Mathematical modeling of dynamic features of inhomogeneous dilatant inclusion deformation

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Abstract. Methods of short-term earthquake prediction and some results of mathematical modeling of fracturing processes in lithosphere are developed. General scientific methodology of short-term prediction is formulated including two sufficient conditions for increasing probability of the forecast of seismic event. Implementations of artificial neural networks for interpretation of radon variations are proposed.

Keywords: inhomogeneous dilatant inclusion, radon variation, earthquake prediction, artificial neural networks

1 Introduction

The goal of our research - construction of a methodology (including a model of the medium), which the key idea is to create an algorithm to determine the probability of seismic event occurrence. To provide short-term earthquake prediction, it is necessary to have an adequate model of the medium. In our case, we propose a multi-phase contrast model, which is a porous permeable water and gas saturated media with inclusions in a solid matrix of uranium-bearing minerals, such as Sierra-Nevada granite massif (Fig. 1). We assume that in the area of the proposed seismic event (during the preparation period) a non-uniform dilatant inclusion develops, which is a set interconnected by permeable channels. The dynamics of each dilatant inclusion depends on the ratio of the rate of tectonic stress increase and coefficients of the hydraulic diffusion of dilatant inclusion area and host rocks. Depending on this rationing, dilatant inclusion boundary conditions may be draining or undraining. For undraining conditions in the dilatant inclusion area, the section of the pore fluid reduced pressure occurs, which in the case of radon-bearing rock increases the radon content in the pores due to injection effects. As a mathematical model of the inhomogeneous dilatant inclusion dynamics, we propose the artificial neural networks.

In the investigation of variables matrix (radon variations) in real-time for a week before the earthquake, the perceptron with the independent variables is formed. We assume that by this time the tectonic energy relaxation process is completed and the interaction of dilatant mesoinclusions stopped. At each point of observation, only processes of uranium-238 radioactive decay occur and radon-222 appears. Therefore, at this moment the amount of emanation depends only on uranium-238 content. Thus, the beginning of the radon emanation independent process fixed by artificial neural networks is most likely a short-term earthquake precursor. Technique of the situation detection consists in the fact that the qualitative characteristics of the seismic hazard is defined once a week. The first rank of danger is the start and end of the stress relaxation process; the second rank of danger is the fixation of independent spatial process of the radon emanation (maximum probability of earthquakes).

2 Mathematical model

In paper, we propose a mathematical model of dynamics of tectonic processes in the Earth's crust at various hierarchical levels and the relationship of these levels to each other. Let us consider three mutually complementary hierarchical levels that adequately describe the dynamics of the earth's crust. The first (lower hierarchical level) is a multiphase contrast model of the earth's crust, which is a porous permeable moisture and gas-saturated medium with inclusions in the solid matrix of uranium-containing minerals. The second hierarchical level is where the inhomogeneous dilatant inclusion is replaced by clusters of mesoinclusions interconnected by permeable channels. The third (the most high hierarchical level) is the regional inhomogeneous dilatant inclusion in the field of variable tectonic stresses, the dynamics of which are described by the Rice-Rudnicki model [4]. Mesoinclusion is an independent element, the process of its dynamics is a single tectonic event. Due to the relationship, one tectonic event can induce another one or be a trigger for numerous further events. The problem of physics-mathematical descriptions of the relationship of events is difficult and ambiguous, at least in seismological practice. Description of the dynamics of the homogeneous dilatant inclusion according to [4] is rather complicated, and for the inhomogeneous inclusion, the difficulties in describing its dynamics within the framework of the ordinary physical model are hundred times more complicated, since we are forced to consider small interacting inclusions.

The paradigm of space-temporal point processes is particularly suited to neural network analysis, which we previously applied to radon variations and their relationship to tectonic processes. This approach proved to be useful in describing the space-temporal dynamics of tectonic processes (regional shift stresses) in Southern California [2]. In our work, we are developing a productive approach to analysis of the coupling of dilatant inclusions by describing their interaction with the help of neural networks. These networks represent a mathematical model of the dynamics of the development of an inhomogeneous dilatant inclusion.

