Comparative analysis of digital radar data processing algorithms

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Abstract A mathematical modeling was performed to obtain and analyze the results of trajectory filtration using linear and nonlinear Kalman filters. It is established that a nonlinear filter and a filter that takes into account the correlation of the errors of the linearized observations have a close efficiency. However, in the case of limited computing resources, it is preferable to use the linear filter. A program was developed in a cross-platform complete integrated development environment Qt Creator on the C++ programming language. This program can be applied to many systems to solve the problem of trajectory filtration after special adjustment.

Keywords: Digital signal processing, Trajectory processing, Kalman filter, linear filter, nonlinear filter

A number of methods of trajectory filtration are known, the main of which are based on modifications of algorithms of the Kalman vector estimation. Application of the nonlinear Kalman filter (NF) can be close to the optimal solution because observations are made in the polar coordinate system and estimation of the parameters of the trajectories to be tracked is carried out in a rectangular coordinate system. However, this approach requires additional computational costs and is very difficult to implement [1–6]. In works [5–7], researches were carried out efficiency of the pathfiltering algorithms that are applied to the two-coordinate radar, as well as an algorithm based on the Kalman nonlinear filter that was applied to a three-coordinate radar. However, the comparative modeling of Kalman's nonlinear and linear filter algorithms for a three-coordinate radar was not considered.

The aim of the research is a comparative analysis of effectiveness of the proposed modifications of linear and nonlinear Kalman filters for various types of trajectories of radar targets.

Operation of most algorithms of the trajectory tracking is based on the use of various mathematical models, with which it is possible to accurately approximate the real motion of the target and the process of its observations by the radar station, and then, in the process of filtration, refine the measurements obtained assessing the degree of their suitability for the model. The combination of optimally selected models of motion and observation underlies most methods of trajectory tracking. Consider the mathematical models of the motion of objects and observations with that applied to a three-coordinate radar station. As a model of motion of the accompanied object, we use the Markov random sequence given up to the stochastic equation [1–4]:

$$\bar{x}_i = \rho_i \bar{x}_{i-1} + \bar{\xi}_i, i = 1, 2, \ldots, \tag{1}$$

where $\bar{x}_i = (x_i \quad y_i \quad z_i \quad v_{xi} \quad v_{yi} \quad v_{zi})^{\mathrm{T}}$; x_i , y_i , z_i are the Cartesian coordinates of the object position; v_{xi}, v_{yi}, v_{zi} are the projections of the velocity on to X, Y, and Z axes, respectively;

$\rho_i = \begin{pmatrix} 1 & 0 & 0 & t_i & 0 & 0 \\ 0 & 1 & 0 & 0 & t_i & 0 \\ 0 & 0 & 1 & 0 & 0 & t_i \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$; t_i is the time, for which the object position has changed;

$\bar{\xi}_i = (0 \quad 0 \quad 0 \quad \xi_{v_{xi}} \quad \xi_{v_{yi}} \quad \xi_{v_{zi}})^{\mathrm{T}}$ is the white Gaussian noise with covariance matrix $V_{\xi_i} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma\sigma_{xi}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma\sigma_{yi}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma\sigma_{zi}^2 \end{pmatrix}$; γ is the relative average change in target speed during the time of flight through the radar coverage area; $\sigma_{xi} = \sqrt{v_{x0}^2 \frac{t_i}{\Delta t}}$, $\sigma_{yi} = \sqrt{v_{y0}^2 \frac{t_i}{\Delta t}}$, $\sigma_{zi} = \sqrt{v_{z0}^2 \frac{t_i}{\Delta t}}$ are the root mean square deviations (RMS) of the projections of velocity; v_{x0} , v_{y0} , v_{z0} are the initial values of the projections of velocity; Δt is the radar scan time.

