Computer Modelling of Nonisothermal Transfer of Moisture during Drying of Wood with the Use of the Computing Environment Mathcad

Alexander G. Gorokhovsky
Ural State Forestry Engineering University
620100, Russia, Ekaterinburg,
goralegr@yandex.ru

Elena E. Shishkina
Ural State Forestry Engineering University
620100, Russia, Ekaterinburg,
elenashishkina@yandex.ru

Sergey V. Zalesov
Ural State Forestry Engineering University
620100, Russia, Ekaterinburg,
Zalesov@usfeu.ru

Victoria V. Savina
Ural State Forestry Engineering University
620100, Russia, Ekaterinburg,
vik_savina@bk.ru

Abstract

This study explores the features of process of drying internal heat and mass transfer in wood in the conditions of essentially nonisothermal transfer of moisture. The process of capillary conductivity of wood is analyzed on the model of colloidal capillary-porous solid. The density of moisture flows caused by the phenomenon of thermal and hydraulic conductivity is determined. Previously experimentally established fact of influence on the process of moisture removal of the current moisture content of wood is analytically confirmed. Dependences of the phase transition criterion taking into account the direction of the temperature gradient and wood moisture are analytically obtained. Recommendations are given to determine the thermogradient coefficient taking into account the asymmetry of moisture flows depending on the direction of the temperature gradient, and also for the correction in the entry form of the equation of moisture transfer with the influence of the effect of thermal and hydraulic conductivity.

Introduction

To describe the transfer of heat and mass of matter is usually used a generalized system of differential equations of transfer [1].

For the simplest case when the total pressure gradients are absent (heat and mass transfer processes in low-temperature convective drying) A. V. Lykov [2] and M. S. Smirnov [3] give the following system of differential equations:

$$\frac{\partial t}{\partial \tau} = a\nabla^2 \mathbf{t} + \frac{\varepsilon\rho}{c}\frac{\partial u}{\partial \tau}, \tag{1}$$

$$\frac{\partial u}{\partial \tau} = a_{м}\nabla^2 u + a_{м}\delta\nabla^2 t. \tag{2}$$

For an unbounded plate, the initial and boundary conditions of the III kind have the form:

$$\mathbf{t}(\mathbf{x_0}, \mathbf{0}) = \mathbf{f}(\mathbf{x}), \tag{3}$$

$$\mathbf{u}(\mathbf{x}, \mathbf{0}) = \boldsymbol{\varphi}(\mathbf{x}), \tag{4}$$

$$-\boldsymbol{\lambda}\frac{\partial t(R,\tau)}{\partial x} + \boldsymbol{\alpha}[\mathbf{t_c} - \mathbf{t}(\mathbf{R},\boldsymbol{\tau})] - (\mathbf{1} - \boldsymbol{\varepsilon})\boldsymbol{\rho}\boldsymbol{\alpha}_{\mathbf{m}}[\mathbf{u}(\mathbf{R},\boldsymbol{\tau}) - \mathbf{u_p}] = \mathbf{0} \tag{5}$$

$$\mathbf{a_m}\frac{\partial \mathbf{u(R,\tau)}}{\partial \mathbf{x}} + \mathbf{a_m}\boldsymbol{\delta}\frac{\partial \mathbf{t(R,\tau)}}{\partial \mathbf{x}} + \mathbf{a_m}[\mathbf{u(R,\tau)} - \mathbf{u_p}] = \mathbf{0} \tag{6}$$

Symmetry condition:

$$\frac{\partial \mathbf{t(0,\tau)}}{\partial \mathbf{x}} = \frac{\partial \mathbf{u(0,\tau)}}{\partial \mathbf{x}} = \mathbf{0} \tag{7}$$

In criterion form of record [1] in (1) and (2) there are three dimensionless criterions: Lu (Lykov'), Ko (Kossovich') and Pn (Posnov'), which, for example, N.I. Gamayunov [4] calls the criteria of non-isothermal transfer. On closer inspection (1) and (2), it is obvious that in non-isothermal moisture exchange, the latter is significantly influenced not only by the thermogradient coefficient δ, but also by the phase transformation criterion ε. It is especially necessary to take into account when calculating the drying parameters based on using the phenomenon of thermal and hydraulic conductivity [1, 5].

In non-isothermal conditions, in the presence of heat and mass transfer potentials, heat and mass transfer in gas mixtures is caused by thermodynamic driving forces. In gas blends moisture transfer occurs under the influence of temperature gradient (thermal diffusion of moisture) [1]. In this case, there is an additional flow of moisture [2]:

$$q_t^{'} = -a_m \delta \nabla t, \tag{8}$$

δ - the coefficient of thermal diffusion, $^0K^{-1}$ (sometimes called the coefficient of Soré);

∇t – the temperature gradient, ^0K.

