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On the Estimation of Accuracy and Stability of 3D Face Modeling Using a Pair of Stereo Cameras

Duy T. Nguyen[*], **Vyacheslav M. Khachumov**[*], **Mikhail V. Khachumov**[*], **Soltan I. Salpagarov**[*], **Konstantin S. Yakovlev**[†]

[*] *Department of Information Technologies*
Peoples' Friendship University of Russia
6 Miklukho-Maklaya str., Moscow, 117198, Russian Federation
[†] *Federal Research Centre "Computer Science and Control" of RAS*
44/2 Vavilova str., Moscow,119333, Russian Federation

Email: nguyenduythanh1410@gmail.com, khachumov_vm@rudn.university, khachumov_mv@rudn.university, salpagarov_si@rudn.university, yakovlev@isa.ru

The study deals with the problem of estimating the accuracy and stability of 3D face models obtained by a stereo pair. The problem of the conditionality of the fundamental matrix, which is a mathematical stereo pair model, is considered. We prove that small changes of stereo camera parameters result in small changes in the solution of the problem of reconstructing three-dimensional coordinates. Several types of three-dimensional reconstruction optimization problems that are based on quality criteria are formulated. The paper also considers the issues of determining an object orientation in a three-dimensional space by position lines. A 3D image coding system utilizing invariant moments is proposed and the theoretical sensitivity of 3D invariants to geometric distortions is investigated. These results are used to obtain scaling invariants. Designing and studying such models is the important step to solve the analysis problem and determine the proximity of images, which is therefore necessary for their clustering and recognition.

Key words and phrases: face recognition, face models, invariant moments, image, reconstruction.

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1. Introduction

There are now many different methods and algorithms for face recognition related both to the identification of local features (lips, nose, facial contours or profile) and methods aimed at analyzing the entire image as a whole [1–3]. Neural networks, Markov chains, elastic graphs, a wavelet analysis, a support vector method and other tools are used as classifiers [4–7]. Almost all approaches have insufficient accuracy if images contain brightness noises, color distortions or when objects move on video sequences. Experiments show that 2D models have limited application, because it is difficult to use them for face recognition if there are different head angles, natural facial expressions, grimaces and other disturbances. Thereby, more and more attention is paid to 3D-models obtained with the use of high-resolution cameras, which allow increasing the accuracy and completeness of recognition [8–10]. 3D models can be adapted to existing images to achieve the best similarity. A 3D image is a piecewise continuous three-variable function $f(x, y, z)$ defined on a compact support $D \subset R \times R \times R$ and having a finite nonzero integral. An example of such a function is the brightness function also known as a halftone image. The digital image typically results from the discretization of the continuous brightness function $f(x, y, z)$ and is stored as a three-dimensional array $I(i, j, k)$, where $i = 0, 1, \ldots, N_x - 1, j = 0, 1, \ldots, N_y - 1$ and $k = 0, 1, \ldots, N_z - 1$. Each element of this array is a pixel with an intensity ranging from 0 to $L - 1$. The L value is typically a power of two (for example, 64, 256) and is called the image depth.

2. The problem of designing 3D face models

Let us consider the issue of designing a 3D model by reconstructing an image and the problem of the conditionality of the fundamental matrix, which is a mathematical stereopair model. The three-dimensional reconstruction reduces to solving the following problem

$$A \cdot x = b, \tag{1}$$

where:

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, A = \begin{bmatrix} T_{11}^1 - T_{14}^1 x^{1*} & T_{21}^1 - T_{24}^1 x^{1*} & T_{31}^1 - T_{34}^1 x^{1*} \\ T_{12}^1 - T_{14}^1 y^{1*} & T_{22}^1 - T_{24}^1 y^{1*} & T_{32}^1 - T_{34}^1 y^{1*} \\ T_{11}^2 - T_{14}^2 x^{2*} & T_{21}^2 - T_{24}^2 x^{2*} & T_{31}^2 - T_{34}^2 x^{2*} \\ T_{12}^2 - T_{14}^2 y^{2*} & T_{22}^2 - T_{24}^2 y^{2*} & T_{32}^2 - T_{34}^2 y^{2*} \end{bmatrix}, b = \begin{bmatrix} T_{44}^1 x^{1*} - T_{41}^1 \\ T_{44}^1 y^{1*} - T_{42}^1 \\ T_{44}^2 x^{2*} - T_{41}^2 \\ T_{44}^2 y^{2*} - T_{42}^2 \end{bmatrix}.$$

