Modern Nonconvex Optimization. Theory, Methods, and Application

Alexander S. Strekalovsky

Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia strekal@icc.ru

Keywords: nonlinear optimization, d.c. optimization, penalized problem, local search, global search.

We consider the general nonconvex problem with the goal function and equality and inequality constraints given by d.c. functions.

We reduce this problem to a problem without nonconvex constraints by the exact penalty approach and present the penalized problem as a d.c. minimization one, i.e. with d.c. goal function which is nonsmooth. Furthermore, relations between the original problem and the penalized problem are investigated.

In addition, an employment the d.c. structure of penalized problem the Global Optimality Conditions (GOCs) are developed and analysed. In particular, we prove that the GOCs possess the constructive property, i.e. when the GOCs are violated, it is possible to find a feasible (in original problem) vector which is better than the point under investigation.

Moreover, it is shown that the point satisfying the GOCs turns out to be a KKT vector in the original problem. It means, that the new GOCs are related to the Classical Optimization Theory.

Besides we establish that the verification of the GOCs consists in a solution of a family of the partially linearized (with respect to the basic nonconvexities of the original problem unified by the exact penalty in one function) problem, and consecutive verification of the principal inequality of the GOCs.

The effectiveness of the GOCs is verified by a number of examples in which the GOCs confirm its ability to escape stationary points and local minima with improving the goal function.

On the base of GOC we propose local and global search methods for the penalized problems and investigate their convergence and other properties.

The testing of the developed Global Search Theory were carried out on a rather large field of test examples and applied problems of different nature. For instance, there were considered quadratic and polynomial extremum problems and fractional optimization problems.

In addition, a seeking for a solution to d.c. equations systems, the search for the Nash equilibrium points, the finding of optimistic and pessimistic solutions to bilevel problems with (d.c.) nonlinear data have been performed.

The results of computational simulations look rather promising even for the case of problems of high dimensions.