# Universal Properties of the General Agent-Based Market Model through Computational Experiments

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**Abstract.** *Research goals*: to synthesize the general view market mathematical model in accordance with new dynamic paradigm of economics, to reveal the universal properties of general view markets.

During our investigation we developed and continuously improved a desktop C# application *Model* for support the research process using computing experiments. Here our task is revelation of the universal properties of general view market as a result of simulation experiments using this software module.
*Results of the research*: the crucial factors which ensure the market stability are the level of agreement in adaptive expectations and the share of planning with adaptive expectations in a market. The increase of naive expectations leads to stability loss, to bifurcations and finally to chaos in general view market. The increase of number of firms also leads to stability loss and finally to chaos in the general view market at appreciable naive expectations. We revealed that the profits ratio and quantity outputs ratio of firms remains almost unchanged in short-run period in general view markets. It seems an important stability factor of many important real markets for which chaotic dynamics is usual.

**Keywords:** agent-based model, heterogeneous type, bifurcation, adaptive expectations.



## 1. Introduction

Information technology in the economy made it possible to model artificial societies and study economic models through the computer simulation. Economics has entered the stage of deep transformation of its bases. In recent years the researchers are renouncing the assumption of perfect rationality as unconditional basis of economic agents' behavior [1]. The neoclassical 'rational man' does not exist in reality: economic agents act according to established rules, without being fully informed and maximizing their own utility [2].

The real economic processes make a clear demonstration that neoclassical "rational man" is not their subject. In real economy "optimal imperfect decisions" are taken by simple and non-expensive calculations, well adapted to frequent repetitions, to evolution: it is more efficient for perfectly rational firm to perform multiple experiments with quantity to estimate the demand function rather than search for nonrecurrent, instantaneous achieving of equilibrium [3]. All it means that the real economy is dynamical system, and real processes of economy are iterative processes of this system.

Now institutional school of economics analyzes economic systems as a result of evolutionary process of participants' interaction [4]. New paradigm of economics is a mix of the nonlinear dynamical system theory and mathematical programming, including game theory and optimal control theory [5]. And the main tool of new economics is simulation modeling grounded on the basis of 3 computer paradigms (object-oriented, dynamic and multi-agent system) [6].

This new economics allows explaining the phenomena which were not keeping within traditional schemes. The evolutionary approach and analysis of the dynamics allow to explain why one type of firm ousts another from the market, why sometimes the economic system is stable, but in other cases is unstable [3, 7]. If the system has multiple equilibriums, the dynamics and evolution is the selection mechanism of best equilibrium according to certain criteria [8]. Traditional static models of competition (e.g., Cournot, Bertrand and Stackelberg) were converted in dynamic models which were investigated on existence, stability and local bifurcations of the equilibrium points [9, 10, 11].

Within the limits of new economics it is natural to study reciprocity relations [12]. Reciprocity or social responsibility implies that the firms not only pursue their selfish goal of increasing profits, but are also ready to sacrifice some of their profits for the benefit of consumers without direct compensation for it by the state [13]. Such targets can be stipulated by the firms' desire to get stable profits in the long run rather than maximal short-run profits [14, 15]. Such forward-thinking firms-reciprocators are considered in this paper. Their objective function is a weighted average of the profits and consumer surplus of their market segment.

Modern development of dynamic paradigm in economics is a wide stream of researches. However it is a stream of examples which are not developing in the general theory; their relations with real markets are often problematic [16]. The traditional method of constructing a scientific theory is first to synthesize and investigate the simplest possible mathematical model. And then we can study complex real systems which are grounded on this basis. This traditional approach is taken as a principle of our research.

This paper is a continuation of our previous works [17], [18]. There the elementary market model corresponding to the new paradigm of economics has been synthesized and investigated. That model describes a simplest market where firms have only one difference in their type when some firms (egoists) are focused exclusively on short-run profits, while others (reciprocators) take into account long-run factors. However it`s not any special, specific market; actually any global market contains such elementary local markets and consists of them. As suggests common sense then dynamics of the global market is stratified on dynamics of such local markets. Therefore it was naturally to state a hypothesis that the derived in [17], [18] properties of the elementary model are universal, i.e. these properties are the properties of general view market including real markets as a special case. Check of this hypothesis makes the project of this work.

The **paper goal** is to synthesize the general view market mathematical model according to the new dynamic paradigm of economics, to reveal the universal properties of general view markets including real markets, to check up the hypothesis about universality of properties of the model [17].

During our investigation we developed and continuously improved a desktop C# application *Model* for support the research process using computational experiments. Our next task is revelation of universal properties of general view market as a result of simulation experiments using this software module.

The paper is organized as follows: in part 2 we synthesize the general view market model; part 3 demonstrates desktop application *Model* for computing experiments; sections 4.1 – 4.3 describe our market model researches using this application, section 4.4 formulates their results; part 5 concludes.

## 2. Agent-Based General View Market Model

In general, almost any microeconomic market model is constructed as follows: 1) $n$ firms operate in the market (to simplify the notation suppose $n = 2$); 2) these firms produce homogeneous products in quantities $x_1(t)$ and $x_2(t)$ in time period $t$; 3) they use adaptive approach, i.e. they try to predict the quantity of their competitor in the next time period; 4) let $x_j^e(t+1)$ is the expected quantity of rival $j$ by a firm $i$ in next period $t+1$ ($i, j = 1, 2$). Then under planning of their quantity $x_i(t+1)$ in the next period the firms solve the following optimization problem:

$$Max\Pi_1(x_1(t+1); x_2^e(t+1)),\ Max\Pi_2(x_1^e(t+1); x_2(t+1)), \qquad (1)$$

where $\Pi_i$, $i = 1, 2$ is a profit function of firm $i$. The assumption about unchangeable quantity of the competitor (i.e. firm $i$ will use $x_j(t)$ instead of $x_j^e(t+1)$ when it solves the optimization problem) is an example of imperfect, bounded rationality in firm's strategies; it is called naive expectations. As a rule these two approaches (adaptive and naive) coexist in the market with a certain probability. Our model is based on these assumptions.