Herein $T_{i,j}^k$, is a coefficient of the mathematical k camera model represented by a 4×4 matrix. We call this matrix fundamental. It is necessary to calculate the condition number to estimate the conditionality of the matrix. The maximum and minimum changes in Ax can be set with the following numbers:

$$Q = \max_{x \neq 0} \frac{\left\| Ax \right\|}{\left\| x \right\|}, \qquad q = \min_{x \neq 0} \frac{\left\| Ax \right\|}{\left\| x \right\|},$$

where:

$$\left\| x \right\|_l = \sum_{i=1}^{n} \left| x_i \right|.$$

Matrix condition number A is defined as $cond(A) = \frac{Q}{q}$ (**cond stands for** conditioned) and shows how close the square matrix is to degeneracy. If matrix A is almost degenerate, then we can expect small changes in A and b to cause significant changes in x.

Let us consider system $A(x+\Delta x) = b+\Delta b$. With a relative change in the right-hand side ($\frac{\|\Delta b\|}{\|b\|}$), relative error $\frac{\|\Delta x\|}{\|x\|}$ can be $cond(A)\frac{\|\Delta b\|}{\|b\|}$. If $q = 0$, then $cond(A) = +\infty$, that is a rank-deficient (degenerate) matrix. The bigger $cond(A)$, the closer matrix A to degeneracy and vice versa – the closer the matrix to the identity matrix, the closer the $cond(A)$ value to 1 and the matrix is far from degeneracy. The study of conditionality is an important link in determining the stability of the solution to the problem of reconstructing 3D information. From this point on, let us assume that the matrix is well-conditioned.

Let fundamental matrix A be well-conditioned.

Proposition 1. If coefficients of matrix A and absolute term column b in the equation (1) are changed for small quantities εA_1 and εb_1, then the reconstruction problem solution obtained by using a least-squares method will change for small value

$$\Delta = O(\varepsilon) = \left\|(A_0 + \varepsilon A_1)^T (A_0 + \varepsilon A_1)\right\| - \left\|A_0^T A_0\right\|.$$

Let us show that if coefficients of matrix A and absolute term column b are changed for small quantities, then the solution obtained by using a least-squares method will change insignificantly. To do this, let us introduce disturbing matrix A_1, disturbing vector b_1 and require singular numbers (i.e. square roots of the eigenvalues) of matrix A_1 to be bounded from above by some constant. In addition to the system (1), let us also consider the system

$$(A_0 + \varepsilon A_1)x_\varepsilon + b_0 + \varepsilon b_1,$$

where ε is a small parameter. Its solution is given by

$$x_\varepsilon = ((A_0^T + \varepsilon A_1^T)(A_0 + \varepsilon A_1))^{-1}(A_0^T + \varepsilon A_1^T)(b_0 + \varepsilon b_1).$$

Let us consider the expression

$$J = (A_0^T + \varepsilon A_1^T)(A_0 + \varepsilon A_1)\Delta,$$

where $\Delta = |x_\varepsilon - x_0|$ is a deviation. Value J can be written as

$$\begin{aligned} J = (A_0^T + \varepsilon A_1^T)(b_0 + \varepsilon b_1) - (A_0^T + \varepsilon A_1^T)(A_0 + \varepsilon A_1)(A_0^T A_0)^{-1} A_0^T b_0 = \\ = A_0^T b_0 + \varepsilon(A_0^T b_1 + A_1^T b_0) + \varepsilon^2 A_1^T b_1 - A_0^T A_0 (A_0^T A_0)^{-1} A_0^T b_0 - \\ -\varepsilon A_1^T A_0 (A_0^T A_0)^{-1} A_0^T b_0 - \varepsilon A_0^T A_1 (A_0^T A_0)^{-1} A_0^T b_0 - \varepsilon^2 A_1^T A_1 (A_0^T A_0)^{-1} A_0^T b_0 = \\ = \varepsilon A_1^T (E - A_0 (A_0^T A_0)^{-1} A_0) b_0 + \varepsilon A_0^T (b_1 - A_1 (A_0^T A_0)^{-1} A_0^T b_0) + O(\varepsilon^2). \end{aligned}$$

