Valid Inequalities for Time-indexed Formulations of the Runway Scheduling Problem⋆

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Abstract. The problem of sequencing and scheduling airplanes landing and taking off on a runway is under consideration. We propose a new family of valid inequalities which are obtained from the study of the single machine scheduling problem polytope.

1 Introduction and problem statement

In this paper we consider the integrated management of departures and arrivals on a single runway, and we will refer to this problem as ADMAN (Arrival and Departure MANagement). In ADMAN, one wants to (jointly) schedule the take-offs and the landings of a set of airplanes F. For each arrival and departure flight $i \in F$, an arrival/departure window T_i is given, and the flight must land/takeoff in this time window, however the departure can be canceled (at high cost). Two successive flights i and j on the runway must be separated by a minimum time interval s_{ij} which depends on the involved airplanes. The *official timetable* provides wanted arrival and departure times. However, when one or more airplanes are delayed, a new schedule must be found, so that (some measure of) the deviation from the official timetable is minimized.

For details on the different approaches for AMAN and DMAN we refer the reader to a recent survey [1]. The great majority of the approaches presented in the literature are heuristic or meta-heuristic - see again [1], but also the literature discussion in the recent paper by [2].

As for MIP approaches, we observe first that ADMAN can be interpreted as a classical single machine scheduling problem with sequence dependent setup times and earliness/tardiness objective function (for a recent paper on this problem see [3]). The runway corresponds to the machine, flights correspond to jobs and time separations between flights on the runway to setup times. In this paper the Time Indexed (TI) formulations is considered. In TI formulations, the time horizon is discretized in small

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time periods. The schedule of a given flight (job) is modeled by a set of binary variables, each associated with a feasible departure or arrival time period.

Let L and D be the sets of arriving and departure flights correspondingly, $L \cup D = F$. Let us associate a binary variable x_{it} with every $i \in F$ and every $t \in T_i$, which is 1 if and only if flight i arrives/departs at time t. Also, with every departure $i \in D$ we associate a binary variable y_i which is 1 if i is dropped and 0 otherwise. Since every flight is assigned (at most) one arrival/departure time in a feasible schedule, for every $i \in F$ and every $k, l \in T_i$, $k \neq l$, we have $x_{ik} + x_{il} \leq 1$.

Consider now two distinct flights $i, j \in F$, and assume that the assignment k and l violates the separation requirement between i and j, that is $-s_{ji} < l - k < s_{ij}$. Then, we have either $x_{ik} = 0$ or $x_{jl} = 0$ in any feasible solution. In turn, this can be expressed by the constraint $x_{ik} + x_{jl} \leq 1$ and we say that the pair (of indices) $\{ik, jl\}$ is an *incompatible pair*. For an instance of the problem, we let I be the set of all incompatible pairs (of indices).

With an instance of the problem, we associate an undirected graph $G(V, E)$ called *conflict graph*. The nodes of G are in one-to-one correspondence to the x variables of the formulation and we have an edge between two nodes whenever the associated variables cannot both assume value 1. More formally, we let

$$V = \{it : i \in F, t \in T_i\}$$

and

$$E = I \cup \{\{ik, il\} : ik, il \in V, k \neq l\}.$$

From the above discussion, it follows that x represents a feasible schedule, if and only if x satisfies:

$$x_{ik} + x_{jl} \leq 1 \qquad \{ik, jl\} \in E \tag{1}$$

A *clique* of an undirected graph is a subset of the nodes such that every two nodes in the subset are adjacent. Incidently, observe that any pair of adjacent nodes is also a clique (of cardinality 2). Let K be a clique of the conflict graph $G(V, E)$, and let x satisfy (1) then it is easy to see that x also satisfies the *clique inequality*:

$$\sum_{it \in K} x_{it} \leq 1 \tag{2}$$

If $K \subseteq V$ is a clique and $u, v \in K$, then the edge $\{u, v\}$ is said to be *covered* by K. An *I-cover* is a set of cliques $K_1, K_2, \ldots,$ such that every edge in I is covered by at least one clique in the set. Let $\mathcal{K} = \{K_1, K_2, \ldots\}$ be a I-cover. It is not difficult to see that x satisfies (1) if and only if x satisfies the system of inequalities:

