational Closure (based on the definition in [16]) can be computed in polynomial time, we show that, for general $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ KBs, deciding instance checking under minimal entailment is coNP-hard (while minimal entailment coincides with Rational Closure for $\mathcal{ALC}$).

The notion of concept in $\mathcal{SROEL}(\sqcap, \times)$, as defined by Krötzsch [21], is extended with typicality concepts. We let N_C be a set of concept names, N_R a set of role names and N_I a set of individual names. A concept in $\mathcal{SROEL}(\sqcap, \times)$ is defined as follows:

$$C := A \mid \top \mid \bot \mid C \sqcap C \mid \exists r.C \mid \exists S.Self \mid \{a\}$$

where $A \in N_C$ and $r \in N_R$. We introduce a notion of *extended concept* C_E as follows:

$C_E := C \mid \mathbf{T}(C) \mid C_E \sqcap C_E \mid \exists S.C_E$

where C is a $\mathcal{SROEL}(\sqcap, \times)$ concept; i.e., extended concepts include typicality concepts $\mathbf{T}(C)$, which can occur in conjunctions and existential restrictions, but $\mathbf{T}$ cannot be nested.

A KB is a triple $(TBox, RBox, ABox)$. $TBox$ contains a finite set of *general concept inclusions* (GCI) $C \sqsubseteq D$, where C and D are extended concepts; $RBox$ (as in [21]) contains a finite set of *role inclusions* of the form $S \sqsubseteq T$, $R \circ S \sqsubseteq T$, $S_1 \sqcap S_2 \sqsubseteq T$, $R \sqsubseteq C \times D$, where C and D are concepts, $R, S, T \in N_R$. $ABox$ contains *individual assertions* of the form $C(a)$ and $R(a, b)$, where $a, b \in N_I$, $R \in N_R$ and C is an extended concept. Restrictions are imposed on the use of roles as in [21].

Following [6, 16], a semantics for the extended language is defined, adding to interpretations in $\mathcal{SROEL}(\sqcap, \times)$ [21] a *preference relation* $<$ on the domain, which is intended to compare the "typicality" of domain elements.

Definition 1. *A $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ interpretation $\mathcal{M}$ is any structure $\langle \Delta, <, \cdot^I \rangle$ where: (i) Δ and $\cdot^I$ are a domain and an interpretation function as in a $\mathcal{SROEL}(\sqcap, \times)$ interpretations; (ii) $<$ is an irreflexive, transitive, well-founded and modular relation over Δ; (iii) the interpretation of concept $\mathbf{T}(C)$ is defined as follows: $(\mathbf{T}(C))^I = Min_<(C^I)$, where $Min_<(S) = \{u : u \in S$ and $\nexists z \in S$ s.t. $z < u\}$. An irreflexive and transitive relation $<$ is* well-founded *if, for all $S \subseteq \Delta$, for all $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ such that $y < x$. It is* modular *if, for all $x, y, z \in \Delta$, $x < y$ implies $x < z$ or $z < y$.*

As in [22], modularity in preferential models can be equivalently defined by postulating the existence of a ranking function $k_{\mathcal{M}} : \Delta \longmapsto \Omega$, where Ω is a totally ordered set and $x < y$ if and only if $k_{\mathcal{M}}(x) < k_{\mathcal{M}}(y)$. Hence, modular preferential models are called $ranked\ models$. In the following we assume that such a ranking function is associated with a model. Satisfiability and models of a KB is defined as usual for DLs.

Definition 2 (Rational entailment). *Let a query F be either a concept inclusion $C \sqsubseteq D$, where C and D are extended concepts, or an individual assertion. F* is entailed by *K, written $K \models_{sroelrt} F$, if for all models $\mathcal{M} = \langle \Delta, <, \cdot^I \rangle$ of K, $\mathcal{M}$ satisfies F.*

Example 1. $TBox$:
$(a)\ \mathbf{T}(Italian) \sqsubseteq \exists hasHair.\{Black\}$ $\quad (b)\ \mathbf{T}(Student) \sqsubseteq MathHater$
$(c)\ \exists hasHair.\{Black\} \sqcap \exists hasHair.\{Blond\} \sqsubseteq \bot$ $\quad (d)\ \mathbf{T}(Student) \sqsubseteq Young$
$(e)\ \exists friendOf.\{mary\} \sqsubseteq \mathbf{T}(Student)$ $\quad (f)\ MathLover \sqcap MathHater \sqsubseteq \bot$