Herein, E is an identity matrix. We shall now notice that matrix $A_0^T A_0$ is well determined. Consequently, matrix $(A_0^T + \varepsilon A_1^T)(A_0 + \varepsilon A_1)$ is also well determined at small values of ε, with

$$\Delta = O(\varepsilon) = \left\|(A_0 + \varepsilon A_1)^T (A_0 + \varepsilon A_1)\right\| - \left\|A_0^T A_0\right\|.$$

Thus, small changes in the parameters of the stereo camera (provided that the fundamental matrix is well-conditioned) lead to small changes in the solution to the problem of reconstructing three-dimensional coordinates. Knowledge of the transforming characteristics of the vision system allows forming a set of equations and reconstructing a three-dimensional point with a certain degree of accuracy when pairs of pixels corresponding to one point on the object surface are recognized on a stereo image. We can suggest different criteria for finding recovered point $N = (x, y, z)$ being an approximation of the problem (1), as it can be seen in Table 1.

Table 1

Types of three-dimensional reconstruction optimization problems

Problem type	Problem setting	Solution
Minimizing root-mean-square deviation N from the solution of system equations (1)	$A_0 x_0 = b_0$	$x = \left(A^T A\right)^{-1} A^T b$
Minimizing the largest deviation	Minimizing the largest deviation from the solution of the system $\begin{cases} a_{x1}x + a_{y1}y + a_{z1}z + b_1 = 0 \\ a_{x2}x + a_{y2}y + a_{z2}z + b_2 = 0 \\ a_{x3}x + a_{y3}y + a_{z3}z + b_3 = 0 \\ a_{x4}x + a_{y4}y + a_{z4}z + b_4 = 0 \end{cases}$ (2)	$\begin{cases} a_{x1}x + a_{y1}y + a_{z1}z + b_1 = \xi \\ a_{x2}x + a_{y2}y + a_{z2}z + b_2 = \xi \\ a_{x3}x + a_{y3}y + a_{z3}z + b_3 = \xi \\ a_{x4}x + a_{y4}y + a_{z4}z + b_4 = \xi \end{cases}$ where ξ is a deviation from the solution of the system.
Minimizing root-mean-square deviation N from the tetrahedron faces	$\delta^2 = \sum_i \frac{(a_{xi}x + a_{yi}y + a_{zi}z + b_i)^2}{a_{xi}^2 + a_{yi}^2 + a_{zi}^2} \to \min\limits_{x,y,z}$	$\begin{cases} x\sum_i \frac{a_{xi}^2}{\Delta_i^2} + y\sum_i \frac{a_{xi}a_{yi}}{\Delta_i^2} + \\ +z\sum_i \frac{a_{xi}a_{zi}}{\Delta_i^2} + \sum_i \frac{a_{xi}b_i}{\Delta_i^2} = 0 \\ x\sum_i \frac{a_{xi}a_{yi}}{\Delta_i^2} + y\sum_i \frac{a_{yi}^2}{\Delta_i^2} + \\ +z\sum_i \frac{a_{yi}a_{zi}}{\Delta_i^2} + \sum_i \frac{a_{yi}b_i}{\Delta_i^2} = 0 \\ x\sum_i \frac{a_{xi}a_{zi}}{\Delta_i^2} + y\sum_i \frac{a_{yi}a_{zi}}{\Delta_i^2} + \\ +z\sum_i \frac{a_{zi}^2}{\Delta_i^2} + \sum_i \frac{a_{zi}b_i}{\Delta_i^2} = 0 \end{cases}$ where $\Delta_i^2 = \Delta_{xi}^2 + \Delta_{yi}^2 + \Delta_{zi}^2$, $i \in \{1, 2, 3, 4\}$.
Minimizing the root-mean-square deviation of one coordinate of point N	For coordinate z: $\rho_z^2 = \sum_i (z - z_i(x, y))^2 \to \min\limits_{x_0}$ $z_i(x, y)$ is the lowest value calculated for each of the system equations (2)	$\begin{cases} x\sum_i \frac{a_{xi}^2}{a_{zi}^2} + y\sum_i \frac{a_{xi}a_{yi}}{a_{zi}^2} + \\ +z\sum_i \frac{a_{xi}}{a_{zi}} + \sum_i \frac{a_{xi}b_i}{a_{zi}^2} = 0 \\ x\sum_i \frac{a_{xi}a_{yi}}{a_{zi}^2} + y\sum_i \frac{a_{yi}^2}{a_{zi}^2} + \\ +z\sum_i \frac{a_{yi}}{a_{zi}} + \sum_i \frac{a_{yi}b_i}{a_{zi}^2} = 0 \\ x\sum_i \frac{a_{xi}}{a_{zi}} + y\sum_i \frac{a_{yi}}{a_{zi}} + 4z + \\ \sum_i \frac{b_i}{a_{zi}} = 0 \end{cases}$