We consider a market of homogeneous product, where exogenous parameter $n(t)$ indicates how many firms operate at time $t$. Each firm produces output $x_i(t)$, where $i = 1, ..., n(t)$. Thus the industry output of the market is $Q(t) = \sum_{i=1}^{n} x(t)$ at time $t$. Product price $P$ is given by isoelastic demand function $P = P(Q) = b(t)/Q$ ($b(t) > 0$). Such kind of demand function as a matter of fact is not a restriction. Really, in a small neighborhood of a market state during the moment $t$ any demand function with elasticity $b(t)$ differs from the isoelastic one a little. Then in short-run period dynamics of a market with such demand function differs a little also. And at a structural stability they are qualitatively (i.e. orbitally) equivalent.

Formally the firm is defined by its objective function. Firm maximizes both its own profit $\pi_X = (P - v) \cdot x - fc$ (where $v$ is the firm's cost per unit in the market, $fc$ is fixed cost) and consumer surplus $CS$ (difference between maximum price which consumer can pay and real price) $CS = \Theta \cdot \left( \int_{\varepsilon}^{Q} P(q)dq - P \cdot Q \right)$, where parameter $\Theta$ specifies the segment of the market, which the firm believes its own and optimizes; $\varepsilon$ is the minimal technologically possible product quantity. Then $CS = \Theta \left( b \cdot \ln(\frac{Q}{\varepsilon}) - \frac{b}{Q} \cdot Q \right) = b\Theta \cdot \left( \ln(\frac{Q}{\varepsilon}) - 1 \right) = b\Theta \cdot \ln \frac{Q}{\hat{\varepsilon}}$, where $\hat{\varepsilon} = \varepsilon \cdot e$ (specific choice of $\varepsilon$ does not affect the model dynamics and so we suppose $\varepsilon = 1$). Then general profit function $\Pi = \Pi_i(t)$ of firm is:

$$\Pi = \alpha \cdot \pi + (1-\alpha) \cdot CS = \alpha \cdot ((P-v) \cdot x - fc) + (1-\alpha) \cdot b\Theta \cdot \ln \frac{Q}{\varepsilon}, \tag{2}$$

where $\alpha = \alpha_i(t)$ is share of short-run own profit $\pi = \pi_i(t)$ in the objective function, $1-\alpha$ is share of consumer surplus $CS$, $fc = fc_i(t)$ is a fixed cost. As a matter of fact $\Pi$ is a weighted average of short-run profit $\pi$ and expected stable long-run profit.

The model of paper [17] is the elementary special case of this general model. There we consider a market of homogeneous product, where $n$ firms operate, among them are $k$ identical reciprocator firms with the same output $x$ and $n-k$ identical selfish firms with the same output $y$.

Dynamic of the model is considered for discrete time $t = 1, 2, \ldots$. Our model is uniquely defined by firms' objective functions and their expectations types. It does not use any additional assumptions or restrictions.

### 2.1 Dynamics Model Equations

In real life both decision making approaches (adaptive and naive) coexist in the market with a certain probability. Let's obtain now the equations of a general market model with the minimum account of adaptive expectations dictated by common sense. According to such expectations firm $i$ suggests that production quantities of its rival $j$ will be equal to $x_j^e(t+1) = \delta_{ij}(t)x_i(t+1) + \chi_{ij}(t)x_j(t)$. Here $\delta_{ij}(t) \geq 0$ and $\chi_{ij}(t) \geq 0$ are parameters, defining shares of naive and adaptive expectations at this planning.

Let $z_i = x_i(t+1) + \sum_{j \neq i} x_j^e(t) = \sum_{j=1}^{n(t)} \delta_{ij}(t)x_i(t+1) + \sum_{j=1}^{n(t)} \chi_{ij}(t)x_j(t)$ is prospective industry output of the market, where $\delta_{ii}(t) = 1$, $\chi_{ii}(t) = 0$. Then the objective function for the firm $i$ has the form $\Pi_i = \alpha_i((\frac{b}{z_i} - v)x_i(t+1) - v_0) + (1-\alpha)b\Theta_i \ln(z_i)$ in accordance with (2) (here $\varepsilon = 1, \alpha_i = \alpha_i(t)$). Then according (1) the point $x_i(t+1)$ of maximum objective function $\Pi_i$ is found from the condition

$$\frac{\partial \Pi_i}{\partial x_i(t+1)} = \alpha_i \left(\frac{bz_i - \sum_{j=1}^{n(t)} \delta_{ij}(t) \cdot bx_i(t+1)}{z_i^2} - v\right) + (1-\alpha_i) b\Theta_i \frac{\sum_{j=1}^{n(t)} \delta_{ij}(t)}{z_i} = 0.$$ Then

$$z_i^2 = \frac{b}{v} \sum_{j=1}^{n(t)} \chi_{ij}(t) x_j(t) + \frac{b\Theta_i}{v} \frac{1-\alpha_i}{\alpha_i} \sum_{j=1}^{n(t)} \delta_{ij}(t) z_i. \qquad (3)$$

