

Software Package for Power Flow Mathematical Modeling with Single-Phase and Three-Phase Power-Energy Flow Model

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Abstract

Nowadays power flow mathematical modeling underlines power system operation. Traditionally, software packages for power flow calculation use a mathematical model based on nodal admittance equation. The authors have developed a software package for calculating power flow using an alternative power-energy flow model. The model has several advantages compared with the traditional approach. In particular, it allows combining single-phase and three-phase network representation. The authors have analyzed the limits of applicability of this method, as well as the errors that it introduces.

1 Introduction

A classical problem of circuit theory is to find all branch currents and all node voltages of an assigned circuit. Typical input data are generator voltages as well as the impedances of all branches. If all impedances are constant, the resulting set of equations that describe the circuit is linear. The power flow problem is conceptually the same problem as solving a steady-state ac circuit [1]. The only, though substantial, difference is the set of input data. Input data for power flow calculation is active and reactive load and active power injection of generators, as well as power lines and transformers describing the transmission network. In this formulation of the problem, it is necessary to solve a system of nonlinear algebraic equations. The result of the calculation is the voltages in the nodes of power system.

Power flow mathematical modeling has wide application in power system engineering. For solving design and operation problems, it is necessary to understand the consequences of changing elements states in power system. In particular, bus voltages must be maintained near nominal values. In addition, currents in elements of power system must not exceed admissible continuous current. There are a number of other aspects in power system operation and control that can be associated with power flow calculation [2].

There are different software packages in which the power flow calculation function is implemented. In addition, this function is implemented in SCADA system [3] for analysis controlled power system. There are various models describing power flow, but models are represented by nonlinear algebraic equations in mathematical point of view. For solve this nonlinear algebraic equations different variations of Newton method are used [4]. Nowadays power flow model based on nodal admittance equations has received the most widespread.

The main problem of this approach is connected with the fact that iteration procedure of Newton method may diverge. It depends on initial conditions and structure of grid. In addition, this method has sufficiently high computational complexity for large grid. This circumstance can be significant when using power flow calculation in control systems, which performs variant calculations.

The authors proposed the formulation of the problem based on the power-energy flow model [5], [6]. Initially, this model was developed to solve the problem of state estimation. Further, the formulation of the problem for power flow calculation was developed. This model has better conditioning than nodal admittance model. This leads to the fact that the Newton method begins to converge more stable and for a smaller number of iterations.

High voltage transmission network are generally modeled using single-phase model. It is believed that power flow of backbone networks are symmetric. In this way, exact consideration of all three phases is not required. In normal condition, this assumption does not lead to a high error. For the analysis of asymmetric condition, such as short circuits, it is customary to use the symmetric component method[7]. In distribution grid, load unbalanced is more significant in comparison with backbone networks and power flow calculation on single-line model lead to a significant error. For this reason, three-phase model of network is used for power flow calculation of distribution grid [8], [9].

Traditionally management of distribution and transmission grids was divided administratively and technically. As a consequence, there was no need for a joint consideration of two types networks. Modern trends in digital smart grid [10] concept are changing this situation. The amount of available information increases, which entails an increase in the load on computing systems. The requirements to the accuracy of calculations are raised which reduces the scope of application of approximate fast calculation methods. Respectively there is a need to combine single-phase model for transmission grid and three-phase model for distribution grid. In this paper, authors proposed approach based on power-energy flow model for solve this problem.

2 Power-energy flow model

In general, approach for compilation equation is the same for single-phase and three-phase models. In proposed model active P^s and reactive Q^s line start power, voltage magnitude V in bus are accepted as unknown variables. Three types of equation describe power flow. In the beginning, they will be described in general form for a single-phase model, and then fundamental differences will be presented for a three-phase model.