$$(i) \ \sum_{l \in T_i} x_{i,l} \leq 1 \qquad i \in F \tag{3}$$

$$(ii) \ \sum_{it \in K} x_{it} \leq 1 \qquad K \in \mathcal{K}$$

We are finally able to write a 0,1 linear programming formulation of the problem:

$$\min \sum_{i \in D} c_i y_i + \sum_{i \in F} \sum_{t \in T_i} q_{it} x_{it}$$

$$(i) \qquad \sum_{t \in T_i} x_{it} = 1, \;\; i \in L$$

$$(ii) \qquad \sum_{t \in T_i} x_{it} + y_i = 1, \;\; i \in D \qquad\qquad (4)$$

$$(iii) \qquad \sum_{it \in K} x_{it} \leq 1, \;\; K \in \mathcal{K}$$

$$(iv) \qquad x_{it} \in \{0,1\}, \; i \in F, t \in T_i \quad y_i \in \{0,1\}, \; i \in D$$

where $\mathcal{K} = \{K_1, K_2, \dots\}$ is a I-cover and the (4.iii) define an I-cover system of inequalities.

Constraints (4.i) ensure that every arrival is assigned an arrival time from its time window, whereas constraints (4.ii) ensure that every departure is either dropped or assigned a departure time from its time window. Finally, constraints (4.iii) are the I-cover inequalities which ensure that the schedule respects separation constraints. The objective function represents the overall cost of the solution. Observe that constraints (3.i) are implied by (4.i) and (4.ii), whereas 3.ii) are precisely (4.iii).

2   Valid inequalities

In order to keep the number of constraints at bay, it is important to carefully select the cliques in the I-cover. In fact, typically most of the constraints (in real-life instances) of (4) will belong to the I-cover system of constraints (4.iii).

One of the original and well studied example of I-cover system of inequalities (see, e.g., [3] and [4]) is given by the family of inequalities:

$$x_{jt} + \sum_{k \in T_i \cap [t - s_{ij} + 1, t]} x_{ik} \leq 1 \qquad i, j \in F, i \neq j, t \in T_j \qquad (5)$$

Note that each clique inequality of system (5) can be strengthened by lifting in a trivial fashion, giving the following system:

$$\sum_{l \in T_j \cap [t - s_{ji} + 1, t]} x_{jl} + \sum_{k \in T_i \cap [t - s_{ij} + 1, t]} x_{ik} \leq 1 \qquad i, j \in F, i < j, t \in T_j \cup T_i \qquad (6)$$

In the sequel, for all $Q \subseteq F$, $|Q| \geq 2$, and all $i \in Q$, we let $s_i(Q) = \min\{s_{ij} : j \in Q \setminus i\}$. A family of clique inequalities valid for (4) has been recently introduced and by Nogueira et al. in [3]:

$$\sum_{i \in F} \sum_{l \in [t - s_i(F) + 1, t] \cap T_i} x_{il} \leq 1 \qquad t \in T. \qquad (7)$$

We refer to the formulation (4) where the I-cover system (4.iii) is given by (5) and (7) as the *Nogueira formulation*.

With the aim of defining a stronger but also more compact time-indexed formulation, we introduce a new family of clique inequalities - that we call (S, t)-*clique inequalities* - generalizing (7):

Proposition 1. *Let $t \in T$, $S \subseteq F$ and, with $|S| \geq 2$ and, for each $i \in S$, let $s_i(S) = \min\{s_{ij} : j \in S \setminus i\}$. The (S, t)-clique inequality:*

$$\sum_{i \in S} \sum_{l \in [t - s_i(S) + 1, t] \cap T_i} x_{il} \leq 1 \tag{8}$$

is valid for (4).

Proof. It follows directly from (7) and from the trivial observation that any constraint which is valid for a subset of flights and time periods, is also valid for the larger sets.

Our preliminary computational experiments show, that with careful selection and separation algorithm for (S, t)-clique inequalities (8) along with some special fixing and lifting procedures, allow us to develop an efficient branch-and-cut method to solve the real world instances to optimality.

References

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