Hence suppose that $d_i = \frac{1}{2} \frac{b\Theta_i}{v} \frac{1-\alpha_i}{\alpha_i} \sum_{j=1}^{n(t)} \delta_{ij}(t)$ we obtain

$$(z_i - d_i)^2 = \frac{b}{v} \sum_{j=1}^{n(t)} \chi_{ij}(t) x_j(t) + d_i^2; \qquad z_i = \sqrt{\frac{b}{v} \sum_{j=1}^{n(t)} \chi_{ij}(t) x_j(t) + d_i^{\,2}} + d_i.$$

Thus we obtain the dynamics equations of general view market model

$$\lambda_i x_i(t+1) = \sqrt{\frac{b}{v} w_i(t) + d_i^2} + d_i - w_i(t) \qquad (i = 1, ..., n(t)), \qquad (4)$$

where $\lambda_i = \lambda_i(t) = \sum_{j=1}^{n(t)} \delta_{ij}(t)$, $w_i(t) = \sum_{j=1}^{n(t)} \chi_{ij}(t) x_j(t)$, $d_i = d_i(t) = \frac{1}{2} \frac{b\Theta_i}{v} \frac{1-\alpha_i}{\alpha_i} \lambda_i(t).$

In this paper we consider all actions, expectations and strategies of firms in short-run period, therefore the equations parameters $\lambda_i$ and $d_i$ are assumed further as constants which are independent of time.

Let the market of homogeneous product consists of $m$ firms' types, each type $l$ including $k_l$ identical firms: $l = 1, ..., m,\ k_1 + ... + k_m = n.$ Then $\lambda_i = \sum_{l=1}^{m} k_l \delta_{il}$, ,

$w_i(t) = \sum_{l=1}^{m} k_l \chi_{il} \cdot x_l(t),$ where $\delta_{il} = \delta_{ij}, \chi_{il} = \chi_{ij}, x_l(t) = x_j(t)$ at all $j$ from type $l$.

Then owing to (4) $x_i(t+1) = x_l(t+1)$ at all $i$ from type $l$. As a result dynamics in the equations (4) has dimension $m$:

$$\lambda_i x_i(t+1) = \sqrt{\frac{b}{v} w_i(t) + d_i^2} + d_i - w_i(t) \qquad (i = 1, ..., m), \qquad (5)$$

where $\lambda_i = \sum_{l=1}^{m} k_l \delta_{il}$, $w_i(t) = \sum_{l=1}^{m} k_l \chi_{il} \cdot x_l(t)$, $d_i = \frac{1}{2} \frac{b\Theta_i}{v} \frac{1-\alpha_i}{\alpha_i} \lambda_i$.

The equations (5) are a special case of (4) and simultaneously their generalization, i.e. they are equivalent to (4) in short-run period.

In particular, in two-dimensional model [17] ($m = 2$) for firm $i$

$$\lambda_i = \sum_{j=1}^{n} \delta_{ij} = 1 + p(k-1) = \lambda_x, \quad w_i(t) = \sum_{j=1}^{n} \chi_{ij} x_j(t) = q(k-1)x(t) + (n-k)y(t) = w_x(t),$$

$$d_i = \frac{1}{2} \frac{b\Theta}{v} \frac{1-\alpha}{\alpha} \sum_{j=1}^{n} \delta_{ij} = \frac{1}{2} \frac{b\gamma}{vk} \frac{1-\alpha}{\alpha} (1 + p(k-1)) = d. \qquad (6)$$

So for two-dimensional model [17] equations (5) have the form

$$\begin{cases} \lambda_x x(t+1) = \sqrt{\dfrac{b}{v} w_x + d^2} + d - w_x \\ \lambda_y y(t+1) = \sqrt{\dfrac{b}{v} w_y} - w_y \end{cases}, \qquad (7)$$

where $\lambda_x = 1 + p(k-1), \quad w_x = q(k-1)x(t) + (n-k)y(t), \quad \lambda_y = 1 + p(n-k-1),$

$$w_y = kx(t) + q(n-k-1)y(t), \qquad d = \frac{1}{2}\frac{1-\alpha}{\alpha}\frac{b\gamma}{vk}(1+p(k-1)).$$

We usually use further the following simplest after two-dimensional version of (5) for the illustrations of results of computational experiments. In this version we consider a market of three firms' types: $k_1$ and correspondingly $k_2$ reciprocator firms, ($k = k_1 + k_2$) and $dn = n - k$ identical selfish firms. Here as well as above $\alpha_1 = \alpha_2 = \alpha$, $\alpha_3 = 1, \delta_{13} = \delta_{23} = \delta_{31} = \delta_{32} = 0, \chi_{ij} = 1 - \delta_{ij}$. Then (5) has the form

$$\lambda_i x_i(t+1) = \sqrt{\frac{b}{v} w_i(t) + d_i^2} + d_i - w_i(t) \qquad (i = 1,2,3), \qquad (8)$$

where $\lambda_1 = 1 + \delta_{11}(k_1 - 1) + \delta_{12}k_2, \quad \lambda_2 = 1 + \delta_{22}(k_2 - 1) + \delta_{21}k_1, \quad \lambda_3 = 1 + \delta_{33}dn,$

$$w_1(t) = \chi_{11}(k_1 - 1)x_1(t) + \chi_{12}k_2x_2(t) + dnx_3(t), \quad w_2(t) = \chi_{22}(k_1 - 1)x_2(t) + dnx_3(t),$$

$$w_3(t) = \chi_{33}(dn-1)x_3(t) + k_1x_1(t) + k_2x_2(t), \quad d_i = \frac{1}{2}\frac{b\Theta_i}{v}\frac{1-\alpha_i}{\alpha_i}\lambda_i \ (i = 1,2), d_3 = 0.$$

## 2.2 Equilibrium Conditions

In a Nash equilibrium point we have $x_i(t+1) = x_i(t) = x_i$ at all $t = 1, 2, ...$ and $i = 1,...,m$. Hence $x_i^e(t+1) = x_i$ at all $i$ and $t$.