At the same time, in our work, using the hypothesis of connectedness (in the physical sense) of dilatant inclusions, decomposition of the tectonic process into a set of successive (or parallel) acts was carried out. Each of them represents the process of development of the dilatant inclusion (the Rice-Rudnicki model) with certain space-temporal and energy scales. From our standpoint, the emerging physical chains linking the dilatant mesoinclusions are porous channels with permeability varying in time. Thus, the proposed model is a cluster of combined dilatant mesoinclusions interconnected by the permeable channels (Fig. 1).

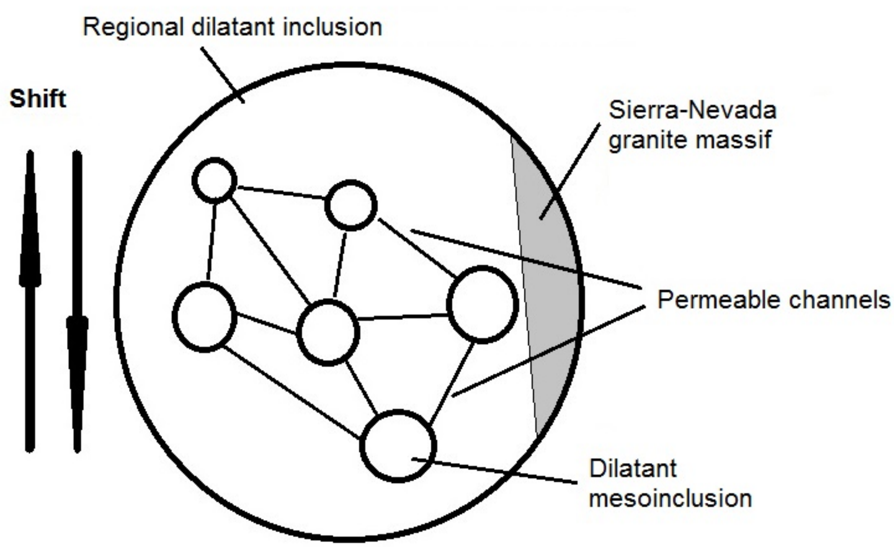

Fig. 1. The model of inhomogeneous dilatant inclusion

3 Experimental data analysis

As an example of analysis of space-time pattern of variations of radon concentration, we considered the possibility of determining the time and place of tectonic events. The problem of any forward-looking research is to determine the dynamic parameters of the anomalous zone of the earth crust, which properties define the conditions of tectonic events occurrence. Thus, we need to interpret the observed data. Interpretation of the radon variations like any other physical field has to begin with its division into normal and anomalous components [2, 3].

We conducted a study of radon exhalation field variations from rock mass obtained by King Chi-Yu on 12 boreholes in the vicinity of the San-Andreas Fault (California, USA) in the period from 1975 to 1980 [1]. In this period, 17 seismic events with magnitude $M > 4$ occurred. The observation points and the earthquake epicenters positions are shown in (Fig. 2). Data of radon exhalation obtained at weekly exposure of track detectors are selected for the analysis.

Based on the analysis of the space-temporal pattern of the radon concentration variation field in Southern California, we studied both the structure of the earth's crust of the prospective indicator zone (the intersection of the San-Andreas and Calaveras faults) and the probability of determining the time and