The presence of a temperature gradient causes a diffusion transfer of vapor under the influence of the gradient of its partial pressure in wet solids in addition to the Soré effect. In addition, the movement of the liquid in the capillary-porous solid in the direction of the heat flow can be caused by the presence of "pinched" air. When the temperature rises, the pressure of the "pinched" air increases and the air bubbles expand. As a result, the liquid in the capillary pore moves in the direction of the heat flow. Therefore, at P = const the total moisture flow is equal to:

$$q^{'} = -a_m \rho_0 (\nabla u + \delta \nabla t). \tag{9}$$

The first member of equation (9) shows the isothermal mass transfer, the second is the process of thermal and hydraulic conductivity.

From the theory of drying it is known that such a phenomenon as thermal and hydraulic conductivity [5] can create an additional flow of moisture, if the temperature gradient is negative value, and thus accelerate the process of moisture removal (drying). But if the temperature gradient is positive value, thermal and hydraulic conductivity can significantly complicate moisture removal. Thus it is absolutely obvious that such inhibiting effect can extend only to process of removal of a liquid moisture (up to its full stop). At the same time, the molecular and molar transport of vaporous moisture inside the capillary-porous solid is not extended due to the different physical nature of these phenomena [6 – 8].

The inhibiting effect of thermal and hydraulic conductivity (non-isothermal flow) can reduce and even stop the liquid flow that has arisen due to the sufficient amount of the jamming pressure, but it can not make it negative. This is due to the following objective phenomena [9, 10]. The motion of the wetting fluid in a single through cylindrical capillary under the action of surface tension forces under laminar regime is determined by the equation [10].

$$\frac{d^2 l}{dt^2} + \frac{1}{l}\left(\frac{dl}{dt}\right)^2 + \frac{8\eta dl}{r^2 \rho_ж dt} + g \cdot \sin\varphi - \frac{2\sigma\cos\theta}{r\rho_ж l} = 0 \tag{10}$$

In the linear approximation, ignoring in equation (10) the first two terms due to their smallness, we get for the horizontal capillary (φ = 0)

$$\frac{dl}{dt} = \frac{2\sigma\cos\theta r}{8\eta l}, \tag{11}$$

for a vertical capillary (φ = 90^0)

$$\frac{dl}{dt} = \frac{r^2}{8\eta l}\left(\frac{2\sigma\cos\theta}{r} - \rho_ж g l\right), \tag{12}$$

l - the length of the liquid column in the capillary;

t – the driving time;

$\rho_ж$ – the liquid density;

r - the radius of the capillary;

φ – the angle of the capillary;

η – the viscosity;

σ – the surface tension.

Equations of fluid motion (10 – 12) do not take into account the effects of clamped air [5]. The appearance of air bubbles when fluid flow in wood a significant role is played by the Jamen' effect (figure 1). It consists in the fact that with a slow filling of capillaries, the formed closed air inclusions cause a sharp increase in the hydrodynamic resistance of the medium. Such a picture is observed at movement of the aerated liquid, for example oil, through the porous medium [11]. Classical works of academician E. F. Votchal [12] were devoted to the study of the effect of Jamen on the movement of fluid through the vessels of living wood, unfortunately not received in the future the necessary development.

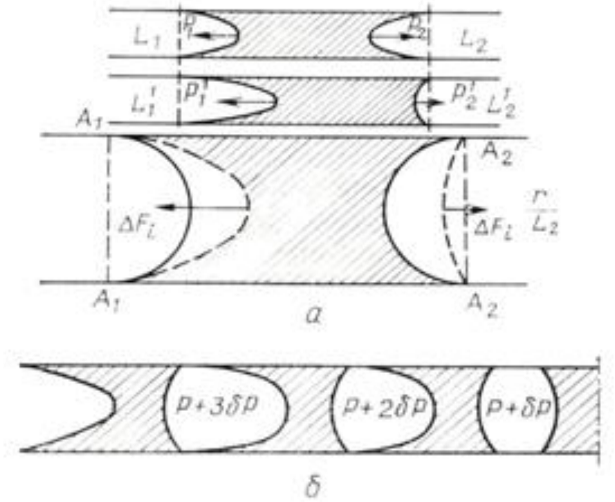

Figure 1: The Effect of Jamen
(a – the meniscuses of the capillary; б – rosaries of Jamen)