$ABox$: $\{Student(mary), friendOf(mario, mary), (Student \sqcap Italian)(mario),$
$\mathbf{T}(Student \sqcap Italian)(luigi), \mathbf{T}(Student \sqcap Young)(paul)\}$

Standard DL inferences hold for $\mathbf{T}(C)$ concepts and $\mathbf{T}(C) \sqsubseteq D$ inclusions. For instance, we can conclude that Mario is a typical student (by (e)) and young (by (d)). However, by the properties of defeasible inclusions, Luigi, who is a typical Italian student, and Paul, who is a typical young student, both inherit the property of typical students of being math haters. In this logic we cannot conclude that all typical young Italians have black hair (and that Luigi has black hair), according to Rational Monotonicity in [22], as we do not know whether there is some typical Italian who is young.

To supports such a stronger nonmonotonic inference, a minimal model semantics (and minimal entailment) is needed to select the interpretations where individuals are as typical as possible (see below).

A normal form can be defined for $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ knowledge bases. A KB is in *normal form* if it admits axioms of a $\mathcal{SROEL}(\sqcap, \times)$ KB in normal form, and axioms of the form: $A \sqsubseteq \mathbf{T}(B)$ and $\mathbf{T}(B) \sqsubseteq C$ with $A, B, C \in N_C$. Extending the results in [1, 21], it is easy to see that, given a $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ KB, a semantically equivalent KB in normal form (over an extended signature) can be computed in linear time.

For normalized KBs, the Datalog materialization calculus for $\mathcal{SROEL}(\sqcap, \times)$ proposed by Krötzsch [21] can be extended to define a polynomial Datalog encoding for instance checking under rational entailment.

Theorem 1. *Instance checking under rational entailment in $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ can be decided in polynomial time for normalized KBs.*

Exploiting the approach presented in [21], a version of the Datalog specification where predicates have an additional parameter can also be used to check subsumption for $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ in polynomial time. This also provides a polynomial upper bound for a rational closure construction analogous to the one in [16].

We now consider the notion of minimal canonical model in [16]. Given a KB K and a query F, let $\mathcal{S}$ be the set of all the concepts (and subconcepts) occurring in K or F together with their complements ($\mathcal{S}$ is finite).

Definition 3 (Canonical models). *A model $\mathcal{M} = \langle \Delta, <, I \rangle$ of K is* canonical *if, for each set of $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ concepts $\{C_1, C_2, \ldots, C_n\} \subseteq \mathcal{S}$ consistent with K (i.e., s.t. $K \not\models_{sroelrt} C_1 \sqcap C_2 \sqcap \ldots \sqcap C_n \sqsubseteq \bot$), there exists a domain element $x \in \Delta$ such that $x \in (C_1 \sqcap C_2 \sqcap \ldots \sqcap C_n)^I$.*

Among canonical models, we select the minimal ones according to the following *preference relation $\prec$ over the set of ranked interpretations*. An interpretation $\mathcal{M} = \langle \Delta, <, I \rangle$ *is preferred to* $\mathcal{M}' = \langle \Delta', <', I' \rangle$ ($\mathcal{M} \prec \mathcal{M}'$) if: $\Delta = \Delta'$; $C^I = C^{I'}$ for all concepts C; for all $x \in \Delta$, $k_{\mathcal{M}}(x) \leq k_{\mathcal{M}'}(x)$, and there exists $y \in \Delta$ such that $k_{\mathcal{M}}(y) < k_{\mathcal{M}'}(y)$.

Definition 4 (Minimal entailment). *$\mathcal{M}$ is a* minimal canonical model *of K if it is a canonical model of K and it is minimal among all the canonical models of K wrt. the preference relation $\prec$. Given a query F, F* is minimally entailed by *K, written $K \models_{min} F$ if, for all minimal canonical models $\mathcal{M}$ of K, $\mathcal{M}$ satisfies F.*

Under minimal entailment, in Example 1 we can conclude $\exists hasHair.\{Black\}(luigi)$ and $\mathbf{T}(Young \sqcap Italian) \sqsubseteq \exists hasHair.\{Black\}$.

Theorem 2. *Instance checking in $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$ under minimal entailment is* CONP-*hard.*

The proof is based on a reduction from tautology checking of propositional 3DNF formulae to instance checking in $\mathcal{SROEL}(\sqcap, \times)^{\mathbf{R}}\mathbf{T}$.