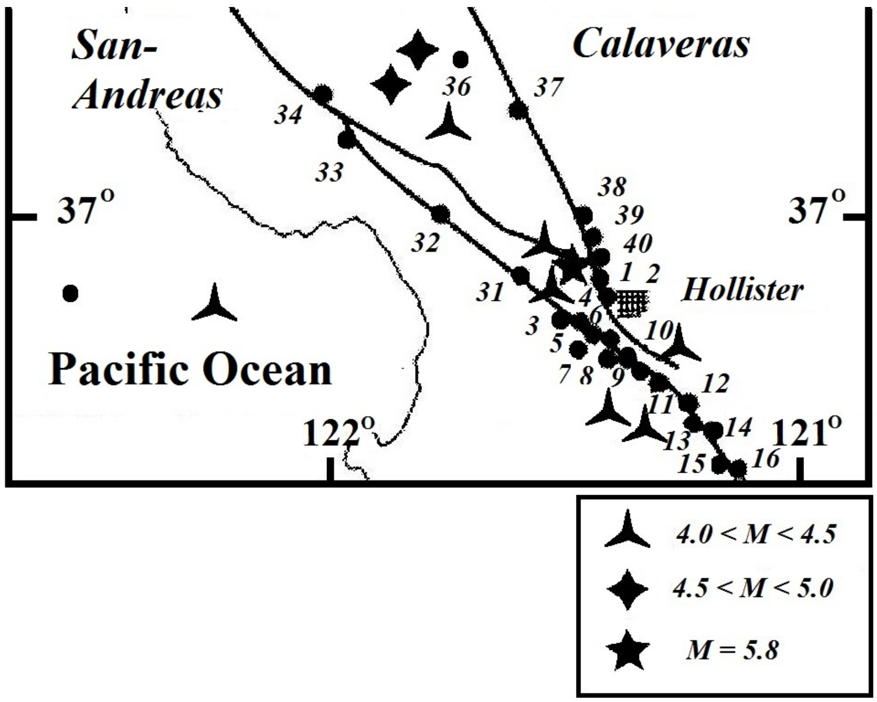

Fig. 2. The observation points and the earthquake epicentres positions

place of the tectonic event from these data. The goal of any predictive research is to determine the dynamic parameters of the investigated anomalous zone of the earth's crust, which determine the conditions for occurrence of a tectonic event. To create a system for forecasting earthquakes, it is necessary to have adequate both static and dynamic models of the earth's crust. To this end, we proposed a model of an inhomogeneous dilatant inclusion (Fig. 1). It is a volume filled with clusters of dilatant mesoinclusions connected by channels with time-varying permeability.

To provide short-term earthquake prediction it is necessary to have mentioned adequate both static and dynamic earth crust models. In our case, we propose a multiphase contrast model, there are a porous permeable water- and gas-saturated earth crust with the uranium-bearing minerals inclusions in a solid matrix. As a result of the proposed model of the medium at each point of observation in tectonically quiet periods, only processes of radioactive decay of the radium-226 isotope and the occurrence of radon-222 were considered. Therefore, at each point of observation the radon emanation amount should depend only from the radium-226 isotope content (Fig. 3).

The dynamic model of a medium is a high permeability dilatant inclusion in the field of variable tectonic stresses. The process of destruction is stabilized at undraining conditions deformation of high permeable moisture saturating inclusion. Such conditions involve significant deformation rate at a relatively low rate of the liquid diffusion. Stabilization of the fracture is associated with the strengthening of a deformable object. In this regard, to explain the features of radon field variations, we use the Rice-Rudnicki model with dilatant strengthening inclusion [4].

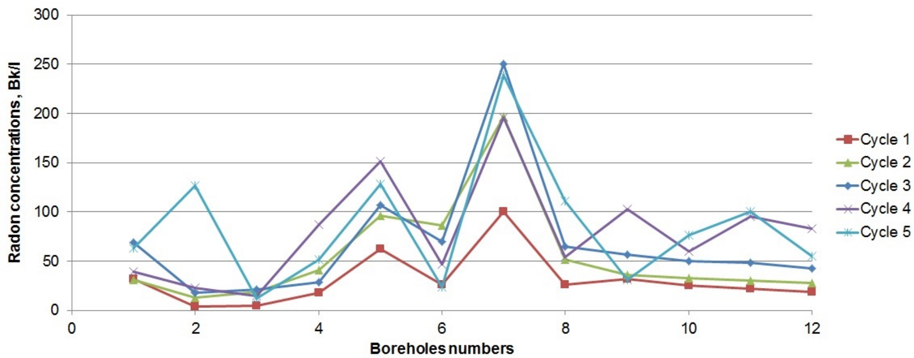

Fig. 3. Space-temporal distribution of radon variation (San-Andreas, Calaveras)

Let us prove the reasonableness of the proposed hypothetical model on the basis of analysis of the experimental material. Figure 3 shows a space-temporal picture of variations of the radon concentration at two intersecting observation profiles. From the analysis of the pattern of variations, two obvious conclusions follow:

1. The zones of maxima of the variations of radon concentration do not change their spatial location.
2. The amplitudes of variations the of radon concentration over time vary by more than two orders of magnitude.

