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Using the Gini coefficient to calculate the degree of consensus in group decision making process

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Nowadays, almost all people in the world can communicate and exchange information on the Internet. The Internet is an open structure where everyone can express their opinions. In social networks millions of users are registered around the world, and the data of their interaction can be used for specific purposes, for instance, in group decision making problems. Therefore, social network is one of the best environments to raise any questions, discuss them and make decisions. The process of reaching consensus and many different approaches to the solution of this problem have already been well studied. In recent years, modeling of the process of reaching consensus in the context of social networks is of special interest. In addition, the development of improved structures for GDM processes and consensus decision making is actual now, as they can be used in new social networking services. Group decision making is the process of selecting the best alternative or a set of alternatives from all possible. In conditions that are far from reality, decision-makers come to full agreement. However, most often such result is impossible. Actually, it is interesting to understand how much the experts had reached agreement through discussion. Calculation of the degree of agreement usually requires the calculation and aggregation of distance measures, which assess how close each expert's preferences to each pair of alternatives. Such calculations can have long time in view of the selected aggregation operator and require the construction of a collective preference matrix before it can be obtained. In this paper we propose to use the Gini coefficient and present the formula, which shows the degree of agreement between the experts. We also offer a method for assessing the effectiveness of consensus reached by decision-makers.

Key words and phrases: group decision making, social network analysis, fuzzy logic, Gini coefficient, consensus level.

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1. Introduction

Social network analysis is a new and actively developing direction in the field of group decision making (GDM). The interest of researchers in this field is due to the fact that it provides a set of explanatory models and analytical tools beyond the scope of conventional quantitative methods. At the same time, a rich mathematical apparatus has been accumulated, which makes it possible to build complex models of social interactions [1].

One of the advantages that the Internet provides is the opportunity for everyone to express their opinions, which allows making group decisions based on the information gathered. It should be noted that social network is the most convenient platform for achieving consensus decisions [2].

Group decision making is applied in many situations in the real world. Firstly, GDM is a process of ranking of alternatives, taking into account the opinions expressed by a group of people [3, 4]. Secondly, it is a process focused on people, with their subjectivity and uncertainty in evaluation. Therefore, in GDM processes commonly fuzzy logic theory is used [5,6]. When we use the apparatus of fuzzy logic, alternatives are classified according to the following principle: the degree of preference is expressed as a value from the interval [0,1] with 0 for less preferable and 1 for more preferable.

Group decision making problems require a high level of consensus among experts before getting a solution. A prospective field of research of group decision making is the study of interpersonal interaction and calculation the degree of consensus agreement [7,8].

The main goal of our work is to evaluate the effectiveness of the consensus reached.

The rest of the paper is organized as follows: in section 2 a model of group decision making in a social network is formally described. Section 3 demonstrates the new coefficient to calculare the degree of agreement among all experts.In section 4 we present the results of numericai example of applying this method. Section 5 concludes the paper.

2. The model of consensus in group decision making process

The goal of the group decision making process is to order different alternatives from a set of alternatives from the best to the worst with the help of the association of some preference degrees, taking into account the opinions of groups of decision-makers [9, 10].

We assume that the GDM process involves the participation of K experts who compare M alternatives pairwise. Then $E = \{e_1, ..., e_K\}$ - a set of experts and $X = \{x_1, ..., x_M\}$ – a set of alternatives, such that $|E| = K < \infty$, where K is the number of experts and $|X| = M < \infty$, where M – the number of alternatives.

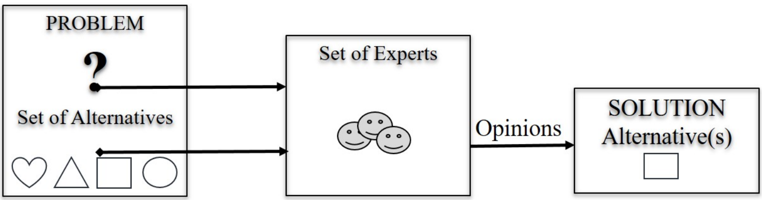

Figure 1. The scheme of GDM process.

Each expert e_k provides a certain preference value $p_{ij}(k)$, which indicates how much the alternative i is better than the alternative j.