2.1 Single-phase model

First type of equation is relationship between voltages of the beginning V_i and the end V_j of line:

$$V_j = V_i - \sqrt{(V_i - \Delta V')^2 + (\Delta V'')^2}$$

Where $\Delta V'$ and $\Delta V''$ are expressed as:

$$\Delta V' = \frac{P_{ij}^s \cdot R_{ij} + Q_{ij}^s \cdot X_{ij}}{V_i}$$

$$\Delta V'' = \frac{P_{ij}^s \cdot X_{ij} - Q_{ij}^s \cdot R_{ij}}{V_i}$$

Where: V_i – voltage of line start, V_j – voltage of line end, P_{ij}^s and Q_{ij}^s – active and reactive line start power, R_{ij} and X_{ij} – active and reactive line resistance.

Second type of equations are active and reactive power balance in bus:

$$\sum (P_{ji}^s - \Delta P_{ji}) + \sum (P_{ik}^s) + P_i = 0$$

$$\sum (Q_{ji}^s - \Delta Q_{ji}) + \sum (Q_{ik}^s) + Q_i = 0$$

Where: P_i and Q_i – load of bus; P_{ik}^s and Q_{ik}^s – start power of lines, that start number coinciding with the node number; P_{ji}^s and Q_{ji}^s – start power of lines, that end number coinciding with the node number; ΔP_{ji} and ΔQ_{ji} – power losses in lines, that end number coinciding with the node number. Power losses are expressed as follows:

$$\Delta P_{ji} = \frac{P_{ji}^{s2} + Q_{ji}^{s2}}{V_j^2} R_{ji},$$

$$\Delta Q_{ji} = \frac{P_{ji}^{s2} + Q_{ji}^{s2}}{V_j^2} X_{ji}$$

Third type of equations use condition that sum of the angle difference is equal to zero in closed loop:

$$\sum \Delta \delta = 0$$

For variables under consideration, this equation can be transformed as follows:

$$\sum atg\left(\frac{\Delta V''}{V_i - \Delta V'}\right) = 0$$

All voltages in single-phase model are phase-to-phase voltages, all power in single phase are the sum of each phase power.

2.2 Three-phase model

In three-phase model, all this type of equations is used for each phase. The main difference is connected with component ΔP_{ji} , ΔQ_{ji} , $\Delta V'$ and $\Delta V''$. Below are the expressions for phase A, expressions for phases B and C can be obtained by analogy. Active power losses is expressed as:

$$\Delta P_a = \frac{P_a^2 \operatorname{Re}[Z_{aa}]}{U_a^2} + \frac{Q_a^2 \operatorname{Re}[Z_{aa}]}{U_a^2}$$

$$+ \frac{(P_a Q_b - P_b Q_a) \operatorname{Im}[e^{-i(120^\circ + \delta_{ab})} Z_{ab}] + (P_a P_b + Q_a Q_b) \operatorname{Re}[e^{-i(120^\circ + \delta_{ab})} Z_{ab}]}{U_a U_b}$$

$$+ \frac{(P_a Q_c - P_c Q_a) \operatorname{Im}[e^{i(120^\circ - \delta_{ac})} Z_{ac}] + (P_a P_c + Q_a Q_c) \operatorname{Re}[e^{i(120^\circ - \delta_{ac})} Z_{ac}]}{U_a U_c}$$

Reactive power losses is expressed as:

$$\Delta Q_a = \frac{P_a^2 \operatorname{Im}[Z_{aa}]}{U_a^2} + \frac{Q_a^2 \operatorname{Im}[Z_{aa}]}{U_a^2}$$

$$+ \frac{(P_a Q_b - P_b Q_a) \operatorname{Re}[e^{-i(120^\circ + \delta_{ab})} Z_{ab}] + (P_a P_b + Q_a Q_b) \operatorname{Im}[e^{-i(120^\circ + \delta_{ab})} Z_{ab}]}{U_a U_b}$$

$$+ \frac{(P_a Q_c - P_c Q_a) \operatorname{Re}[e^{i(120^\circ - \delta_{ac})} Z_{ac}] + (P_a P_c + Q_a Q_c) \operatorname{Im}[e^{i(120^\circ - \delta_{ac})} Z_{ac}]}{U_a U_c}$$

