Strictly Positive Fragments of Modal and Description Logic

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Abstract. In this talk we will advocate the use of weak systems of modal logic called strictly positive. These can be seen as fragments of polymodal logic consisting of implications of the form $A \to B$, where A and B are formulas built-up from T (truth) and the variables using just conjunction and the diamond modalities. The interest towards such fragments independently emerged around 2010 in two different areas: in description logic and in the area of proof-theoretic applications of modal logic.

From the point of view of description logic, strictly positive fragments correspond to the OWL 2 EL profile of the OWL web ontology language, for which various properties of ontologies can be decided in polynomial time. In the area of proof-theoretic applications, these fragments emerged under the name reflection calculi, as they proved to be a convenient tool to study the independent reflection principles in arithmetic and to calculate proof-theoretic ordinals of formal systems.

Thus, in two different areas strictly positive languages and logics proved to combine both efficiency and simplicity, and sufficient expressive power. In this talk we discuss general problems around weak systems of this kind and describe some of their applications.

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