The obtained values form the preference matrix $P_k = (p_{ij}(k)),_{i,j\in 1,\dots,M}$ for the expert e_k, where $p_{ij} = 1$ reflects the maximum preference, $p_{ij} = 0$ - the minimum preference. And $p_{ij} = 1 - p_{ji}$, $p_{ii} = 0.5$.

The GDM process contains following stages.

1. **Providing preferences.** Experts decide which alternatives are most appropriate and share their opinions to the system. One of the most commonly used techniques for implementing this process is to allow users to compare alternatives pairwise.
2. **Aggregation of information** received from all experts. Individual preferences of experts are aggregated into the matrix of collective preferences. The selected aggregation operator summarizes the preferences or reflects properties contained in the preferences of experts. In this stage we prefer to use the matrix of averaged estimate $\overline{\boldsymbol{P}}=(\overline{p}_{ij})_{i,j=1,\dots,M}$, the elements of which are calculated as follows
$$\overline{p}_{ij} = \frac{\sum\limits_{k=1}^{K} p_{ij}(k)}{K}. \tag{1}$$
Also OWA operators can be used for this purpose. The obtained result contains the general opinion of all experts. It is usually presented in the form of a square matrix, where each position $\overline{p}_{ij}$ contains the preference of the alternative x_i over x_j.
3. **Exploitation of the information** received from stage 2. This step forms the final ranking of alternatives. Results can be presented as a choice of the best alternative or as a rating of all alternatives.

We can perform the ranking of alternatives using, for instance, operators GDD (Quantifier Guided Dominance Degree) and GNDD (Quantifier Guided Non-Dominance Degree). These operators are aimed to calculate the rating of alternatives or create lists of ranked alternatives using the collective preference matrix obtained in the previous step. Commonly alternatives are evaluated using an average value between GDD and GNDD. The operator GDD shows how the estimated alternative dominates all the others, i.e. how much this alternative is better than all the others.

The calculation of GDD is carried out according to the following formula

$$GDD_i = \sum_{j=1}^{M} \overline{p}_{ij}, i = 1, ..., M. \tag{2}$$

The GNDD operator can be calculated as follows

$$GNDD_i = \sum_{j=1}^{M} 1 - \max\{\overline{p}_{ji} - \overline{p}_{ij}, 0\}, i = 1, ..., M. \tag{3}$$

Averaging the values of GDD and GNDD, we obtain the RV (Ranking Value):

$$RV_i = \frac{GDD_i + GNDD_i}{2}, i = 1, ..., M. \tag{4}$$

When a decision is defined as a value of RV_i for each alternative (4), it is interesting to understand how much each of the experts agree with its, in other words, to know the degree of agreement among the experts.

4)**Calculating the consensus level** among experts in a group decision making process is an important part of this process. Thanks to this, experts know whether they have reached agreement or, on the contrary, their opinions are too far from each other. Therefore, consensus measures help to decide whether to continue the discussion or they have already reached an agreement.

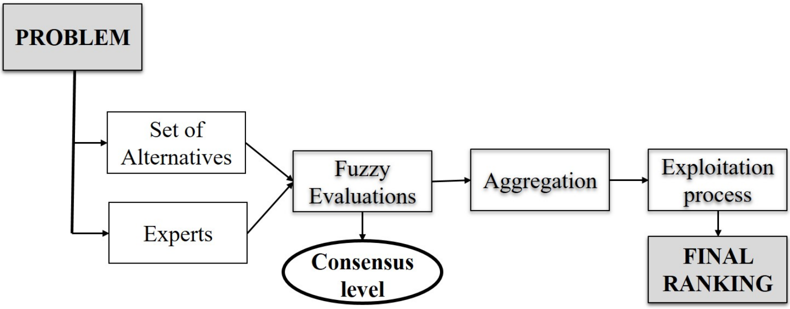

Figure 2. Group decision making model.

3. The Gini coefficient for calculating the degree of agreement among experts

In most real-world scenarios, a complete consensus is practically unreachable. Due to some differences in the level of knowledge and personal interests of decision-makers, a full agreement is reached in rare cases. To understand how much experts were close to each other in expression opinions on the set of alternatives, we propose to use the Gini coefficient.

Traditionally, this coefficient is used in the economy and social policy for differentiation of incomes of the population. It is necessary to find the value at which the distribution of the economic variable deviates from the ideal value (equal distribution of wealth at all people). For measuring a certain statistical spread or dispersion are used.