For KBs which only allow typicality concepts to occur on the left hand side of typicality inclusions, and are in the language of $\mathcal{ALC}$, the result in [16] guarantees that all minimal canonical models of the KB assign the same ranks to concepts, namely, the ranks determined by the rational closure construction. This is not true, however, for general KBs in $\mathcal{SROEL}(\sqcap, \times)$.

References

1. Baader, F., Brandt, S., Lutz, C.: Pushing the $\mathcal{EL}$ envelope. In: Proc IJCAI 2005. pp. 364–369 (2005)
2. Baader, F., Hollunder, B.: Priorities on defaults with prerequisites, and their application in treating specificity in terminological default logic. J. of Automated Reasoning 15(1), 41–68 (1995)
3. Bonatti, P.A., Faella, M., Petrova, I., Sauro, L.: A new semantics for overriding in description logics. Artif. Intell. 222, 1–48 (2015)
4. Bonatti, P.A., Lutz, C., Wolter, F.: The Complexity of Circumscription in DLs. J. of Artificial Intelligence Research 35, 717–773 (2009)
5. Booth, R., Casini, G., Meyer, T., Varzinczak, I.J.: On the entailment problem for a logic of typicality. In: Proc. IJCAI 2015. pp. 2805–2811 (2015)
6. Britz, K., Heidema, J., Meyer, T.: Semantic preferential subsumption. In: Brewka, G., Lang, J. (eds.) Proc. KR 2008. pp. 476–484 (2008)
7. Casini, G., Meyer, T., Varzinczak, I.J., , Moodley, K.: Nonmonotonic Reasoning in Description Logics: Rational Closure for the ABox. In: Proc. DL 2013. pp. 600–615 (2013)
8. Casini, G., Straccia, U.: Rational Closure for Defeasible Description Logics. In: Proc. JELIA 2010. pp. 77–90. LNAI 6341, Springer (2010)
9. Donini, F.M., Nardi, D., Rosati, R.: Description logics of minimal knowledge and negation as failure. ACM Transactions on Computational Logic (ToCL) 3(2), 177–225 (2002)
10. Eiter, T., Ianni, G., Lukasiewicz, T., Schindlauer, R.: Well-founded semantics for description logic programs in the semantic web. ACM Trans. Comput. Log. 12(2), 11 (2011)
11. Eiter, T., Ianni, G., Lukasiewicz, T., Schindlauer, R., Tompits, H.: Combining answer set programming with description logics for the semantic web. Artif. Intell. 172(12-13), 1495–1539 (2008)
12. Giordano, L., Gliozzi, V.: Encoding a preferential extension of the description logic *SROIQ* into *SROIQ*. In: Proc. ISMIS 2015. LNCS, vol. 9384, pp. 248–258. Springer (2015)
13. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: Preferential Description Logics. In: Proceedings of LPAR 2007. LNAI, vol. 4790, pp. 257–272. Springer-Verlag (2007)
14. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: ALC+T: a preferential extension of Description Logics. Fundamenta Informaticae 96, 1–32 (2009)
15. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: A NonMonotonic Description Logic for Reasoning About Typicality. Artificial Intelligence 195, 165–202 (2013)
16. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: Semantic characterization of rational closure: From propositional logic to description logics. Artif. Intell. 226, 1–33 (2015)
17. Gottlob, G., Hernich, A., Kupke, C., Lukasiewicz, T.: Stable model semantics for guarded existential rules and description logics. In: Proc. KR 2014 (2014)
18. Ke, P., Sattler, U.: Next Steps for Description Logics of Minimal Knowledge and Negation as Failure. In: Proc. DL 2008. CEUR Workshop Proceedings, vol. 353 (2008)
19. Knorr, M., Hitzler, P., Maier, F.: Reconciling OWL and non-monotonic rules for the semantic web. In: ECAI 2012. p. 474479 (2012)
20. Kraus, S., Lehmann, D., Magidor, M.: Nonmonotonic reasoning, preferential models and cumulative logics. Artif. Intell. 44(1-2), 167–207 (1990)
21. Krötzsch, M.: Efficient inferencing for OWL EL. In: Proc. JELIA 2010. pp. 234–246 (2010)
22. Lehmann, D., Magidor, M.: What does a conditional knowledge base entail? Artificial Intelligence 55(1), 1–60 (1992)
23. Motik, B., Rosati, R.: Reconciling Description Logics and rules. J. ACM 57(5) (2010)
24. Straccia, U.: Default inheritance reasoning in hybrid KL-ONE-style logics. In: Proc. IJCAI 1993. pp. 676–681 (1993)