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ПРИМЕНЕНИЕ НОВЫХ МЕТОДОВ ПОСТРОЕНИЯ МНОГОСЛОЙНЫХ ПОЛУЭМПИРИЧЕСКИХ МОДЕЛЕЙ К ЗАДАЧЕ НЕЛИНЕЙНОГО ИЗГИБА КОНСОЛЬНОГО СТЕРЖНЯ*

Аннотация

Данная статья посвящена новой методологии, которую мы разработали для построения моделей на основе гетерогенной информации, включающей дифференциальные уравнения и дополнительные данные. Основная идея заключается в разработке нового класса математических моделей, в рамках которого появляются дополнительные возможности для построения уточняемой модели на основе сочетания классических и новых методов. Мы строим данные модели путём применения классических формул численного интегрирования дифференциальных уравнений к интервалу с переменным верхним пределом. Этот подход был проверен на задаче нелинейного изгиба консольного металлического стержня. В качестве исходного дифференциального уравнения мы взяли дифференциальное уравнение для прямого идеально упругого бесконечно тонкого стержня. Для измерений мы взяли реальный стержень, который не является ни идеально упругим, ни бесконечно тонким, ни прямым. В результате применения наших методов, мы получили приближённую модель, для которой точность описания реального стержня выше, чем у точного решения исходного дифференциального уравнения. Наши исследования важны для долгосрочного прогнозирования состояния и поведения строительных балок и сжимающих элементов подъёмных кранов и других реальных механизмов.

Ключевые слова

Новые методы построения; многослойные модели; нелинейный изгиб консольного стержня; изгиб металлической трубки.

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NEW METHODS OF MULTILAYER SEMIEMPIRICAL MODELS IN NONLINEAR BENDING OF THE CANTILEVER

Abstract

This article focuses on a new methodology that we have developed for building models based on heterogeneous information, which includes differential equations and additional data. The basic idea is to develop a new class of mathematical models, in which there are additional opportunities to build and Refine models based on the combination of classical and new methods. We build these models by applying the classical formulas for the numerical integration of differential equations to the interval with a variable upper limit. This approach was tested on the problem of nonlinear bending of a cantilever metal rod. As the original differential equation we get the differential equation for elastic perfectly straight, infinitely thin rod. For our measurements we took the actual rod that is neither perfectly elastic or infinitely thin nor straight. The result of the application of our methods, we

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obtained the approximate model for which the accuracy of the description of the real terminal is higher than the exact solution of the original differential equation. Our research is important for long-term forecasting of the status and behavior of construction beams and compression elements of cranes and other real mechanisms.

Keywords

New methods of building; multilayer models; nonlinear bending of the cantilever; bending of the metal rod.

Introduction

Modeling of complex technical objects is often hampered by insufficient knowledge of the processes occurring in them. As a result, the structure and coefficients of the differential equations describing these processes are not known accurately, the boundary conditions are known even less accurately. The problem of identifying equations and boundary conditions from the results of observations (the inverse problem) is usually much more complicated than the direct problem of solving a differential equation with boundary conditions. One of the methods for solving such problems is the approach based on the construction of the neural network model of the object by differential equations and additional data [1-8].

However, the training of neural networks requires a fairly large computational cost. To accelerate these processes, a new class of multi-layer models was developed [9-12], with which it is possible to do it without a complicated training procedure. In this paper, this approach is tested on the task of modeling a real object using our experiments.

The essence of the approach is to apply the known recursive formulas for the numerical integration of differential equations to an interval with a variable upper limit. As a result, an approximate solution is obtained as a function of this upper limit. In this paper, this approach is applied to the problem of modeling the shape of the bend of a directly cantilevered metal rod. In this case, attempts to select the coefficients of the equation in such a way that its exact solution corresponds to the experimental data with an acceptable accuracy did not lead to success. The developed methods can be used for long-term forecasting of the behavior of building beams, various structural elements of load-lifting machines and mechanisms taking into account the real picture of wear, aging and corrosion of metal.

Material and methods

The measurements were performed with a straight rod made of an aluminum alloy 940 mm long with a circular cross-section with a diameter of 8 mm and a mass of 126 grams. One end of the rod was tightly fixed. At the other end of the tube we weighted weights 100 grams, 200 grams and so up to 1900 grams. The positions of the rod were fixed with increasing mass, which acted on the unattached end of the object under study. The tube was photographed after attaching and detaching of every weight.