The use of this model allows one to simulate motion of an object of various degrees of complexity: from rectilinear uniform to motion with large accelerations and maneuvering. If $\gamma = 0$, then $v_i = v_0$, that is, the speed does not change. At $\gamma = 0.01$, the speed will change by 1%, etc. Thus, it is possible to estimate the operation of the filter under different conditions.

As a model for observing an object using a three-coordinate radar, we use the following expression:

$$\bar{z}_i = h(\bar{x}_i) + \bar{n}_i, \tag{2}$$

where $\bar{z}_i = (z_{R_i} \quad z_{\alpha_i} \quad z_{\beta_i})^{\mathrm{T}}$; z_{R_i} are the distance observations; z_{α_i} are the bearing observations; z_{β_i} are the elevation observations; $\bar{n}_i = (n_{R_i} \quad n_{\alpha_i} \quad n_{\beta_i})^{\mathrm{T}}$ are the additive noise with zero mean and covariance matrix $V_n = \begin{pmatrix} \sigma_R^2 & 0 & 0 \\ 0 & \sigma_\alpha^2 & 0 \\ 0 & 0 & \sigma_\beta^2 \end{pmatrix}$;

$h(\bar{x}_i) = \begin{pmatrix} \sqrt{x_i^2 + y_i^2 + z_i^2} \\ \arctan \frac{y_i}{x_i} \\ \arctan \frac{z_i}{\sqrt{x_i^2 + y_i^2}} \end{pmatrix}$; σ_R , σ_α , σ_β are the RMS deviations of the source of observations by distance, bearing and elevation, respectively.

Estimation of the trajectory of a moving object consists in determining the numerical values of the parameters of its motion. Consider the NF algorithm [4–6]. Using observations (2), we calculate the estimation of the parameters of the motion of the target

$$\hat{\bar{x}}_i = \hat{\bar{x}}_{эi} + P_i H_i^{\mathrm{T}} V_n^{-1} \left(\bar{z}_i - \bar{h}_{эi}\right), \tag{3}$$

where $\hat{\bar{x}}_{эi} = \rho_i \hat{\bar{x}}_{i-1}$ is the vector of prediction values in Cartesian coordinates at the i-th step; $\bar{h}_{эi}$ is the vector of prediction values in polar coordinates at the i-th step.

The covariance matrix of estimation errors is calculated by formula [4]

$$P_i = P_{эi} - P_{эi} H_i^{\mathrm{T}} \left(H_i P_{эi} H_i^{\mathrm{T}} + V_n\right)^{-1} H_i P_{эi},$$

where $P_{эi} = \rho_i P_{i-1} \rho_i^{\mathrm{T}} + V_{\xi_i}$ is the covariance matrix of prediction errors

$$H_i = \frac{dh(\bar{x}_{эi})}{d\bar{x}_{эi}} = \begin{pmatrix} x_{эi}\Big/\sqrt{x_{эi}^2 + y_{эi}^2} & y_{эi}\Big/\sqrt{x_{эi}^2 + y_{эi}^2} & z_{эi}\Big/\sqrt{x_{эi}^2 + y_{эi}^2} & 0\;0\;0 \\ -y_{эi}\Big/\left(x_{эi}^2 + y_{эi}^2\right) & x_{эi}\Big/\left(x_{эi}^2 + y_{эi}^2\right) & 0 & 0\;0\;0 \\ -\frac{x_{эi} z_{эi}\big/\sqrt{x_{эi}^2+y_{эi}^2}}{x_{эi}^2+y_{эi}^2+z_{эi}^2} & -\frac{y_{эi} z_{эi}\big/\sqrt{x_{эi}^2+y_{эi}^2}}{x_{эi}^2+y_{эi}^2+z_{эi}^2} & \frac{\sqrt{x_{эi}^2+y_{эi}^2}}{x_{эi}^2+y_{эi}^2+z_{эi}^2} & 0\;0\;0 \end{pmatrix}.$$

The described algorithm is the most difficult both in the implementation and in adjustment [5–7]. However, this approach makes it possible to take fuller account of the nature of the observational models.