3. Experimental designing and the 3D model orientation determinantion

Let the three-dimensional object be given by a set of N points with coordinates (x_k, y_k, z_k), $k = 1, \dots, N$, to which weights f_k corresponding, for example, to brightness are assigned. A 3D human face model is exemplified in Figure 1

Figure 1. Variants of the face image orientation

Various affine transformations generally performed in homogeneous coordinates are used to manipulate the model. The weights of points affect the object center of mass, hence changing them may lead to an inordinary displacement trajectory of the center of mass and a change in the orientation of the object. It is necessary to draw a straight spatial line to determine the position and orientation of the 3D object [11–13]. Let the position line go through the centroid of the object's point system:

$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{\sum_{i=1}^{N} f_i x_i}{\sum_{i=1}^{N} f_i}, \frac{\sum_{i=1}^{N} f_i y_i}{\sum_{i=1}^{N} f_i}, \frac{\sum_{i=1}^{N} f_i z_i}{\sum_{i=1}^{N} f_i} \right),$$

where $(\bar{x}, \bar{y}, \bar{z})$ are the object centroid coordinates. The position line forms angles α,β,γ with reference axes OX, OY, OZ and its equation can be written as $\frac{x-\bar{x}}{l} = \frac{y-\bar{y}}{m} = \frac{z-\bar{z}}{n}$, where: l, m, n are the angular coefficients of a straight line in space to be determined as a result of the problem solution; x, y, z are the coordinates of an arbitrary object point. The distance from the k object point to the desired line is calculated by

$$d_k^2 = \frac{[(x_k - \bar{x})m - (y_k - \bar{y})l]^2 + [(y_k - \bar{y})n - (z_k - \bar{z})m]^2 + [(z_k - \bar{z})l - (x_k - \bar{x})n]^2}{l^2 + m^2 + n^2}.$$

We introduce

$$A = \sum_{i=1}^{N} (x_i - \bar{x})^2 f_i, B = \sum_{i=1}^{N} (y_i - \bar{y})^2 f_i,$$
$$C = \sum_{i=1}^{N} (z_i - \bar{z})^2 f_i, D = \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y}) f_i,$$
$$E = \sum_{i=1}^{N} (x_i - \bar{x})(z_i - \bar{z}) f_i, F = \sum_{i=1}^{N} (y_i - \bar{y})(z_i - \bar{z}) f_i.$$

Let A, ..., F be called moments of inertia and be subsequently used as constant coefficients.To solve the problem, we have to find partial derivatives of function S of variables l, m, n and equate them to zero. After the necessary transformations we get the following system:

$$\begin{cases} ((B + C)l - Dm - Cn)m = ((C + A)m - Dl - Fn)l, \\ ((C + A)m - Dl - Fn)n = ((A + B)n - El - Fm)m, \\ ((A + B)n - El - Fm)l = ((B + C)l - Dm - Cn)n. \end{cases}$$