As a mathematical model, the equation of a large static deflection of a thin homogeneous physically linear elastic rod is used under the action of distributed q and concentrated p forces [14].

$$\frac{d^2\theta}{dz^2} - \frac{mgL^2}{D}\left(\frac{m_i}{m} + z\right)\cos\theta = 0 \quad (1)$$

Where D and L are the constant flexural rigidity and length of the rod; θ – is the angle of inclination of the tangent; $z = 1 - s/L$ - s –the natural coordinate of the curved axis of the rod, measured from the seal, m – the mass of the rod, m_i _ the mass of the load. In the experiment performed, the distributed and concentrated forces were the weights of the rod and the load at the end.

The boundary conditions have the form:

$$\left.\frac{d\theta}{dz}\right|_{z=0} = 0; \quad \left.\theta\right|_{z=1} = \theta_0$$

The angle θ is related to the coordinates of the points on the rod by the equalities:

$$\frac{dx}{ds} = Cos(\theta); \frac{dy}{ds} = Sin(\theta); \quad (2)$$

To obtain the equation (1) we used the equation of small pure bending of an ideal straight rod of infinitesimal length in the projection on the tangent to the line of large deflection.

Equations (1) and (2) describe the object in question inaccurately – the rod has a yield zone near the seal, has geometric and physical errors. By our methods, an approximate model is constructed by the equation (in the problem under consideration, these are equations (1 and 2)), the parameters of which are refined from the measurement data.

We rewrite equation (1) in the form:

$$\frac{d^2\theta}{dz^2} = a\left(\mu_i + z\right)\cos\theta \qquad (3)$$

where $a = \frac{mgL^2}{D}$, $\mu_i = \frac{m_i}{m}$.

As indicated earlier, equation (3) describes the process of bending a rod with a large error. This statement was confirmed by numerical experiments. To build a more adequate model we move from the system (2 – 3) to its approximate parametric solution $x(s,\theta_0,a)$ and $y(s,\theta_0,a)$. Parameters θ_0, a can be found by the method of least squares, by minimization of:

$$\sum_{j=1}^{N}(x(s_i,\theta_0,a)-x_i)^2 + (y(s_i,\theta_0,a)-y_i)^2 \qquad (4)$$

Here N - number of points at which measurements were taken, $\{x_i, y_i\}$ - the coordinates of the points at which measurements were taken corresponding to the marks on the rod at distance s_i from the embedding. In this case, s_i are not known in advance. To search for them the length of the rod is divided into 100 parts and as s_i we take a number corresponding to the minimum value of the corresponding summand in the sum (4). Due to the fact that the number s_i is not known beforehand minimization (4) was conducted using the random search method [16].

To define functions $x(s_i,\theta_0,a)$, $y(s_i,\theta_0,a)$ we used method [9-12]. Its essence with respect to equation (3) is that the known formulas for the numerical solution of differential equations should be applied not to the interval $[0,1]$ but to an interval with a variable upper limit $[0,z]$. In this case, instead of a table of numbers, we get a function $\theta(z,\theta_0,a)$,, and the parameters of the problem θ_0, a are among its arguments.

From $\theta(z,\theta_0,a)$ we go over to the original Cartesian coordinates, integrating (2) according to the Simpson formula for a variable-length interval:

$$x(s) = \frac{s}{6M}\left(1+\cos\left[\theta(1-s/l)\right]+4\sum_{i=1}^{M}\cos\left[\theta\left[\frac{1-s/l}{2M}(2i-1)\right]\right]+2\sum_{i=1}^{M-1}\cos\left[\theta\left[\frac{1-s/l}{M}i\right]\right]\right),$$

$$y(s) = \frac{s}{6M}\left(\sin\left[\theta(1-s/l)\right]+4\sum_{i=1}^{M}\sin\left[\theta\left[\frac{1-s/l}{2M}(2i-1)\right]\right]+2\sum_{i=1}^{M-1}\sin\left[\theta\left[\frac{1-s/l}{M}i\right]\right]\right).$$

We applied this formula in calculations for $M = 10$.