One of the important simplifications laid down in the Kalman filter is the assumption of the linear character of the equations of motion and observation. To reduce computational costs, it is proposed to use a linear filter, to which input linearized observations arrive. To take into account nonlinear dependencies and to achieve acceptable accuracy, we perform the following transformations:

$$\bar{z}_i' = \begin{pmatrix} z_{x_i} \\ z_{y_i} \\ z_{z_i} \end{pmatrix} = \begin{pmatrix} z_{R_i} \cos z_{\alpha_i} \cos z_{\beta_i} \\ z_{R_i} \sin z_{\alpha_i} \cos z_{\beta_i} \\ z_{R_i} \sin z_{\beta_i} \end{pmatrix},$$

where $z_{x_i}, z_{y_i}, z_{z_i}$ are the observations of Cartesian coordinates.

As a result of the transformations, the observation model takes the following form:

$$\bar{z}_i' = C\bar{x}_i + \bar{n}_i', \tag{4}$$

where $C = \begin{pmatrix} 1\;0\;0\;0\;0\;0 \\ 0\;1\;0\;0\;0\;0 \\ 0\;0\;1\;0\;0\;0 \end{pmatrix}$ is the conversion matrix, $\bar{n}_i' = (n_{x_i} \quad n_{y_i} \quad n_{z_i})^{\mathrm{T}}$ is the additive noise with zero mean and covariance matrix $V_{n_i}' = \begin{pmatrix} \sigma_{nx_i}^2 & B_{xy_i} & B_{xz_i} \\ B_{xy_i} & \sigma_{ny_i}^2 & B_{yz_i} \\ B_{xz_i} & B_{yz_i} & \sigma_{nz_i}^2 \end{pmatrix}$, where B_{xy_i} , B_{xz_i} , B_{yz_i} are the covariance of observations;

$$B_{xy_i} = M\left\{n_{x_i} n_{y_i}\right\} = \frac{1}{2}\sin 2z_{\alpha_i}\left(\sigma_R^2\cos^2 z_{\beta_i} + R_i^2\sigma_\beta^2\sin^2 z_{\beta_i} - R_i^2\sigma_\alpha^2\cos^2 z_{\beta_i}\right),$$

$$B_{xz_i} = M\{n_{x_i} n_{z_i}\} = \frac{1}{2}\cos z_{\alpha_i}\sin 2z_{\beta_i}\left(\sigma_R^2 - R_i^2\sigma_\beta^2\right),$$

$$B_{xz_i} = M\{n_{x_i} n_{z_i}\} = \frac{1}{2}\sin z_{\alpha_i}\sin 2z_{\beta_i}\left(\sigma_R^2 - R_i^2\sigma_\beta^2\right);$$

variances of observation inaccuracies in Cartesian coordinates are

$$\sigma_{nx_i}^2 = \sigma_R^2\cos^2 z_{\alpha_i}\cos^2 z_{\beta_i} + R_i^2\sigma_\alpha^2\sin^2 z_{\alpha_i}\cos^2 z_{\beta_i} + R_i^2\sigma_\beta^2\cos^2 z_{\alpha_i}\sin^2 z_{\beta_i},$$

$$\sigma_{ny_i}^2 = \sigma_R^2\sin^2 z_{\alpha_i}\cos^2 z_{\beta_i} + R_i^2\sigma_\alpha^2\cos^2 z_{\alpha_i}\cos^2 z_{\beta_i} + R_i^2\sigma_\beta^2\sin^2 z_{\alpha_i}\sin^2 z_{\beta_i},$$

$$\sigma_{nz_i}^2 = \sigma_R^2\sin^2 z_{\beta_i} + R_i^2\sigma_\beta^2\cos^2 z_{\beta_i}.$$

To estimate the parameters of the objects motion with the account of the next observation, we use the following expression:

$$\hat{\bar{x}}_i = \hat{\bar{x}}_{эi} + P_i C^{\mathrm{T}} V_{n_i}^{\prime -1}\left(\bar{z}_i' - C\bar{x}_{эi}\right), \tag{5}$$

In this case, the covariance matrix of estimation errors can be found from the following expression:

$$P_i = P_{эi}\left(E + C^{\mathrm{T}} V_{n_i}^{\prime -1} C P_{эi}\right)^{-1},$$

where E is the unit matrix.