**Proposition 1**. There is unique Nash equilibrium point in a general market model (5).

**Proof**. In an equilibrium point $z_i = x_i(t+1) + \sum_{j \neq i} x_j^e(t) = \sum_{j=1}^{m} x_j = z$ at all $i = 1,...,m$.

Therefore owing to (3)

$$z^2 = \frac{b}{v}\sum_{j=1}^{n}\chi_{ij}(t)x_j + \frac{b\Theta_i}{v}\frac{1-\alpha_i}{\alpha_i}\sum_{j=1}^{n}\delta_{ij}z = \frac{b}{v}\sum_{l=1}^{m}k_l\chi_{il}\cdot x_l + \frac{b\Theta_i}{v}\frac{1-\alpha_i}{\alpha_i}\lambda_i z \ (i = 1,...,m),$$

where $\lambda_i = \sum_{l=1}^{m} k_l\delta_{il}$, $x_l = x_i$ at all $i$ from type $l$, $l = 1,...,m$. Hence

$$\sum_{l=1}^{m} k_l\chi_{il}\cdot x_l = a_i \qquad (i = 1,...,m), \qquad (9)$$

where $a_i = \frac{v}{b} z^2 - \frac{1-\alpha_i}{\alpha_i} \Theta_i \lambda_i z_i$. Since matrix of types parameters ($k_l \chi_{il}$) is nonsingular $m \times m$ matrix on construction then the system of linear equations (9) has one and only one solution, Q.E.D. .

For two-dimensional system (7) this Nash equilibrium point is the same, as in [17] and also is set by the same formula.

**Proposition 2**. There is unique Nash equilibrium point in a dynamical system (7):

$$y^* = \frac{\frac{b}{v}(kG + q(n-k-1))}{(kG + (n-k))^2} \qquad x^* = Gy^* = \frac{\frac{b}{v}(k + q(1/G)(n-k-1))}{(k + (1/G)(n-k))^2}, \qquad (10)$$

where function $G = G(p,q,n,k,\alpha) = \frac{p(n-k) + q(\alpha + (1-\alpha)\frac{n-k}{k})}{(2\alpha - 1)(1 + p(k-1))}$ .

**Proof**. Since (6) equation (3) has the following form for any reciprocator firm $i$

$$z_i^2 = \frac{b}{v}(q(k-1)x(t) + (n-k)y(t)) + \frac{1}{2}\frac{1-\alpha}{\alpha}\frac{b\gamma}{vk}(1 + p(k-1))z_i. \qquad (11)$$

For any selfish firm equation (3) takes the form $z_i^2 = \frac{b}{v}(kx(t) + q(n-k-1)y(t))$. But in the Nash equilibrium point $x(t+1) = x(t) = x_i(t) = x$, $y(t+1) = y(t) = y_j(t) = y$ at all $i$, $j$ and $t = 0,1,...$ . Then since (11) we get

$$(kx + (n-k)y)^2 = \frac{b}{v}(kx + q(n-k-1)y) =$$

$$= \frac{b}{v}(q(k-1)x + (n-k)y) + \frac{1}{2}\frac{1-\alpha}{\alpha}\frac{b\gamma}{vk}(1 + p(k-1))z_i. \qquad (12)$$

From second equation (12) we obtain the response function

$$\frac{x}{y} = \frac{p(n-k) + q(\alpha + (1-\alpha)\frac{n-k}{k})}{(2\alpha - 1)(1 + p(k-1))} = G \; .$$

To calculate the coordinates of a fixed point, we substitute the expression of $y$ through $x$ in the first equation (12), Q.E.D. .

In (10) by the data we get $x^* > 0, y^* > 0$. In view of the following proposition 3 it also ensures nonsingularity of a matrix (9) in proposition 1.

## 3. Desktop Application *Model* for Computing Experiments

During our research we developed desktop application *Model* to support the research process using computational experiments with dynamic systems. The main purpose of the application is to provide the best service for research cycle: hypothesis → experiment → hypothesis. It’s impossible to realize new idea with new device immediately, at once after it appearance for natural experiments. However here we can do it using application window with the appropriate tools. The results of new experiment give rise to new ideas, which we can check immediately using new windows and so on. Therefore intensive researches with multidimensional dynamical systems during this work have demanded efforts for computational speedup of the application. The goal of *Model* is the highest possible support for research process.

*Model* is a C# application created on the basis of the graphical interface of the System.Drawing and System.Windows.Forms C# system libraries. All calculations related to the model are localized in the *calc* method, which makes it easy to modify the equations of the model or move to other models.

*Model* application additionally uses *Open Maple* to work with differential equations and 3D graphs. *Open Maple* is access interface to Maple computational core from various programming languages: C#, Java, Visual Basic etc. In addition to the above standard namespaces is also used the System.Runtime.InteropServices namespace, which allow us to make links to the Maple dynamic linking core library - maplec.dll.

The following figure demonstrates the main application window which automatically appears when you open it.