These equations can be rewritten as follows:

$$\begin{vmatrix} i & j & k \\ l & m & n \\ Al+Dm+En & Dl+Bm+Fn & El+Fm+Cn \end{vmatrix} =$$
$$= \begin{pmatrix} l \\ m \\ n \end{pmatrix} \times \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix} \cdot \begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0.$$

Herein,$(\times), (\cdot)$ are the signs of vector and scalar products. Thus, the system can be rewritten as

$$w \times Iw = 0, |w| = 1, \quad (3)$$

where $w = \begin{pmatrix} l & m & n \end{pmatrix}^T, I = \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix}.$

Proposition 2. The system (3) has a solution if and only if $Iw = \lambda w$, where λ is some number.

The area of the parallelogram spanned by vectors w and Iw is equal to zero if and only if these vectors are collinear. Equation $Iw = \lambda w$ has a nonnegative solution if and only if λ is an eigenvalue of matrix I. The matrix of the operator is symmetric and well determined; consequently, its eigenvalues are real [14]. The eigenvectors are orthogonal and correspond to the directing vectors of the ellipsoid determining the object orientation in space. The lengths of the ellipsoid axes correspond to the eigenvalues. Moreover, the maximum characteristic number corresponds to the desired direction of the position line. The vector corresponding to the second largest eigenvalue determines the object's rotation direction; the third eigenvector determines the rotation by an angle around the main axis.

4. 3D image coding utilizing invariant moments

Let $\rho(x, y, z)$ be the function describing the brightness value of points with coordinates (x, y, z) in a 3D space. It is required to build moments invariant to the group of affine transformations for correct correlation of images [15, 16]. For a discrete case (a digital image), the moments about mean can be calculated as follows:

$$\mu_{lmn} = \sum_X \sum_Y \sum_Z (x - \bar{x})^l (y - \bar{y})^m (z - \bar{z})^n \rho(x, y, z),$$

where X, Y, Z is an image pixel coordinate determination area; $(\bar{x},\bar{y},\bar{z})$ is the object's centroid. In accordance with the analysis carried out by using different sources, the following moments were selected [17–20]:

$$I_1 = \mu_{200} + \mu_{020} + \mu_{002},$$
$$I_2 = \mu_{200}\mu_{020} + \mu_{200}\mu_{002} + \mu_{020}\mu_{002} - \mu_{101}^2 - \mu_{110}^2 - \mu_{011}^2,$$
$$I_3 = \mu_{200}\mu_{020}\mu_{002} - \mu_{002}\mu_{110}^2 - \mu_{020}\mu_{101}^2 - \mu_{200}\mu_{011}^2 +$$
$$+2\mu_{110}\mu_{101}\mu_{011} - \mu_{011}^2 - \mu_{101}^2,$$