In essence, The effect of Jamen is an integral expression of the influence of internal boundary conditions on the motion of a liquid through a complex capillary-porous medium.
We can describe the filtration of a liquid as it moves through a porous medium by Darcy's law:

$$\bar{u} = \frac{c}{\eta} \cdot \frac{dP}{dx} \tag{13}$$

or when it reaches the capillary Poiseuille's formula:

$$\bar{u} = \frac{r^2}{8\eta} \cdot \frac{dP}{dx} \tag{14}$$

c – the permeability coefficient;
$\frac{dP}{dx}$ – the pressure gradient.
At a sufficiently stable temperature, the viscosity coefficient can not change its value depending on the structure of the medium. The change in the filtration rate of the liquid $\bar{u}$ can be attributed to the appearance of a foreign phase – gas bubbles, causing a change in the internal boundary conditions of the medium. This is most clearly seen from the law of Darcy: at constant η and $\frac{dP}{dx}$ [11] there is a decrease in u by reducing the effective value of the permeability coefficient.
Poiseuille's formula in its standard form does not apply to the case of non-continuous fluid flow and the discontinuity can be represented as [9]

$$\bar{u} = \frac{r^2}{8\eta} \cdot \frac{dP}{dx} \cdot \frac{1}{1+\alpha}, \tag{15}$$

where α – the correction factor depending on the number of gas bubbles, the coefficient of surface tension in relation to the material of the capillary walls.
It should be noted that formally reducing $\bar{u}$ can also be attributed by increasing the effective value of the viscosity coefficient η of the liquid, due to its "carbonation", considering the permeability coefficient with the same value.
However, according to N.I. Osnach [9, 15] reduction of filtration rate in the presence rosaries of Jamen in wood capillaries occurs due to the joint effect of both reducing conductivity and increasing the effective viscosity value, but with a significant predominance of the first factor. The gas bubble formed for one reason or another in the capillary will have a significant resistance to the movement of the liquid. The chain of bubbles - rosaries of Jamen - can completely close the capillary.

Research methods

During the study, a specially planned computational experiment was implemented. At the same time, the simulation of wood heat and mass transfer processes was carried out in the Mathcad-14 computing environment using special software [13]. Analysis of moisture transfer processes was carried out using a physical model of colloidal-capillary-porous solid for coniferous wood [14]. In addition, the appropriate mathematical apparatus was used to analyze the integral-differential equations.

Research results

Analysis of the effect of Jamen in wood capillaries showed the following.
For example, the capillary formed a chain of rosaries of Jamen containing n bubbles. Consider it i-th gas bubble.
In dynamic mode, the left and right meniscus will have different curvature (figure 1). We determine the value of the additional resistance generated at the boundaries of the i-th liquid drop.
The pressure P_i, which in the literature is usually called Laplace' at each point of the curved surface will be directed towards the center of the curvature of the corresponding elementary platform $\Delta\alpha_i$.
The total force of Laplace pressure on the surface of the meniscus A_1A_1 is

$$f'A_1A_1 = \iint \sigma \left(\frac{1}{r_1} + \frac{1}{r_2}\right)_{11} \sin \nu_{11} d\alpha, \tag{16}$$

where r_1, r_2 – accordingly, the radii of the smallest and largest curvature of the surface element..
For the surface of the meniscus A_2A_2 power Laplace' pressure equal

$$f''A_2A_2 = \iint \sigma \left(\frac{1}{r_1} + \frac{1}{r_2}\right)_{11} \sin \nu_{11} d\alpha. \tag{17}$$

Thus, the backpressure force associated with the i-th drop in the promotion of the latter

$$\Delta F_i = f'A_1A_1 - f''A_2A_2 = \iint \sigma \left\{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)_1 \sin \nu_1 - - \left(\frac{1}{r_1} + \frac{1}{r_2}\right)_{11} \sin \nu_{11}\right\} d\alpha. \tag{18}$$

To calculate (18), it is necessary to know the subintegral expression as a function of independent variables, which will be very difficult to change during the deformation of the meniscus when it moves along with the liquid along the capillary. As a result, the expression (18) becomes virtually non-numbered.
However, it is possible to estimate the amount of backpressure.
Однако оценить величину противодавления возможно.
Let's assume in the first approximation that each bubble introduces some additional resistance

$$\delta P_i \approx \frac{2\sigma}{r_i}, \tag{19}$$

where r_i - the value of the mean radius of curvature of the meniscus drops.
When the whole chain starts to move, the total capillary back pressure will be $\Delta P_{\text{доп}} = n\frac{2\sigma}{r_i}$.
For wood capillaries (model colloidal capillary-porous solids [16], the equivalent radius of early tracheids pine) characteristic size