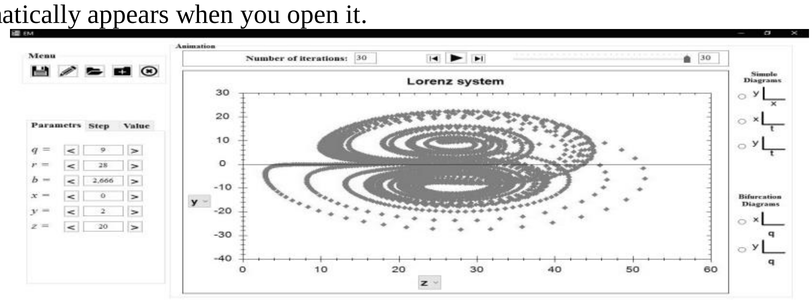


**Fig. 1.** Main window of the *Model* application

In the center of the window is located a two-dimensional projection of Lorentz system's attractor. In fig. 1 above in the left corner are the application menu buttons. From left to right: 1. *Save* button is used to save current model which is displayed on the screen with all the given parameters' values and settings under the chosen user name. 2. *Edit* button is used to modify the current model. 3. *Open* button demonstrates a list of saved models' names with the date of their last modification, which allows you to select and open a window of any of them. 4. *Add* button is served to define new models. 5. *Delete* button gives possibility to delete the current model (depicted on the screen) from the list.

The following fig. 2 shows the application window for market model of this paper.

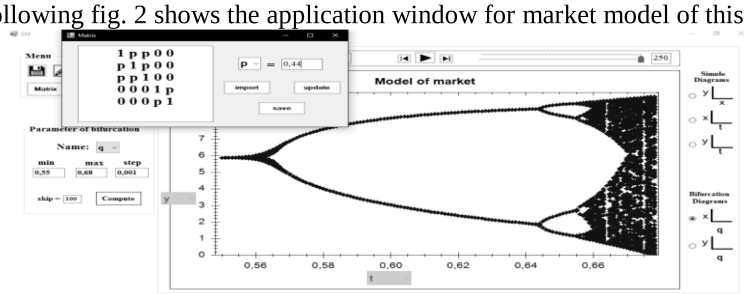


**Fig. 2.** *Model* application window for general view market model

On the right are 5 types of graphs, which are used most often; their examples are pointed out later in the paper. We can set model parameters and the initial values of the model trajectory using counters on the left. After these settings the graph of given model automatically appears in the center of the window. The number of iterations we can be set on the scroll bar above the graph. In the center of the window is also displayed the animation of the selected path when the button (near the scroll bar) is pressed.

When you click *Step* button on the left, you can set step of changing for a list of parameters. If you click *Value* button, you can obtain the table with coordinates of model trajectory for given iterations.

But the main tool to support computational investigations in *Model* application is easy modification of a current model after pressing of *Edit* button (fig. 3). Modification window is located over the current model window, which allows using both windows at the same time. After left click on the model equation in the field *The dynamical system* will move to the field *Equation*, where it can be changed. After pressing *Add* the modified equation will return back. Similar procedure can be done with parameters. We can also add new equations and parameters and delete the previous ones. In the field *System name* we can specify the name of the new model modification. After clicking *Save* button, new model falls into the saved list. If you click *Change*, the new modification will be saved under the name of the current model, which is deleted. When you click *Back*, the modification is temporarily suspended and we return to the current window. *View* button displays information about the model (equations, parameters and settings).

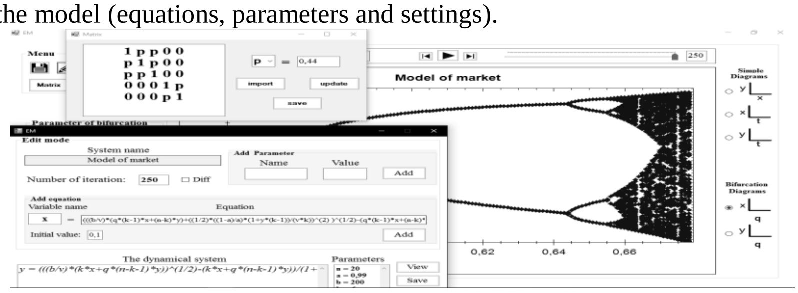


**Fig. 3.** *Model* application window for modifying the current model

# 4. Investigation of General View Market Model via Computing Experiments

## 4.1. Dependence of General View Market Model on Number of Firms

According to [18] with number of firms increase a market moves from stability to chaos. Whether so it for the model of this paper? Let in system (8) $k_1 = k_2 = 10$, $b = 200$, $v = 2$, $\alpha_1 = \alpha_2 = 0.99$, $\delta_{11} = \delta_{22} = \delta_{33} = 0.5$, $\delta_{12} = \delta_{21} = 0.12$, $\Theta_1 = \Theta_2 = 0.1$.