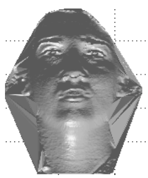
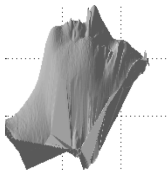
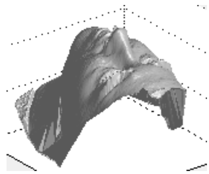
$$F_1 = \mu_{003}^2 + 6\mu_{012}^2 + 6\mu_{021}^2 + 6\mu_{030}^2 + 6\mu_{102}^2 + 15\mu_{111}^2 - 3\mu_{102}^2\mu_{120} + 6\mu_{120}^2 -$$
$$-3\mu_{021}\mu_{201} + 6\mu_{201}^2 - 3\mu_{003}(\mu_{021} + \mu_{201}) - 3\mu_{030}\mu_{210} + 6\mu_{210}^2 -$$
$$-3\mu_{012}(\mu_{030} + \mu_{210}) - 3\mu_{102}\mu_{300} - 3\mu_{120}\mu_{300} + \mu_{300}^2,$$
$$F_2 = \mu_{200}^2 + \mu_{020}^2 + \mu_{002}^2 + 2\mu_{110}^2 + 2\mu_{101}^2 + 2\mu_{011}^2,$$
$$F_3 = \mu_{200}^3 + 3\mu_{200}\mu_{110}^2 + 3\mu_{200}\mu_{101}^2 + 3\mu_{110}^2 + 3\mu_{101}^2\mu_{020} + 3\mu_{101}^2\mu_{002} +$$
$$+\mu_{020}^3 + 3\mu_{020}\mu_{011}^2 + 3\mu_{011}^2\mu_{002} + \mu_{002}^3 + 6\mu_{110}\mu_{101}\mu_{011},$$
$$F_4 = \mu_{300}^2 + \mu_{030}^2 + \mu_{003}^2 + 3\mu_{210}^2 + 3\mu_{201}^2 + 3\mu_{120}^2 + 3\mu_{102}^2 + 3\mu_{021}^2 + 3\mu_{012}^2 + 6\mu_{111}^2,$$
$$F_5 = \mu_{300}^2 + 2\mu_{300}\mu_{120} + 2\mu_{300}\mu_{102} + 2\mu_{210}\mu_{030} + 2\mu_{201}\mu_{003} + \mu_{030}^2 +$$
$$+2\mu_{030}\mu_{012} + 2\mu_{021}\mu_{003} + \mu_{003}^2 + \mu_{210}^2 + 2\mu_{210}\mu_{012} + \mu_{201}^2 +$$
$$+2\mu_{201}\mu_{021} + \mu_{120}^2 + 2\mu_{120}\mu_{102} + \mu_{102}^2 + \mu_{021}^2 + \mu_{012}^2.$$

Proposition 3. Moments $I_1, \ldots, F_5$ are 3D rotation and translation invariants.

The established limited set of 3D invariants can be directly applied to face recognition. Table 2 shows the calculated values of the selected group of invariants for three positions of the chosen mathematical 3D face surface model.

Table 2

An example of application of 3D invariants: rotation, shift

	Variants of 3D face image orientation (shifts and rotations)		
Moment			
I_1	1	1	1
I_2	0.222	0.222	0.222
I_3	0.013	0.013	0.013
F_1	6.255e-06	6.255e-06	6.255e-06
F_2	0.555	0.555	0.555
F_3	0.371	0.371	0.371
F_4	5.525e-06	5.525e-06	5.525e-06
F_5	5.038e-06	5.038-06	5.038e-06

The sensitivity of 3D invariants to linear image distortions is established. An evaluation of the theoretical sensitivity of 3D invariants to geometric distortions is presented in Table 3

Table 3

The sensitivity of 3D invariants

Moment	I_1	I_2	I_3	F_1	F_2	F_3	F_4	F_5
Sensitivity	δ^2	δ^4	δ^6	δ^6	δ^4	δ^6	δ^6	δ^6

The results can be used to obtain invariants for scaling. To do this, we can use: $\delta = \sqrt{\mu_{200} + \mu_{020} + \mu_{002}}$ and perform the valuation by dividing moments $I_1, ..., F_5$ by the corresponding values of the coefficients from Table 3.

Proposition 4. Normalized moments $I_1, ..., F_5$ are 3D invariants of the operations of rotate, tranlate and scale.
Table 4 exemplifies the sensitivity values of the invariants and the scaling coefficients for the chosen 3D face model.

Table 4

An experimental study of δ as the scaling facto

Scaling coefficient	Pre-scaling δ	After-scaling δ	δratio
2	10099.4	20198.8	2
3	10099.4	30298.2	3
0.5	10099.4	5049.69	0.5
0.25	10099.4	2524.85	0.25

All the calculations were carried out by using the software of the Matlab modeling system. The study of the moments' properties shows that it is possible to solve human face recognition problems utilizing their geometric invariants.

5. Conclusions

Stereo pair-based evaluations of the stability of three-dimensional image reconstruction against fluctuations are obtained provided that the fundamental matrix is initially well conditioned. The statements of various optimization problems of three-dimensional reconstruction based on quality criteria are given. A system of 3D invariants is defined and their stability against image fluctuations is investigated. The developed algorithms are expedient for using as a part of systems searching for faces from photos.

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