$$r_i = 2 \cdot 10^{-5} \text{ м},$$

the surface tension of the liquid at T = 353 ^{0}K: $\sigma_i \approx 60 \cdot 10^{-3}$ Н · м.
Then the back-pressure produced by one bubble $P_i \approx 6000$ Н .
Researchers [9, 10] cautiously suggest that the presence of 50 ... 100 air bubbles in the capillary makes it impervious.
However, we can argue that the presence of already one bubble makes the capillary impervious (when the liquid moves inside), since taking into account the area of the capillary, the backpressure is

$$\delta P_д \approx 5 \cdot 10^{12} \ \text{Н}/_{\text{м}^2}.$$

Taking into account the impossibility of moisture movement through the capillaries into the wood under the influence of a temperature gradient, the following results were obtained in the computational experiment.
Table 1 shows the calculated total density of moisture flow in different directions of the temperature gradient (figure 2).

Table 1: Estimated flux density of moisture, given temperature gradient

№	The direction of the temperature gradient	The flux density (x10^{-7}), кг/м^2·с with a wood moisture of, %				
		5	10	20	30	> 30
1	Positive	0,38	0,6	1,03	1,4	2,07
2	Negative	0,44	0,76	1,35	1,94	6,7
3	The Isothermal transfer	0,41	0,68	1,19	1,67	4,25

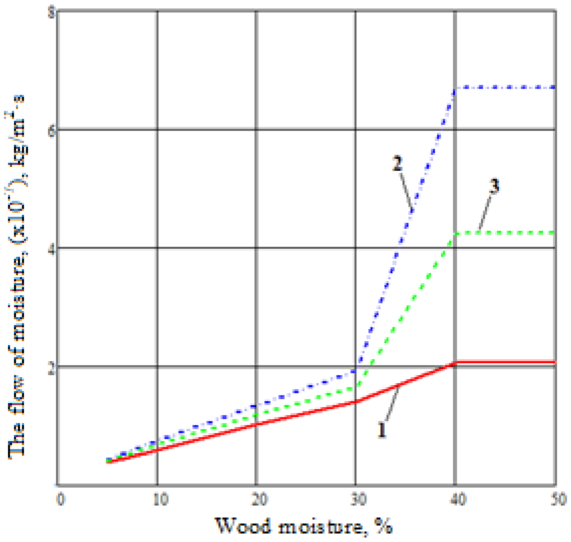

Figure 2: Influence of the direction of the temperature gradient on the process of removing moisture from wood
1 – the temperature gradient is positive;
2 – the temperature gradient is negative;
3 – the Isothermal transfer

An analysis of the data in table 1 and figure 2 shows that:
1. The total flow of moisture also depends significantly on the moisture content of the wood.
2. The most significant thermal and hydraulic conductivity affects the removal of free moisture. If the moisture content of the wood is less than 20%, its influence becomes insignificant. It is necessary to consider it at construction of conditions of drying of wood.

A very important parameter in the calculation of heat and mass transfer processes is the criterion of phase transition ε. Analysis of the system of differential equations of heat and mass transfer (1-2) shows that the special importance of knowledge of the exact magnitude ε becomes in solving the problems of non-isothermal transfer, as in (1) ε largely determines the temperature of the wood. Moreover, the decrease ε, other things being equal, increases the temperature of the wood.

Table 2 shows the calculated values of the phase transition criterion. Also of note are the following:
1. The phenomenon of thermal and hydraulic conductivity significantly affects the value of the phase transition criterion ε. This is due to the fact that the change in the direction of the temperature gradient leads to a significant decrease (or increase) in the flow of liquid moisture, which changes the vapor-liquid ratio, and hence the value ε.
2. The free moisture in the positive direction of the temperature gradient is removed in the form of steam (ε = 1,0), that is, the inhibitory effect of the positive temperature gradient is so high that it overlaps the flow of moisture caused by the wedging pressure.

Table 2: Calculated values of the phase transition criterion ε

№	The direction of the gradient temperatures	The value of ε with a wood moisture of, %				
		5	10	20	30	> 30
1	Positive ($\varepsilon_1(u)$)	0,5	0,616	0,728	0,805	1,0
2	Negative ($\varepsilon_2(u)$)	0,43	0,487	0,555	0,614	0,31
3	The Isothermal transfer	0,463	0,544	0,630	0,677	0,487

The obtained information about the magnitude ε will be used in the future. When solving differential equations of heat and mass transfer of the form (1-2), it is necessary to present the value of the phase transformation criterion as

$$\varepsilon = k_1(\Delta t) \cdot \varepsilon_1(u) + k_2(\Delta t) \cdot \varepsilon_2(u) \tag{20}$$

where $k_1(\Delta t), k_2(\Delta t)$ – coefficients depending on the direction of the temperature gradient;
$\varepsilon_1(u), \varepsilon_2(u)$ – the value of the phase transition criterion (table 2).
Figure 3 shows the values of the coefficients $k_1(\Delta t), k_2(\Delta t)$.