Since concentrations of the radioactive elements of the uranium-235 type at the observation points are practically unchanged (with the exception of the long-period decay process), it can be assumed that the time variations of the radon concentration are related to the processes of gas transfer in the active medium. The invariance of the position of the extrema of variations of the radon concentration in space suggests that in these zones the sources (sinks) in the earth's crust are stipulated by the redistribution of radon in space. It remains to confirm presence of one more element(s), which is an integral part of the model of the inhomogeneous dilatant inclusion. These are permeable channels connecting dilatant mesoinclusions. Their presence is confirmed by analysis of the long-period variations of the radon concentration (Fig. 4). This figure shows the dynamics of variations of the radon concentration over a period of more than

4 years (averaged curve). From analysis of the presented data, it follows that, apparently, there is a system of interconnected sources (sinks), *i.e.* presence of permeable channels was proved. Thus, in the earth's crust in the zone of the intersection of the San-Andreas and Calaveras faults in Southern California, there is an inhomogeneous permeable dilatant inclusion, each element of which obeys the laws described by the Rice-Rudnicki model and reacts to the change in the dynamic situation of the investigation region [4].

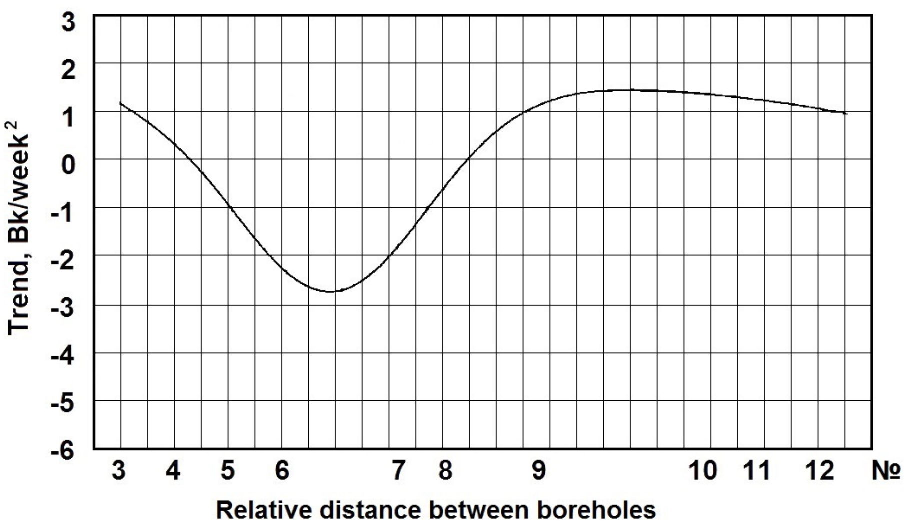

Fig. 4. Dynamics of the depression funnels evolution by the radon exhalation data (by the authors data)

4 Dynamic model

The amount of energy generated by the origin of a large earthquake in the form of elastic waves is so enormous that it would be difficult to conceive that this much amount having been stored up within a small confined volume of the earths crust until the outbreak takes place. Since the material, of which the earths crust is made up, has the limit of strength that is finite, a huge volume of the earth crust has to be needed for this much energy to be stored up in it. It is so because such a limit is exceeded in each part of the stress volume. Let us call this volume the earthquake volume. The earthquake energy or the earthquake volume has also a certain upper limit.

In the article published in 1956, Tsuboi proposed to calculate the energy E of the largest possible earthquake [5]. First of all, he has assumed that the energy

E is given by

$$E = \frac{1}{2} e x^2 V, \tag{1}$$

where the notations used have the following meanings: e is the effective elastic constant of the crustal material, x is the ultimate strain of the crustal material, V is the earthquake volume.

Expression (1) amounts to mean that the material within the earthquake volume will be uniformly strained until this uniform strain reaches the value x everywhere at the same time within the volume. Further, at the time of earthquake occurrence, the stored energy E is sent out from the whole volume by some kinds of mechanism. Evidently, this uniform strain hypothesis cannot be strictly true, but the essential point of it, which the writers would like to emphasize, is that x is assumed not to differ notably from one part to another within the volume. The assumption that x uniform everywhere can be only the first approximation in the formulation of the hypothesis.

