Simulation of Xenon Transition Processes Based on Data of Reactor Experiments and Metric Analysis

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The report presents a new scheme for modeling xenon transient processes, based on the use of accumulated information on the parameters of the core state in the reactor operation. The proposed scheme uses an interpolation algorithm for functions of many variables, constructed on the basis of metric analysis. Approbation of the developed algorithm for searching for the most probable set of model parameters was carried out using the example of solving a series of model problems. The search for a posteriori probability density was carried out by the Monte Carlo method according to the Markov chain scheme. To construct a posteriori probability density, the construction of a Markov chain involving several hundred thousand links is required. For each link of the Markov chain, it is necessary to calculate the values of the functionals. When constructing the Markov chain, the values of the sets of functionals corresponding to different sets of parameters were determined by using the interpolation algorithm between the reference points, based on the metric analysis scheme. To improve the accuracy of the interpolation procedure, on one hand, and to maintain an acceptable volume of calculations, on the other hand, a procedure was developed to supplement the set of reference points with new calculation points. The main idea of such a procedure is that the asymptotic density of points in the Markov chain corresponds to the density of the unknown a priory probability density. The scheme allows us to refine the values of the initial parameters of the calculation model on the basis of the information accumulated during the operation of the reactor and, thereby, to calculate the transient processes with greater accuracy.

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1. Introduction

Many researchers developed procedures for using experimental data on xenon transients to refine the values of the physical parameters of the calculation model. Such procedures relate to the solution of inverse problems of mathematical physics, which have been intensively developed in recent decades [1].

The simplest options for restoring the values of the parameters of the core model are usually based on the method of least squares. However, their practical use for solving the problems of restoring the parameters of the core model on the basis of experimental data on xenon transient processes faces a number of problems. This is due, in particular, to the nonlinearity of xenon processes. In addition, using the method of least squares does not take into account the available information on the accuracy of the calculation of individual parameters.

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2. Simulation scheme

To solve the problem of restoring the parameters of the model on the basis of experimental data on xenon transient processes, we propose to use the Bayes method [2, 3] in this paper, which makes it possible to construct a posteriori (taking into account the available experimental information) the probability density of the values of the required parameters, taking into account the nonlinearity and possible degeneracy of the tasks. For a nonsingular problem, we can assume that the combination of the unknown parameters, which corresponds to the maximum probability density, is the desired set of model parameters. For a singular problem there is a whole set of possible sets of model parameters that are realized for equal or close probability densities. Using the Bayes method, the available (a priori) information on the accuracy of calculation of individual parameters is taken into account. Since the purpose of the work is to restore the integral parameters of the active one, for practical implementation of the method it is necessary to distinguish several functionals from the distribution of energy release, the values of which essentially depend on the desired parameters.

To date, effective procedures for constructing the posterior probability density have been developed. One of the promising is the DRAM algorithm [4, 5], which was used in the developed procedure. However, practical implementation of the Bayes method requires carrying out multiple (several hundred thousand) calculations of xenon transients. Therefore, a direct solution to the problem under consideration without preliminary modifications is impossible because of the limited computing resources.

To reduce the number of direct calculations of transient processes, an interpolation procedure based on metric analysis schemes was used [3], which allows to restore calculated values of the monitored functionals from the energy release field at a small computational cost, taking into account the nonlinearity of the process under consideration with a limited amount of preliminary calculations.

In this paper, we present the results of solving test problems that demonstrate the operability of the developed procedures.

To simulate the xenon transients, the NOSTRA software was used [6].

To reduce the number of direct calculations of transient processes, an interpolation procedure based on metric analysis schemes was used [2, 3], which allows one to restore calculated values of the controllable functionals from the energy release field with small calculation costs, taking into account the nonlinearity of the process under consideration with a limited amount of preliminary calculations.

Approbation of the developed algorithm to search for the most probable set of model parameters was carried out using the example of solving a series of model problems. We denote the set of recoverable deviations of the model parameters from their nominal values by the vector $\vec{\theta}$, and the set of functionals by the vector $\vec{f}$.

At the first stage of solving the model problem, a xenon transition process was simulated, initiated by varying the power and moving the working group of the regulating bodies. The values of the reactivity coefficient for the fuel temperature, the coefficient of reactivity for the density of the coolant and the value of the xenon-135 absorption cross section in this calculation deviated from their nominal values corresponding to the constant file data and the magnitude of the deviations were specified by the vector $\vec{f}^{\exp}$. The functionals from the energy release field obtained during this calculation were considered as their "experimental" values $\vec{f}^{\exp}$. Calculations at the first stage were carried out with the help of the NOSTRA software [6].

At the second stage of the solution of the model problem, the corresponding deviations in the parameter values of the model were reconstructed using the Bayes method on the basis of using the specified "experimental" values of the functionals from the energy release $\vec{f}^{\text{exp}}$, the error in determining the "experimental" values of the functionals and the a priori error of the model parameters.

The calculated values of the functionals from the energy release for an arbitrary set of deviations of the reactivity coefficient from the fuel temperature, the reactivity coefficient for the coolant density and the xenon-135 absorption cross section from their nominal values were determined by direct calculations using the NOSTRA software (~ 100 calculations) and by interpolation methods based on schemes of metric analysis.

At the third stage of solving the problem, the restored and actual values of the model parameters were compared.