The degree of agreement (consensus level) among the participants of the group decision making process is one of the most important indicators. The measurement of this coefficient is a necessary task before making a final decision.

In GDM problems, the measurement of the degree of agreement determines as well as a high degree of similarity in the distribution of preference values.

The Lorenz curve in the economy makes it possible to define the degree of income inequality population. So it can also demonstrate unequal distribution in any system. It should be stressed that the Gini coefficient is intimately connected with the Lorentz curve and is equal to the ratio of the area of the figure bounded by the line of absolute equality and the Lorentz curve, to the area of the entire triangle under the line of absolute equality. Define A - area of the region between line of absolute equality and the curve, B - area of the region under the curve, C - area of the region above the curve as shown in Fig. 3:

$$IG_{ij} = \frac{A}{A+B}. \tag{5}$$

Theorem 1.

Theorem 1. *Gini consensus coefficient for each pair of alternatives (i,j) is defined as*

$$IG_{ij} = 1 - \frac{2}{K}\sum_{k=1}^{K-1} h_{ij}(k). \tag{6}$$

Proof.

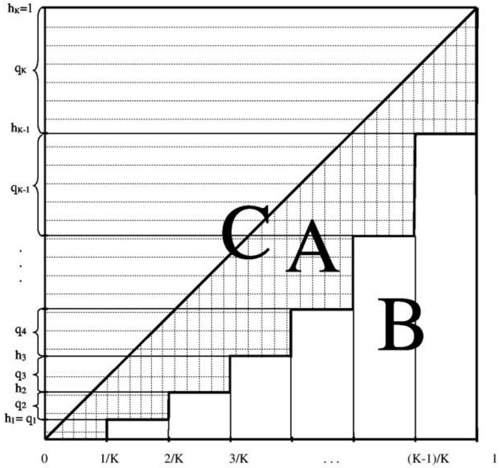

Figure 3. Lorenz Curve.

Proof. Define $h_{ij}(k) = \sum\limits_{l=1}^{k} q_{ij}(l)$, where

$$q_{ij}(k) = \begin{cases} 0, & \text{if } \sum\limits_{l=1}^{K} p^{\sigma}_{ij}(l) = 0, \\ \dfrac{p^{\sigma}_{ij}(k)}{\sum\limits_{l=1}^{K} p_{ij}(l)}, & \text{elsewhere}, \end{cases} \tag{7}$$

Then by the definition [7] and formula (7) the Gini coefficient corresponds to the formula (5), where $C + B = 1$, $A + B = C - A = 0.5$.

From equalities

$$C = \sum_{k=1}^{K} \frac{k}{Kq_{ij}(k)} = \frac{1}{K} \sum_{k=1}^{K} kq_{ij}(k) = \frac{1}{K} \sum_{k=1}^{K} k(h_{ij}(k) - h_{ij}(k-1)) = 1 - \frac{1}{K} \sum_{k=1}^{K-1} h_{ij}(k)$$

we get that

$$IG_{ij} = \frac{C - 0.5}{0.5} = \frac{2}{K} \sum_{k=1}^{K} kq_{ij}(k) - 1 = 1 - \frac{2}{K} \sum_{k=1}^{K-1} h_{ij}(k). \tag{8}$$

□

$IG_{ij} = 0$ shows the perfect agreement, where all evaluations of experts are the same, and $IG_{ij} = 1$ represents the maximum difference between evaluations.

Table 1. Set of alternatives.

x_i	**Mobile OS**
x_1	Tizen
x_2	Windows 10 Mobile
x_3	Lineage OS
x_4	Android
x_5	iOS
x_6	Fire OS
x_7	Kai OS

4. Case study

In this section we give a numerical example of the proposed coefficient.

Let us consider such process, in which 5 experts $E = \{e_1, e_2, e_3, e_4, e_5\}$ participate and there are 7 alternatives $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$. We have conducted a survey to determine which of the 7 below listed mobile operating systems are most convenient for using according to 5 experts.

Thus, taking into account the notation introduced earlier, for the analysis we have the initial data presented in Table 1.