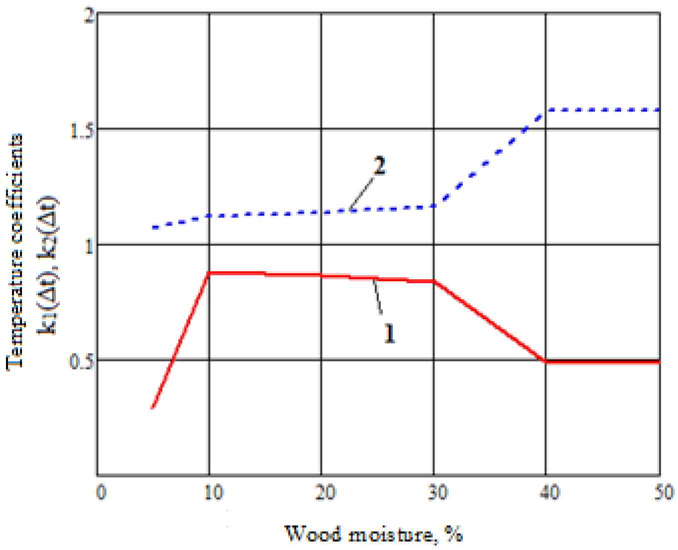

Figure 3: Dependence of the temperature coefficients from the temperature gradient:
1 – the temperature gradient is positive;
2 – the temperature gradient is negative

Calculation formulae:

$$k_1(\Delta t) = -5 \cdot 10^{-3}\Delta t^3 + 1{,}018 \cdot 10^{-3}\Delta t^2 + 0{,}55\Delta t + 0{,}399 \tag{21}$$

$$k_2(\Delta t) = 5 \cdot 10^{-3}\Delta t^3 + 1{,}018 \cdot 10^{-3}\Delta t^2 - 0{,}55\Delta t + 0{,}399 \tag{22}$$

$$\varepsilon_1(u) = 42{,}054u^5 - 94{,}787u^4 + 75{,}965u^3 - 27{,}191u^2 + 5{,}229u + 0{,}298 \tag{23}$$

$$\varepsilon_2(u) = -78{,}103u^5 + 170{,}324u^4 - 125{,}386u^3 + 34{,}667u^2 - 2{,}778u + 0{,}506 \tag{24}$$

The actual non-isothermal transfer is calculated by solving the second equation of the system of differential heat and mass transfer equations (2). The following circumstances should be taken into account (table 3).

Table 3: Coefficients of the change of the flow moisture

№	The direction of the gradient temperatures	The rate of change of flow with a wood moisture of, %				
		5	10	20	30	> 30
1	Positive ($k_{\delta 1}(u)$)	0,296	0,882	0,865	0,838	0,487
2	Negative ($k_{\delta 2}(u)$)	1,073	1,118	1,134	1,162	1,576

1. When solving the system, there is some asymmetry of the moisture flows when the direction of the temperature gradient vector changes.
2. Equation (2) takes the form of

$$\frac{\partial u}{\partial t} = a_m \frac{\partial^2 u}{\partial x^2} + a_m \delta k_\delta \frac{\partial^2 t}{\partial x^2} \tag{25}$$

where k_δ – the coefficient taking into account the asymmetry of moisture flows.

$$k_\delta = k_1(\Delta t) \cdot k_{\delta 1}(u) + k_2(\Delta t) \cdot k_{\delta 2}(u) \quad (26)$$

$$k_{\delta 1}(u) = -84{,}154u^5 + 185{,}116u^4 - 139{,}868u^3 + 42{,}552u^2 - 5{,}434u + 1{,}112 \quad (27)$$

$$k_{\delta 2}(u) = 98{,}812u^5 - 217{,}104u^4 + 163{,}626u^3 - 49{,}458u^2 + 6{,}208u + 0{,}862 \quad (28)$$

Conclusion

Strictly mathematically confirmed the hypothesis about the impossibility of the movement of moisture through the capillaries into the wood under the action of the temperature gradient.
The refined values of the phase transition criterion ε and the temperature coefficient δ, which can be used in modeling the processes of non-isothermal moisture transfer in the process of drying wood by solving a system of differential equations of heat and mass transfer, are obtained. At the same time, it is necessary to take into account some asymmetry of moisture flows when changing the direction of the temperature gradient vector.

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