3. Numerical Results

In order to evaluate the effectiveness of the developed algorithm, series of test problems were solved, in each of which, when the model parameters were replaced by $\vec{\theta}^{\text{exp}}$, a xenon transition process was modeled and a set of functionals $\vec{f}^{\text{exp}}$ was calculated. Next, a set of deviations of the model parameters $\vec{\theta}^{\text{res}}$ was determined, which corresponded to the maximum probability density for the functionals obtain $\vec{f}^{\text{exp}}d$.

The values of the functionals from the energy release were based on the axial offset of the energy release. The axial offset of the energy release is defined as the difference in energy release in the upper and lower parts of the core, referred to the energy release of the entire core.

Four functionals were considered as functionals from the energy release: the damping decrement of the deviation of the axial offset of the energy release from its stationary value, the time between the achievements of the axial offset of the energy release of the maximum values, the difference in the values of the axial offset of the energy release at neighboring points of the maximum and minimum, and the value of axial offset at one of the instants. As parameters of the model, deviations from the nominal values of the reactivity coefficients for the fuel temperature, coolant temperature and relative displacement of the absorption cross section were considered.

Through direct modeling of the xenon transient in the NOSTRA software and with the help of metric analysis of the deviations of the model parameters by values $\vec{\theta}$ from their nominal values, the calculated values of the functionals were determined. The indicated procedure in the operator form can be written in the form $\vec{f}^{\text{cal}} = \tilde{A}(\vec{\theta})$, where $\tilde{A}(\vec{\theta})$ is a nonlinear operator. The problem of determining parameters $\vec{\theta}$ is a classical inverse problem and Bayes' method is applicable to its solution.

The change in the axial offset of energy release in the case of xenon oscillations is shown in Fig. 1 for the initial and restored values of the parameters.

Table 1 shows the results of the calculation of functionals and their comparison with empirical values.

Fig. 2–4 show the distributions of conditional probability densities for two of the parameters, provided that the third parameter belongs to the interval indicated in the figure.

4. Conclusion

The refinement of the parameters of the computational model using simple mathematical schemes, for example, the method of least squares, faces problems of nonlinearity and degeneracy. In this paper, the Bayes method was used to solve the problem, using metric analysis and the DRAM algorithm. The obtained numerical results

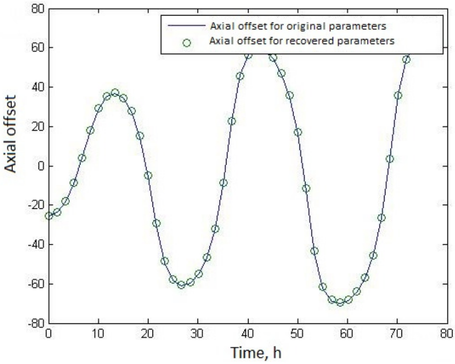

Figure 1. Dependence of axial offset of energy release on time

Table 1

The values of the energy release functionals for the “experimental” values $f^{\exp}$ and for the restored parameter values $\vec{f}^{\mathrm{res}}$

Parameter	$\vec{f}^{\mathrm{res}}$	$\vec{f}^{\exp}$	Relative error
Swing range	95.70	97.89	0.022
Decrement damping	-22.42	-22.49	0.003
The period of oscillations	28.95	28.88	0.002
Axial offset at $t = 5$ hours	-23.59	-23.84	0.010

show the possibility of restoring the parameters of the computational model ensuring the matching of the calculated values of the functionals with their empirical values.

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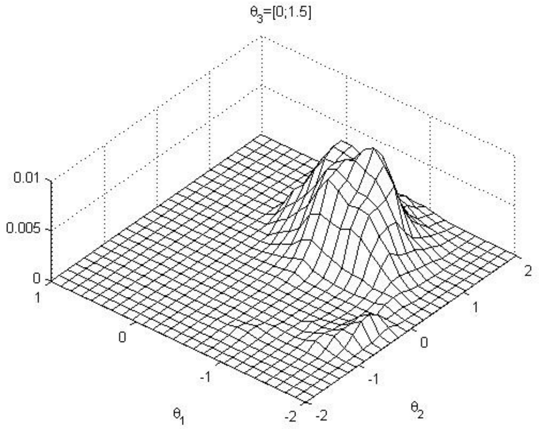

Figure 2. Conditional probability density of parameters (θ_1, θ_2)

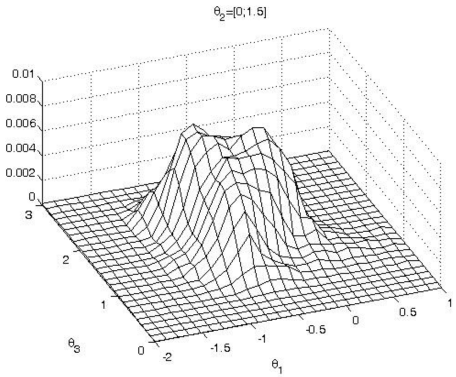

Figure 3. Conditional probability density of parameters (θ_1, θ_3)

6. The NOSTRA software (version 5.0). Attestation passport of the software.

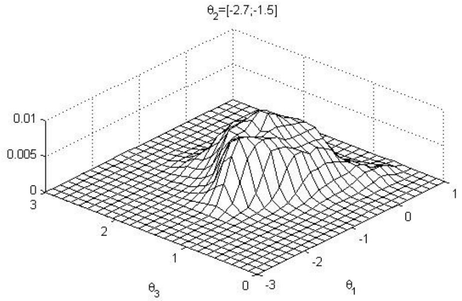

Figure 4. Conditional probability density of parameters (θ_1, θ_3)