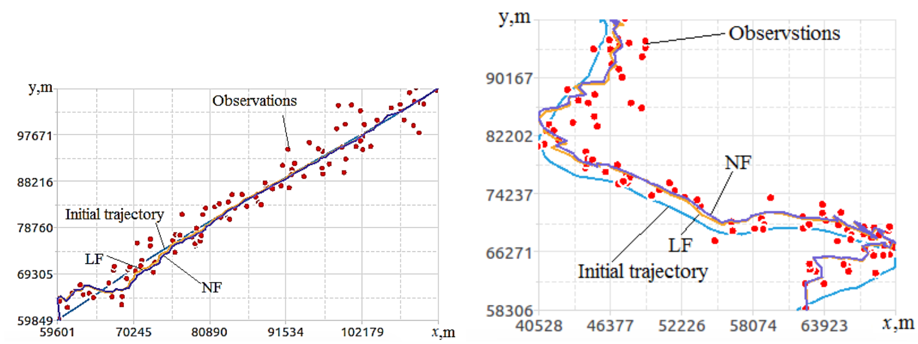
This approach is easier for implementation and adjustment and requires $O\left(N^N\right)$ fewer multiplication operations, where N is the number of observed parameters.

To perform a comparative analysis and research of the effectiveness of the proposed modifications of linear and nonlinear Kalman filters, a mathematical model was implemented in the computer algebra system Wolfram Mathematica. Based on the results obtained in [5–7], a program was developed that allows one

- to simulate various trajectories of motion;
- to simulate observations of a three-coordinate radar with specified accuracy characteristics;
- to perform estimation of observations from the radar using the linear and nonlinear implementation of the Kalman filter;
- to plot graphics of a true trajectory, observations and results of RLI processing.

This program is implemented in the C++ programming language in a free, cross-platform Qt creator complete integrated development environment using solutions from the Boost class library collection. Thus, owing application of these solutions, the configuration of the proposed computational module will not be required if it will be integrated into any automated radar processing system.

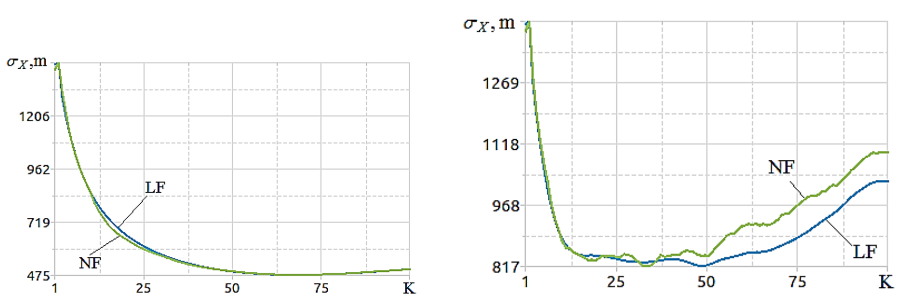
In Figure 1, the initial trajectory of the object's motion, the observation of the radar located at the origin, and the results of the trajectory processing using each filter are presented. The simulation results are shown for the motion of an object with the same modulo velocities along each axis. It is clear that each of the proposed algorithms allows one to get results that are close to the real trajectory of the target.



a) b)

Fig 1. Initial trajectory, observations and filtration results

In Figure 1a, the simulation results for $\gamma = 0.01$ are shown. In this case, the efficiency of NF and LF filters practically coincides (Fig. 2a).