Thus, local parts of the earthquake volume characterized by their x values store and emit according to (1) only a portion of the seismic energy. Consequently, they are sources of both foreshock and aftershocks. So, for example, we shall consider both foreshocks and aftershocks as a manifestation of dynamic processes in mesoinclusion previously, it was believed that the sources of both foreshocks and aftershocks are the entire volume of the earthquake preparation zone. Formalization of the dynamics of processes in the inhomogeneous dilatant inclusion and the construction of neural networks on this basis is a step forward to the theory of tectonic events forecasting and a new tool for analyzing the irregular structure of the tectonic field in seismically dangerous regions. Perhaps, the proposed approach will lead to a new explanation of such established concepts as foreshocks and aftershocks.

We have formulated the following principles of artificial intelligence for automatically prediction of the earthquake time and place [2, 3].

1. Thus, in the case of two-factor solutions, seismic alarm automatically declared. At the same time, the monitor continuously displays the factors curves, by which morphology you can define the possible location of the earthquake with accuracy equal to the discreteness of the observations.
2. Since, uranium 238 (source of radon-222) is uniformly distributed in the test array, we should expect that in this case the radon field variations will be normally distributed. To analyze the situation, artificial neural networks are applied. For neural network modeling, we applied the information interface with the universal analysis software STATISTICA Neural Networks. This makes it possible to solve effectively the problem by means of networks of various types: single-layer and multi-layer perceptrons. We performed the analysis of relationships between variables (variations of radon exhalation) using artificial neural networks. The original data matrix (changing in real-time) is a matrix whose rows are the spatial variables and variations of the columns are variable in time. From the set of possible variants, we chose the three-layered perceptron with 6 neurons in the intermediate layer. In

the perceptron, 11 inputs are the number of variables (radon variations) analyzed at each borehole.

In the study of the matrix of variables in real-time for a week before the earthquake, the perceptron with independent variables is formed. *I.e.*, at this time at each point of observation only processes of uranium-238 radioactive decay occur and radon-222 exhalation will be normally distributed. Therefore, at each point of observation, the emanation amount depends on the content of uranium-238 and does not depend on radon redistribution between points of observation. Thus, with the help of neural networks, complex dynamic processes are simulated that act in an inhomogeneous dilatant inclusion.

We have created a technique of rank recognition of a situation that occurred at any time in the vicinity of controlled earthquake source. On the basis of factor analysis, a technique of separation of radon signal at its normal and anomalous (predictive) components is developed that enabled us to formulate the principles of artificial intelligence for automatical prediction the earthquake time and place. When approaching tectonic events are observed the space-temporal pattern of the field variations of radon should not be related to changes in the structure of the pore space rock massif. Therefore, the study of the matrix of variables in real time with the help of neural networks for the week before the earthquake marked the formation of the perceptron with modified architecture that characterizes the independent of variables from each other. Apparently, this is a short-term earthquake precursor.

5 Conclusions

1. An original model of the crustal block at the intersection of the San-Andreas and Calaveras faults was constructed based on the hypothesis of the existence in the Earth's crust of Southern California of an inhomogeneous dilatant inclusion, which is a set of clusters of dilatant mesoinclusions, interconnected by permeable channels.
2. Analysis of the experimental data on the study of the space-temporal distribution of variations of radon concentration showed that the dynamic zones of the extrema apparently coincide spatially with the mesoinclusions that form the main dilatant volume.
3. The dynamic relationship of these mesoinclusions is derived from an analysis of the space-temporal characteristics of regional long-period variations of the radon concentration.
4. It has been suggested that the source of both foreshocks and aftershocks is not the entire volume of the earthquake preparation zone, but only the local sections of the volume (mesoinclusions).
5. On the basis of the factor analysis and neural network modeling, the basic principles of artificial intelligence are formulated for automatical prediction of the time and place of possible earthquake. Two rank technique is developed for recognizing the situation that occurs at any time in the vicinity of the controlled section of the crust. The technique consists in the fact that once

a week by the method of factor analysis the qualitative characteristics of seismic hazard are determined. In our case, the first rank of danger is the beginning and end of the process of stress relaxation; the second rank of danger is fixing the time of independent radon exhalation process at each point of observation by methods of neural network modeling (the maximum probability of an earthquake).

6 Acknowledgements

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