As a result of the substitution, we obtain the dependences $x(s,\theta_0,a)$ and $y(s,\theta_0,a)$. Parameters θ_0, a, as it was mentioned above, are found by minimization of expression (4).

We present the results of calculations for above mentioned modification [9-12] of implicit Euler method [15] with one step. As a result, we obtain an approximate equation $\theta(z) \cong \theta_0 + z^2 a(\mu_i + z)\cos(\theta(z))$, from which we find an approximate solution:

$$\theta(z) \cong 2\frac{\theta_0 + z^2 a(\mu_i + z)}{\sqrt{1+2z^2 a(\mu_i+z)(\theta_0 + z^2 a(\mu_i+z))}+1}, \qquad (5)$$

where $\theta_0 = \theta(0)$ the angle of the rod at its end. Substitution (5) into Simpson formulas let us to obtain dependence $x(s,\theta_0,a)$ и $y(s,\theta_0,a)$. There are no restrictions on the parameters θ_0 and a.

In fact, accuracy of the given solution is higher, when the parameter a is lower, but in case of an approximate type of equation (3) we are interested not in a small error in the solution of this equation, but in the accuracy of the compliance with measurement data.

Calculations. Let's give the results of calculations for three values of the mass of the cargo $m_1 = 0$ grams, $m_2 = 700$ grams и $m_3 = 1500$ grams. Figure 1 compares the measurement data and the results of calculations using formula (5) for $m_1 = 0$ grams.

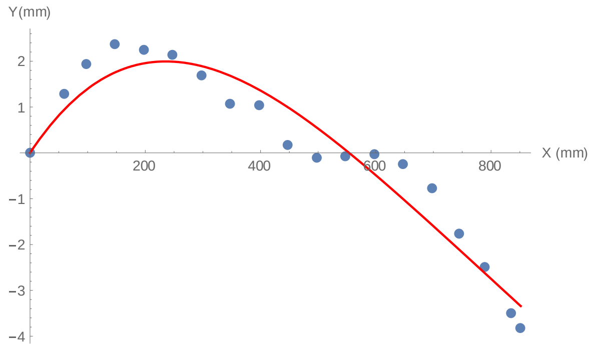

Figure 1. *Theoretical and experimental rod deflection curves obtained by using formula 5 for the mass of the cargo* $m_1 = 0$ *grams*

The standard deviation of the measurement results from the theoretical curve $\{x(s,\theta_0,a), y(s,\theta_0,a)\}$ is 2.25 mm. The error lies within the measurement error.

On the figure 2 there is a comparison of measurement data and calculation results by formula (5) for $m_2 = 700$ grams.

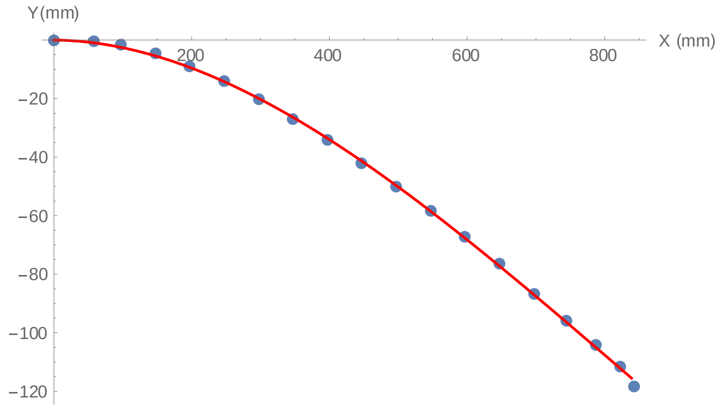

Figure 2. *Theoretical and experimental rod deflection curves, obtained by using formula 5 for mass* $m_2 = 700$ *grams*

The standard deviation of the measurement results from the theoretical curve $\{x(s,\theta_0,a), y(s,\theta_0,a)\}$ is 2.05 mm. The error lies within the measurement error.

On the figure 3 there is a comparison of measurement data and calculation results by formula (5) for $m_3 = 1500$ grams.