Experts' preferences are presented in the following matrixes

$$\boldsymbol{P_1} = \begin{pmatrix} 0.5 & 0.4 & 0.8 & 0.5 & 0.2 & 0.7 & 0.7 \\ 0.6 & 0.5 & 1 & 0.5 & 0.3 & 0.8 & 0.5 \\ 0.2 & 0 & 0.5 & 0.1 & 0 & 0.5 & 0.2 \\ 0.5 & 0.5 & 0.9 & 0.5 & 0.4 & 0.9 & 0.8 \\ 0.8 & 0.7 & 1 & 0.6 & 0.5 & 1 & 0.7 \\ 0.3 & 0.2 & 0.5 & 0.1 & 0 & 0.5 & 0.4 \\ 0.3 & 0.5 & 0.8 & 0.2 & 0.3 & 0.6 & 0.5 \end{pmatrix}$$

,

$$\boldsymbol{P_2} = \begin{pmatrix} 0.5 & 0.3 & 0.8 & 0.5 & 0.1 & 0.5 & 0.5 \\ 0.7 & 0.5 & 0.7 & 0.4 & 0.3 & 0.5 & 0.6 \\ 0.2 & 0.3 & 0.5 & 0.3 & 0.2 & 0.5 & 0.4 \\ 0.5 & 0.6 & 0.7 & 0.5 & 0.5 & 0.8 & 0.7 \\ 0.9 & 0.7 & 0.8 & 0.5 & 0.5 & 8 & 0.9 \\ 0.5 & 0.5 & 0.5 & 0.2 & 0.2 & 0.5 & 0.5 \\ 0.5 & 0.4 & 0.6 & 0.3 & 0.1 & 0.5 & 0.5 \end{pmatrix}$$

,

...

$$P_5 = \begin{pmatrix} 0.5 & 0.6 & 0.6 & 0.4 & 1 & 0.6 & 0.7 \\ 0.4 & 0.5 & 0.8 & 0.5 & 0.2 & 0.7 & 0.5 \\ 0.4 & 0.2 & 0.5 & 0.2 & 0.1 & 0.5 & 0.5 \\ 0.6 & 0.5 & 0.8 & 0.5 & 0.6 & 0.8 & 0.9 \\ 0 & 0.8 & 0.9 & 0.4 & 0.5 & 0.6 & 0.9 \\ 0.4 & 0.3 & 0.5 & 0.2 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.5 & 0.5 & 0.1 & 0.1 & 0.3 & 0.5 \end{pmatrix}$$

,

which result in the following matrix of averaged estimates, calculated by (1):

$$\overline{P} = \begin{pmatrix} 0.5 & 0.48 & 0.66 & 0.38 & 0.52 & 0.66 & 0.58 \\ 0.52 & 0.5 & 0.72 & 0.42 & 0.3 & 0.6 & 0.58 \\ 0.34 & 0.28 & 0.5 & 0.24 & 0.26 & 0.44 & 0.44 \\ 0.62 & 0.58 & 0.76 & 0.5 & 0.5 & 0.7 & 0.84 \\ 0.48 & 0.7 & 0.74 & 0.5 & 0.5 & 0.68 & 0.8 \\ 0.34 & 0.4 & 0.56 & 0.3 & 0.32 & 0.5 & 0.64 \\ 0.42 & 0.42 & 0.56 & 0.16 & 0.2 & 0.36 & 0.5 \end{pmatrix}$$

.

Then using GDD operator (2) and GNDD (3), we get the following results
$GDD_1 = 3.78;$
$GDD_2 = 3.64;$
$GDD_3 = 2.5;$
$GDD_4 = 4.5;$
$GDD_5 = 4.5;$
$GDD_6 = 3.06;$
$GDD_7 = 2.62;$
$GNDD_1 = 5.72;$
$GNDD_2 = 6.44;$
$GNDD_3 = 5;$
$GNDD_4 = 7;$
$GNDD_5 = 6.96;$
$GNDD_6 = 5.72;$
$GNDD_7 = 5.12.$
The ranking of alternatives is given by (4):
$RV_1 = \frac{3.78+5.72}{2} = 4.75;$
$RV_2 = \frac{3.64+6.44}{2} = 5.04;$
$RV_3 = \frac{2.5+5}{2} = 3.75;$
$RV_4 = \frac{4.5+7}{2} = 5.75;$
$RV_5 = \frac{4.5+6.96}{2} = 5.68;$
$RV_6 = \frac{3.06+5.72}{2} = 4.39;$
$RV_7 = \frac{2.62+5.12}{2} = 3.87.$
So, we get the following ranking list: $x_4, x_5, x_2, x_1, x_6, x_7, x_3$.
According to this list, the best mobile operating systems in terms of convenience are Android, IOS and Windows.
Then applying the formula (6) or (8) to the preference matrixes, we have