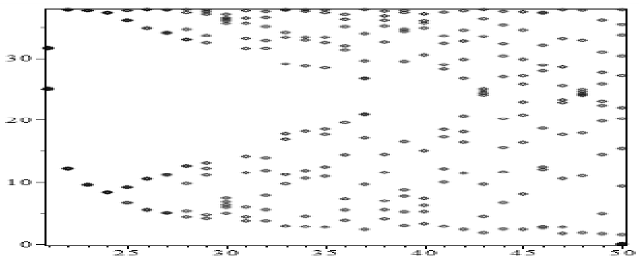


**Fig. 4.** The bifurcation diagram of dependence of quantity $x_1$ on $n$ .

Here the horizontal axis represents the number of firms $n$ from 20 to 50; the ordinate axis represents the quantity $x_1(t)$ of first reciprocator firm on attractor of the trajectory. The path has the equilibrium stable state at $n = 20$. However as we can see at $n = 21$ bifurcation occurred and instead of equilibrium point there is a stable cycle. There values of $x_1$ are approaching the point $x_1* \approx 40$ for even $t$ and the point $x_1* \approx 10$ for odd $t$. By doubling the lag between iterations only even or only odd iterations will be considered, and thus either point $x_1* \approx 40$, or $x_1* \approx 10$ respectively would be the equilibrium stable state. Stable cycle has four points for $n = 25$ (fig. 4). There was a new cycle doubling (flip) bifurcation. Calculations show that with parameter $n$ increase doubling bifurcations continue following Sharkovskii's order. At $n = 45$ there is a state of dynamic chaos (fig. 4).

Process of division of stable equilibrium on some directions will clear up, if during it we trace profit changes. *Model* tools allow us to demonstrate the dependence between reciprocator firm's profit $\pi$ and number of firms $n$ for the same parameter values that in bifurcation diagram 4 above.

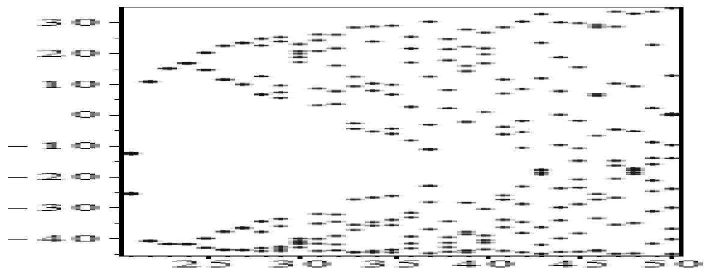


**Fig. 5.** The bifurcation diagram of dependence of profit $\pi$ on the number of firms $n$ .

It appears that the real choice here is unique and depends on quantity output. The smaller quantity output the bigger the firm's profit. Moreover, the profit for bigger output direction varies around zero and often converts into a loss. But quite unexpected is the effect well visible in a fig. 5: firm's profit in chaotic state is on average greater than in stable state. This example illustrates typical, many times investigated via *Model* behavior of dynamics of the general view market model with increasing number of firms.

Analysis of computing experiments for model (7) in [18] show, that such behavior arises provided that firms in the market are not identical, reciprocators and egoists are also presented enough there. How can we generalize such condition for the general view market?

Let in system (8) $\delta_{12} = \delta_{21} = 0.5$ instead of 0.12 above saving all other parameters. Then in (8) disappear difference between first and second types of reciprocators, they unite in one type. Such system has stable equilibrium at all $n$. By $\delta_{12} = \delta_{21} = 0.4$ the whole attractor is a cycle of an order 2 at all $n$. By $\delta_{12} = \delta_{21} = 0.2$ it is a cycle of an order 4 at all $n$. By $\delta_{12} = \delta_{21} = 0.14$ a state of dynamic chaos arises at $n = 140$. At $\delta_{12} = \delta_{21} = 0.12$ we return to fig. 4, where chaos arises by $n = 45$.

But the less value of $\delta_{12} = \delta_{21}$ the greater difference between types of reciprocators and so the market is more heterogeneous. All our computing experiments lead to the following conclusion. The more difference (segregation) between firms i.e. the more types of firms are in a market, the faster this market directs to complex dynamics and to chaos due to increase of firms' number.

### 4.2. The Crucial Factors which Ensure Stability in General View Market

Apparently the main assumption of the traditional neoclassical economics is the idea of automatic stabilization and market order due to increasing the number of independent firms and achievement of perfect competition. This is realization of Adam Smith's 'invisible hand' [19]. Then how stability is possible in real markets with the effects revealed in the previous section?

We found [17] that adaptive behavior is the main tool that ensures the stability of model (7). While increasing of number of firms directs a market to complex dynamics and finally to chaos the increase of adaptive expectations acts in an opposite direction. Due to increase of adaptive expectations predictability and stability of market becomes stronger; due to increase of naive expectations the market loses stability and chaos grows. Whether it is true for multidimensional model of this paper?

Let $k_1 = k_2 = 10$, $b = 200$, $v = 2$, $\alpha_1 = \alpha_2 = 0.99$, $\delta_{12} = \delta_{21} = 0.12$, $\Theta_1 = \Theta_2 = 0.1$, $\delta_{33} = 0.5$ as above. But now $n = 35$ and $q = \chi_{11} = 1 - \delta_{11} = \chi_{22} = 1 - \delta_{22}$ is a variable parameter of following bifurcation diagram. Here $q$ is the parameter of share in output of a market planned under naive expectations.

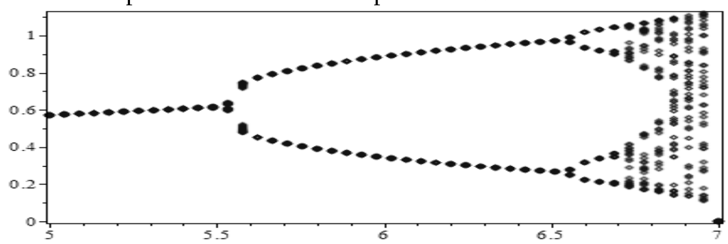


**Fig. 6.** The bifurcation diagram of dependence of quantity $x_1$ on $q$ in the system (8).

Here the ordinate axis represents the quantity $x_1(t)$ of first reciprocator firm on attractor of the trajectory; the horizontal axis represents the parameter value of $q$ multiplied by 10. This rescaling is done for the sake of clarity. In figure the same behavior that in [18]. And common sense prompts too, that increase of naive expectations conducts to chaos. However, in multidimensional model it is incorrectly to estimate a share of planning with naive expectations by the use of parameter $q = \chi_{11} = \chi_{22}$ of this example. Apparently we should estimate it by ratios of parameters $\delta_{ij}$ and $\chi_{ij}$ on all $i$ and $j$. Formal definition will be given in section 4.4.