Voltage drop components are expressed as:

$$\Delta V' = \frac{P_a \operatorname{Re}[Z_{aa}] + Q_a \operatorname{Im}[Z_{aa}]}{U_a} + \frac{Q_b \operatorname{Im}[e^{-120i^\circ - i\delta_{ab}} Z_{ab}] + P_b \operatorname{Re}[e^{-120i^\circ - i\delta_{ab}} Z_{ab}]}{U_b} + \frac{Q_c \operatorname{Im}[e^{120i^\circ - i\delta_{ac}} Z_{ac}] + P_c \operatorname{Re}[e^{120i^\circ - i\delta_{ac}} Z_{ac}]}{U_c}$$

$$\Delta V'' = \frac{-Q_a \operatorname{Re}[Z_{aa}] + P_a \operatorname{Im}[Z_{aa}]}{U_a} + \frac{P_b \operatorname{Im}[e^{-120i^\circ - i\delta_{ab}} Z_{ab}] - Q_b \operatorname{Re}[e^{-120i^\circ - i\delta_{ab}} Z_{ab}]}{U_b} + \frac{P_c \operatorname{Im}[e^{120i^\circ - i\delta_{ac}} Z_{ac}] - Q_c \operatorname{Re}[e^{120i^\circ - i\delta_{ac}} Z_{ac}]}{U_c}$$

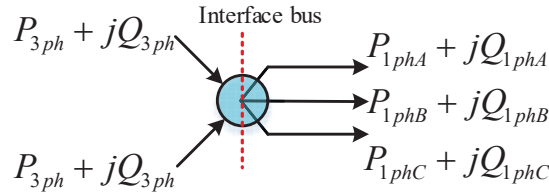
All voltages in three-phase model are phase-to-ground voltages, all power in three-phase are power of one single phase.

3 Combination single-phase and three-phase model

For combination two this model, the further approach is proposed. Firstly, buses with step-down substation that connects distribution and transmission grids are used for interface point. Voltages in interface point are described as:

$$U_{ph-ph} / \sqrt{3} = U_{phA} = U_{phB} = U_{phC}$$

These equations reflect the basic assumption that in interface point conditions is balanced.



In addition to voltage equalities, combination of single-phase and three-phase power in bus balance equation are introduced as:

$$P_{3ph} = P_{phA} + P_{phB} + P_{phC}$$

$$Q_{3ph} = Q_{phA} + Q_{phB} + Q_{phC}$$

4 Software package and numerical calculation

To analyze the proposed approach, a software package was developed in the Wolfram Mathematica [11] environment. The choice of this environment has greatly accelerated the development process, since Wolfram Mathematica contains all the necessary mathematical methods. In addition to standard numerical methods for the described mathematical approach, a library of work with graphs is required. Methods to search all closed loops in graph and the search for the spanning tree were used. The developed package can be used as a standalone application. It is also possible to compile libraries of the C language, which allows it to be integrated into other software products.

To analyze the algorithms underlying the software package, a test network was used, which is presented in Figure 1. The network combines both a single-phase representation for the backbone network and a three-phase representation of the distribution network.