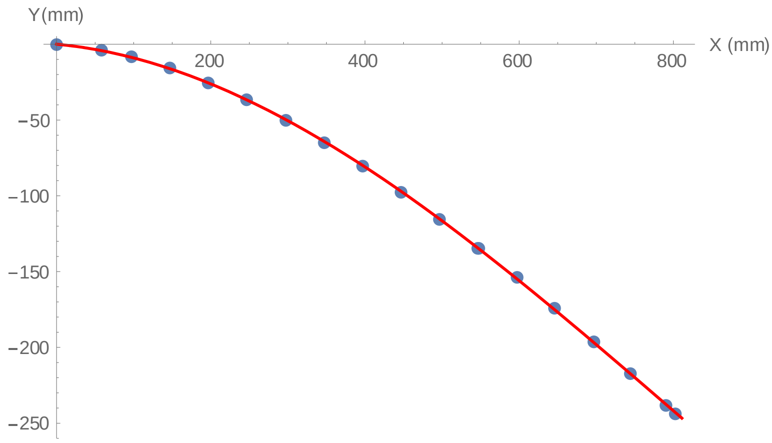

Figure 3. *Theoretical and experimental rod deflection curves, obtained by using formula (5) for the mass of the cargo 1500 grams*

The standard deviation of the measurement results from the theoretical curve $\{x(s,\theta_0,a), y(s,\theta_0,a)\}$ is 3.39 mm. The error lies within the measurement error.

We note that similar results were obtained for other values of the mass of the cargo. Interest is the possibility of predicting the deflection of the rod, depending on the weight of the load. For this we studied the dependence of the parameters θ_0, a and l from μ_i. For the formula (5) the following results were obtained:

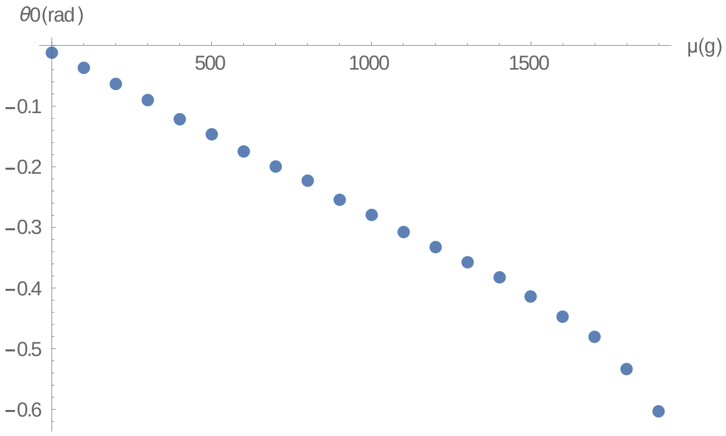

Figure 4. *Dependence* θ_0 *from the mass of the cargo* μ_i

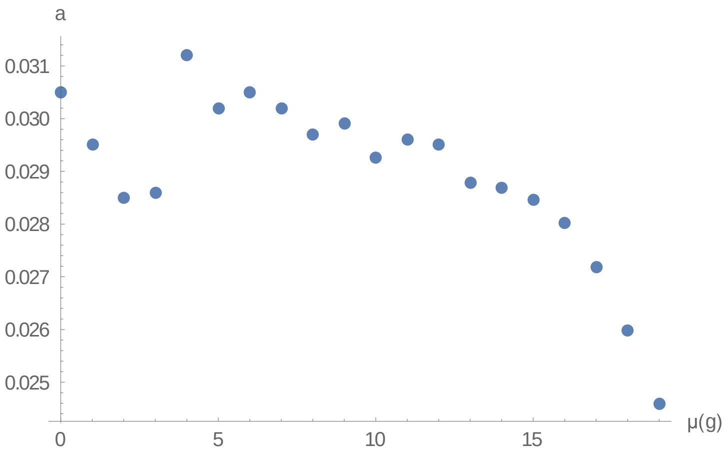

Figure 5. *Dependence* a *(mm) from the mass of the cargo* μ_i

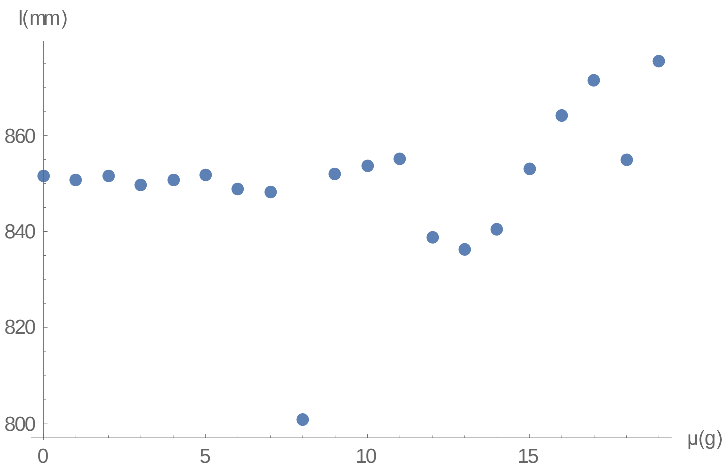

Figure 6. *Dependence* l *from the mass of the cargo* μ_i

The data from the graph show that parameters a and l practically do not change almost at all intervals of the change in the mass of the cargo and angle θ_0 varies linearly. Let's compare the experimental data with the results of calculations using formula (6) for $a = 0.03$, $l = 850$ и θ_0, calculated on the base of the linearly dependence, that is built by first two dots of the graph 4 for the mass of the cargo 100 gram.

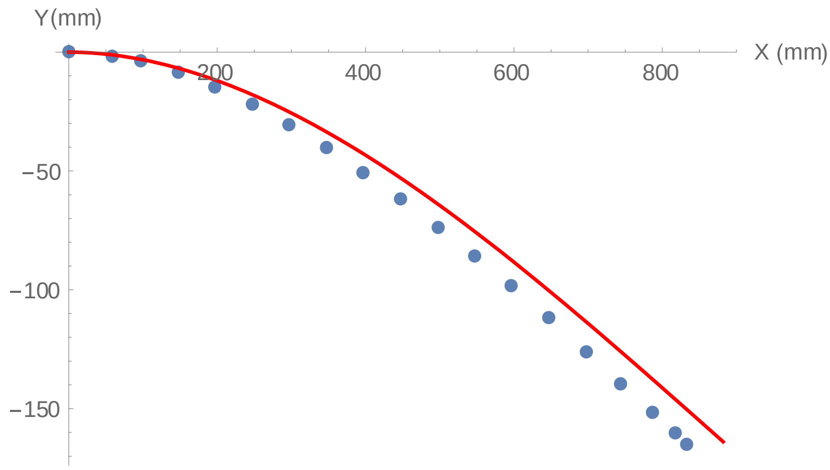

Figure 7. *Theoretical and experimental rod deflection curves, obtained by using formula (5) for* $a = 0.03$, $l = 850$ *and* θ_0, *calculated on the base of the linear dependence, that is built by first two dots of the graph 4 for the mass of the cargo 100 gram*

It can be seen from the graph that the discrepancy between the theoretical curve and experimental data differs no more than 10 mm, which can be considered a satisfactory result.

Results and Discussion

The fundamental difference between our method and traditional numerical methods is the obtaining of a function instead of a table of numbers, while the parameters of the problem naturally enter into the number of arguments of this function. This circumstance makes it possible to use the obtained models as components of cognitive systems, as they are easily adapted when new information about the modeled object appears.

A typical situation for practice is the situation when the results of observations of a real object contradict the mathematical model obtained on the basis of an attempt to apply known physical laws. In this situation, man often seek to refine the physical model of the object and obtain differential equations that reflect the processes occurring in it more accurately.

In particular, we can try to refine the coefficients of the equations. To solve such problems, there are a number of approaches, one of which is the use of neural networks [4, 7]. Sometimes no choice of parameters allows us to reflect the experimental data with reasonable accuracy, then it is necessary to change the structure of the model, which can lead to a sharp complication of differential equations and does not always ends successfully in a reasonable time. These difficulties often lead to the fact that the model of the object is constructed empirically by interpolation from experimental data.

Our method allows us to apply an intermediate approach, which consists in obtaining approximate semi-empirical formulas based on an inaccurate differential model and measurement results. Known theorems on the error of numerical methods [15] allow us to state that we can obtain an arbitrarily exact approximation to the solution of a differential equation by using a partition into a sufficiently large number of intervals. Our approach allows us to obtain formulas without the use of interpolation, which can be refined from the experimental data.

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