Computing experiments and common sense also testify that in multidimensional systems it is incorrectly to estimate adaptation only by the use of a share of planning with naive expectations. Let's consider an example. Let $k_1 = k_2 = 10$, $b = 200$, $v = 2$, $\alpha_1 = \alpha_2 = 0.99$, $\delta_{11} = \delta_{22} = \delta_{33} = 0.5$, $\delta_{12} = \delta_{21} = 0.2$, $\Theta_1 = \Theta_2 = 0.1$. At such values of parameters all trajectories of dynamical system (8) are drawn to stable equilibrium at all $n$. Let's now move away values $\delta_{11}$ and $\delta_{22}$ from their average 0.5 on quantity $\Delta = \delta_{11} - 0.5 = 0.5 - \delta_{22}$. Other parameters we save unchanged. Then at $0 \le \Delta \le 0.2$ the attractor consists of stable cycles. At $\Delta = 0.2$ there are cycles of an order 3.

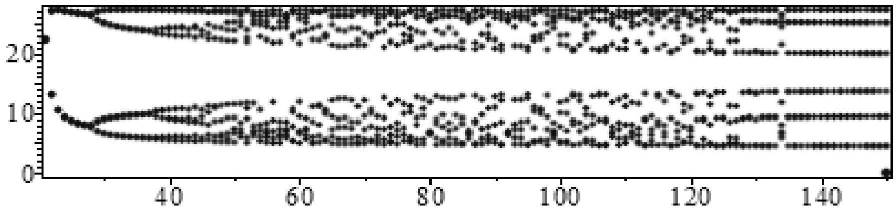


**Fig. 7.** The bifurcation diagram of dependence of quantity product $x_1$ on $n$ at $\Delta = 0.2$.

Such order of a cycle means that there has already been passed all Sharkovskii's order of conditions and there is a dynamic chaos at $\Delta > 0.2$. We observe the similar trends if average of values $\delta_{11}$ and $\delta_{22}$ move away from $\delta_{33}$ or if $\delta_{12}$ move away from $\delta_{21}$.

Numerous computing experiments and common sense testify that stability of the market critically depends on agreement of adaptive expectations of firms at planning. In particular, it depends on how close are all parameters $\delta_{ij}$ and respectively all $\chi_{ij}$. In addition we note that condition in the end of section 4.1 is only a special case of this condition: the more types of firms in a market the lower there level of the agreement of adaptive expectations.

### 4.3. The Stability Factor of Market in Chaotic State

This part reveals the factor that ensures the stability of the market in a complex and even chaotic dynamics. If any type of firms increases their profit more quickly than their rivals then these firms will survive and expand their type among all firms [20].

In model (5) the ratio of profit of firm $i$ from type $l$ at period $t$ $\pi_l(t) = (P(t) - v)x_i(t)$ to profit of firm $j$ from type $k$ $\pi_k(t) = (P(t) - v)x_j(t)$ at the same time period is:

$$\lambda_{lk}(t) = \frac{\pi_l(t)}{\pi_k(t)} = \frac{(P(t) - v)x_i(t)}{(P(t) - v)x_j(t)} = \frac{x_i(t)}{x_j(t)}.$$

This is the unexpected finding of our research [18] during computing experiments. In model (7) $\lambda_{lk}(t)$ is adiabatic invariant of a dynamical system, i.e. it is almost independent on $t$ at $t > 2$ for all acceptable values of parameters. Direct generalization of this fact on model (5) proves to be true by all already made computational researches. For example consider the phase curve that corresponds to trajectory with dynamic chaos in fig. 4.

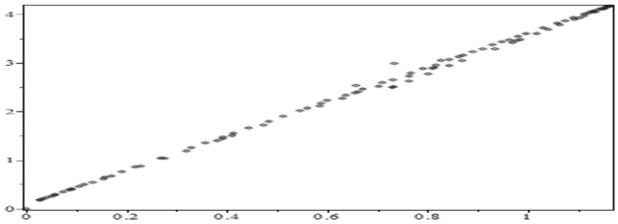

**Fig. 8.** Projection of phase curve of trajectory from fig. 4 at $n = 45$ to a plane $x_1x_3$.

The more chaotic dynamics, the more densely populated points on phase curve. But anyway it almost coincide with line segment, whose slope is equal to $\lambda_{lk}(t)$. We can suppose that rare small deviations from a straight line on fig. 8 are just technical failures at calculations. But look now on next fig. 9 with phase curve of trajectory of system (8) at parameters $n = 100$, $k_1 = k_2 = 10, b = 200$, $v = 2, \alpha_1 = \alpha_2 = 0.99, \delta_{11} = 0.68$, $\delta_{22} = 0.32, \delta_{33} = 0.5, \delta_{12} = \delta_{21} = 0.12, \Theta_1 = \Theta_2 = 0.1$.