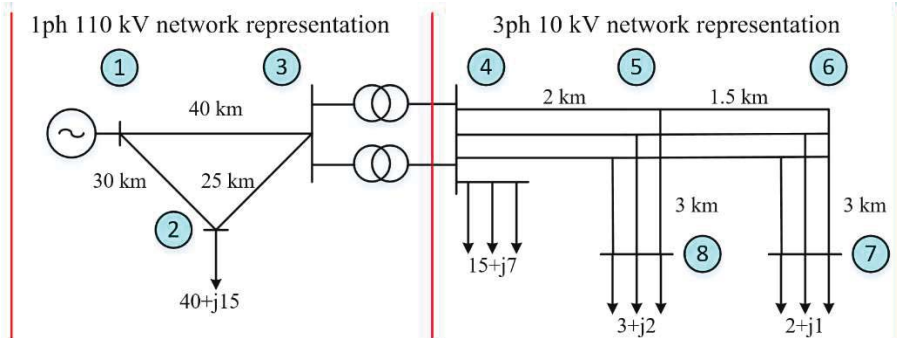


Figure 1: Test system

Firstly, power flow calculation in three phases with regard to mutual induction was analyzed. For this, a set of asymmetrical loads was generated, the maximum amplitude values of the loads are shown in Figure 1. Then, based on these loads, the model without mutual induction and the reference model were compared. An exact three-phase model was used as a reference calculation. The results of the calculation of the modes are shown in Figure 2. In this case, each point in Figure 1 corresponds to the steady-state power flow, the standard deviation of the voltage in the reference calculation is plotted along the X-axis, and the error resulting from the calculation is on the Y-axis. As you can see, the error does not exceed 5 percent, if the standard deviation of the voltage does not exceed 3%. These data can be considered the limit of applicability of the three-phase model.

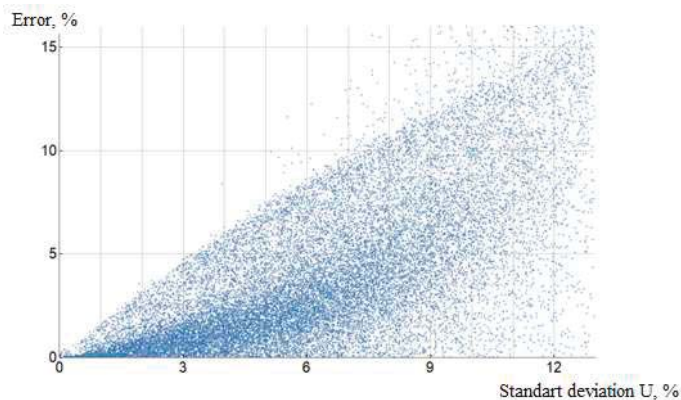


Figure 2: Calculation error when neglecting mutual induction

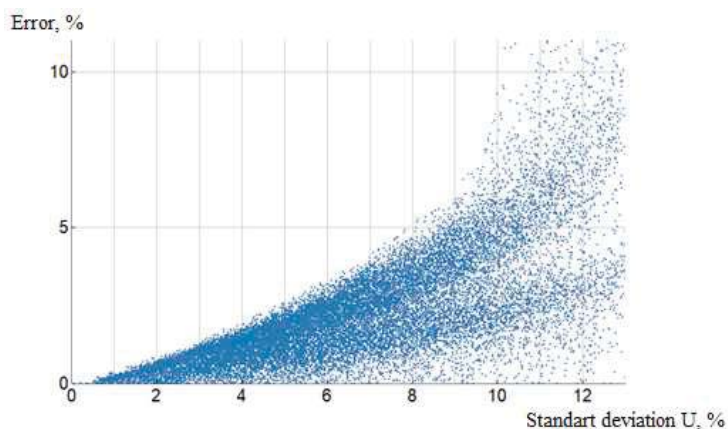


Figure 3: Error of combination of two models

Further, the error of the method was analyzed, which assumes that part of the network is modeled in one phase, and part in three phases. The calculation results are shown in Figure 3. In this case, each point on the X-axis corresponds to the standard deviation of the voltage at the interface point, the Y-axis shows the error of the calculation results. As you can see, the proposed method does not give an error of more than 5% if at the interface point the standard deviation does not exceed 10%.

5 Conclusion

The article described a software package for calculating power flow of electric power systems. It is based on a power-energy flow model. The model allows, depending on the available information and the requirements for the problem being solved, applying the phase or linear form of the described model. The transition from one model to another is carried out without a significant change in the design model itself. Within this model, assumptions are used at the interface point between the single-phase and three-phase network representation. Error analysis was performed on a test system. Calculations showed if at the interface point the differences between the voltages do not exceed 10% percent, then the proposed approach does not lead to a significant error.

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