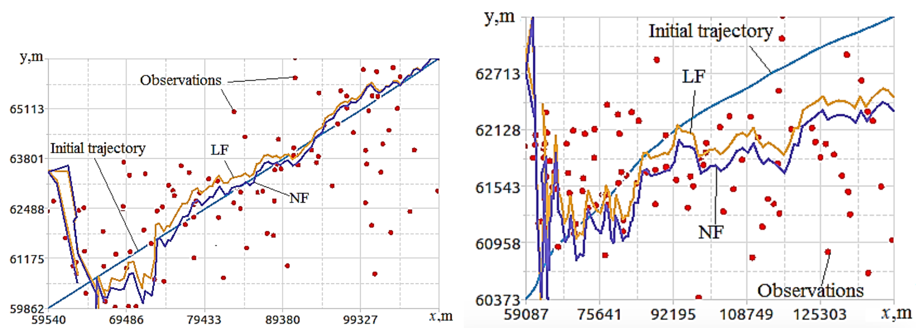


a) b)

Fig 2. RMS errors in estimating the coordinate x

In Figure 1b, the simulation results for $\gamma = 0.3$ are shown. In this case, the target maneuvers more intensively and moves with greater acceleration. From the graphs of the RMS coordinate x, shown in Fig. 2b, we can conclude that the LF has greater we can conclude that the LF has greater efficiency, since the steady-state LF value is lower. It should also be noted that with the removal of the target from the radar, RMS of the proposed algorithms increases.

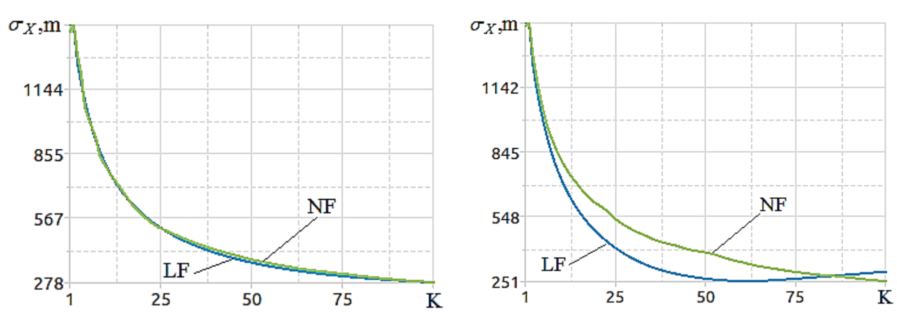
In Figure 3, the motion of an object with a higher velocity along the x axis is shown.



a) b)

Fig 3. Initial trajectory, observations and filtration results

In Figure 3a, the simulation results for $\gamma = 0.01$ are shown. The algorithms of NF and LF have practically the same efficiency (Fig. 4a). In Figure 3b, the simulation results for $\gamma = 0.1$ are shown. In this case, the LF has a greater efficiency since the steady-state value of the RMS of the LF is lower (Fig. 4b).



a) b)

Fig 4. RMS errors in estimating the coordinate x

Modeling of the trajectory estimation processes based on observations of objects with different nature of motion allows us to draw the following conclusions.

1. In modeling of various trajectories of motion, the proposed algorithms for estimating trajectory parameters have close accuracy, while LF has smaller requirements for computational resources. It should also be noted that when simulating target moves with more intensive maneuvering, the algorithm based on the Kalman linear filter is more effective.
2. From the point of view of minimizing errors in estimating, the LF algorithm is more preferable, but the NF algorithm requires more operations and more complicated implementation and adjustment than LF.
3. The difference in the RMS is also affected by the intensity of the target maneuvering, which is determined by the magnitude of the coefficient γ in the motion model (1); it was found out that with increasing γ LF scoring also increases.
4. Application of the solutions described in development of the program allowed us to achieve high performance of the program's computational module.

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