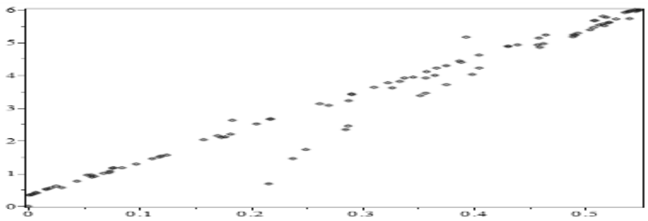

**Fig. 9.** Projection of phase curve with less level of agreement.

Here deviations from a straight line are already indisputable. The cause of difference from the previous example that here parameters $\delta_{11} = 0.68$ and $\delta_{22} = 0.32$ considerably deviate from their average $0.5 = \delta_{33}$. As it is noted in the previous section, it means reduction of level of agreement of adaptive expectations in the market, the key factor of stability in a market. All computing experiments show that if this level increases the value $\lambda_{lk}(t)$ comes nearer to a constant.

### 4.4. Universal Properties of General View Market Model

Let's formulate the formal statements which are clearing up derived results of computing researches. First of all let’s formalize the key concept of level of agreement in adaptive expectations in a market.

Let $x_{ik}^e(t+1)$ is quantity of firm $k$ expected by a firm $i$, $Q_i^e(t+1)=\sum_{k=1}^{n} x_{ik}^e(t+1)$ is prospective industry output of a market expected by a firm $i$ during next time period $t+1$. For firms $i$ and $j$ we put $\varepsilon_{ij}=\max_t \frac{\left|Q_i^e(t+1)-Q_j^e(t+1)\right|}{Q(t)}$, where $Q(t)$ is industry output of the market in period $t$. The value $\varepsilon_{ij}$ characterizes disagreement in adaptive expectations of firms $i$ and $j$. Value $\varepsilon=\max_{i,j}\varepsilon_{ij}$ we will call the level of disagreement in adaptive expectations in the market. Thus value $1-\varepsilon$ we will call the level of agreement in adaptive expectations in the market.

**Proposition 3**. The ratio of profits $\lambda_{lk}(t)$ is equal to a constant with accuracy $\pm 3\varepsilon$ at all $t>2$ for any fixed values of parameters of model (5).

Owing to this statement dynamics of a general view market model is stratified on dynamics of the local markets (7) from [17] with accuracy of the order $\varepsilon$. That is why all derived in [17], [18] and considered above properties of the local markets are generalized on the general view market of this paper. This fact explains universality of their properties. The formal reduction of following statements to results from [17], [18] is also based on this statement.

Let firm $i$ suggests that production quantities of its rival $j$ will be equal to $x_j^e(t+1)=\delta_{ij}(t)x_i(t+1)+\chi_{ij}(t)x_j(t)$ during next time period $t+1$, where $\delta_{ij}(t)\geq 0$ and $\chi_{ij}(t)\geq 0$, $i,j=1,...,n$. Then the value $\Lambda=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\chi_{ij}}{\chi_{ij}+\delta_{ij}}$ we will call the share of planning with naive expectations and the value $1-\Lambda$ we will call the share of planning with adaptive expectations in the market. Thus $\Lambda=0$ if in the market there are no naive expectations, and $\Lambda=1$ at total using naive expectations for planning.

**Proposition 4**. At $\Lambda=0$ the unique Nash equilibrium of proposition 1 is stable for all possible values of parameters of a general view market model (5).

**Proposition 5**. At $\Lambda=1$ the unique Nash equilibrium of proposition 1 is unstable for sufficiently large number of firms $n$ and all other acceptable values of parameters of model (5) if $\left|\frac{k_l}{n}\right|>3\varepsilon$ and $\left|\frac{k_l}{n}-\frac{3}{4}\right|>3\varepsilon$ for all types of firms $l=1,...,m$, where $\varepsilon$ is the level of disagreement in adaptive expectations in the market.

**Proposition 6**. In a general view market model (5) flip bifurcations (cycle doubling bifurcations) occur following all Sharkovskii's order and finally chaos state occur with an increase of $\Lambda$ from $0$ to 1.

**Proposition 7**. In a general view market model (5) flip bifurcations occur and finally chaos state occur with an increase of number of firms in the market provided sufficiently large $\Lambda<1$.

As model (5) is equivalent to a general view market model (3) in short-run period, so actually propositions 3 – 7 describe universal properties of general view markets, including real markets as particular case.

## 5. Conclusion

Thus we have synthesized the heterogeneous agent-based model of general view market according to new economics paradigm as intersection of dynamic system theory, mathematical programming and game theory.

During our investigation we developed and continuously improved a desktop application *Model* for support the research process using computing experiments. As a result of simulation experiments via *Model* application we have revealed the following universal properties of general view market, including real markets. They are derived by generalization and specification of the basic properties of model [17].

The crucial factors which ensure the market stability are the level of agreement in adaptive expectations and the share of planning with adaptive expectations in a market. If no any firm use naive expectations in the market there is unique Nash equilibrium which is stable for all acceptable values of parameters. The increase of naive expectations leads to stability loss, to flip bifurcations and finally to chaos in general view market.

The increase of number of firms also leads to stability loss, to bifurcations and finally to chaos in the general view market at appreciable naive expectations. It appears that really the choice of equilibrium at these bifurcations is unique.

We revealed that the profits ratio and quantity outputs ratio of firms remains almost unchanged in short-run period in general view markets. It seems an important stability factor of many important real markets for which chaotic dynamics is usual.

In the further researches we plan to trace demonstrations of these universal properties on examples of real markets in details.

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