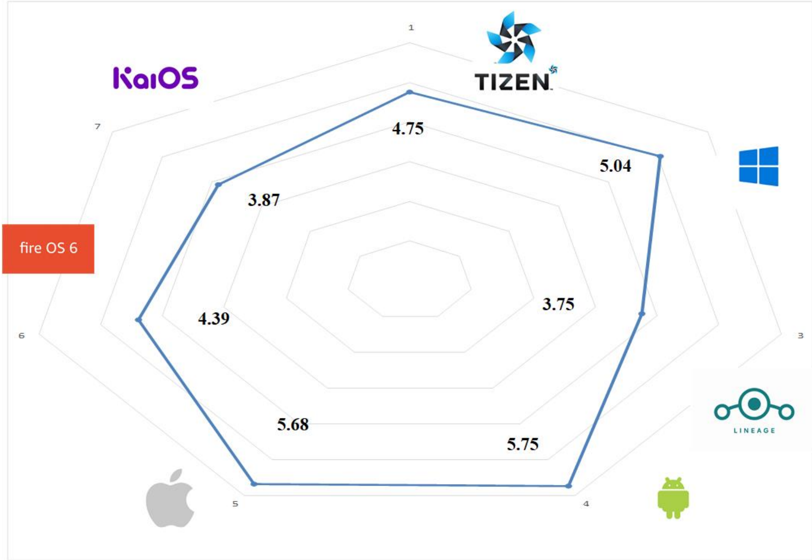

Figure 4. The expert preferences in mobile operating systems.

$$
\boldsymbol{IG} = \begin{pmatrix}
0.5 & 0.37 & 0.3 & 0.37 & 0.58 & 0.28 & 0.31 \\
0.35 & 0.5 & 0.33 & 0.33 & 0.31 & 0.31 & 0.27 \\
0.39 & 0.54 & 0.5 & 0.54 & 0.72 & 0.31 & 0.36 \\
0.3 & 0.3 & 0.31 & 0.5 & 0.27 & 0.32 & 0.25 \\
0.62 & 0.25 & 0.38 & 0.27 & 0.5 & 0.37 & 0.26 \\
0.36 & 0.36 & 0.29 & 0.5 & 0.55 & 0.5 & 0.38 \\
0.35 & 0.3 & 0.33 & 0.45 & 0.44 & 0.5 & 0.5
\end{pmatrix}
$$

.

Considering that it is necessary to obtain a degree of expert consensus on all alternatives, we get

$$
\overline{IG} = \frac{\sum\limits_{i=1}^{M} \sum\limits_{j=1}^{M} IG_{ij}}{2 \sum\limits_{r=1}^{M} (M - r)}, i \neq j.
$$

Thus, the average degree of agreement is $\overline{IG} = 0.37$.

The Gini coefficient indicates that the experts have reached a high level consensus, because of this coefficient is closer to 0 than to 1. Therefore, we can conclude that the experts' estimates are near equal to each other.

5. Conclusions

In this paper a new consensus coefficient proposed in [7, 8] for GDM problems with fuzzy preferences relations was studied. We have investigated the process of group decision making and analyzed the consensus degree to which the experts agree with the decision. For that we built a model of consensus in group decision making, made a rating of alternatives, got a formula for measuring the degree of agreement of experts involved in the GDM process. Also we have proved a formula for measuring the degree of experts' agreement and calculated the consensus level.

Results showed consistent and acceptable behavior of the proposed coefficient, which justifies its use as a valid consensus measure to solve the problems of group decision making.

It should be noted that the Gini coefficient has some interesting advantages. Firstly, it is possible to calculate the degree of consensus without using distances measures. In this case, the use of this coefficient will replace the time-consuming calculations that can not be avoided in the classical theory [1]. Secondly, the Gini does not depend on various aggregation operators that can be applied to obtain a collective preference matrix. And also, the value of the coefficient will not change, even if the final decision changes. This is a kind of evolution of the degree of agreement, independent of the aggregation operator.

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