UDC 621.39

On the performance measures of LTE radio access procedure under massive M2M communications

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Providing the evolution from current wireless systems to fifth generation (5G) network is to support massive Machine-to-Machine (M2M) wireless communications in radio access network. Performance analysis of the random access channel (RACH) is a top issue within the M2M-connection in LTE networks, because prior the data transmitting, the session initiation procedure, which perform the connection initiation for user equipment, could overload the channel dealing with burst arrival of connection requests. The purpose of this paper is to continue the analysis of RACH initiation procedure using discrete Markov chain model, and to investigate the dependence of average delay time from preamble processing time. The simulation model is obtained, which allows for estimating the influence of preamble collision on the success access connection initiation in radio access network.

Key words and phrases: LTE-advanced, 5G, machine-type communications, random access channel, collision, access success probability, access delay, Markov chain, session initiation procedure, mathematical model.

In: K. E. Samouylov, L. A. Sevastianov, D. S. Kulyabov (eds.): Selected Papers of the 12th International Workshop on Applied Problems in Theory of Probabilities and Mathematical Statistics (Summer Session) in the framework of the Conference "Information and Telecommunication Technologies and Mathematical Modeling of High-Tech Systems", Lisbon, Portugal, October 22–27, 2018, published at `http://ceur-ws.org`

1. Introduction

The basic concept of the transition from modern wireless systems to 5G-technologies is to support the massive machine-to-machine(M2M)and Internet-of-things (IoT) devices' connections and still provide a number of promising highly demanded services. According to ETSI [1], potential M2M devices and applications capable of generating and transmitting data autonomously in IoT network are:

1. intelligent devices;
2. smart city;
3. intelligent networks;
4. e-health;
5. connected cars;
6. smart households and energy management;
7. remote industrial process control.

At the same time, the main tasks for observing the required performance indicators are the ability to scale the network, improve energy efficiency and reduce the cost of sensory user devices. Such technologies as Radio Frequency Identification, Zigbee, Bluetooth Low Energy and Low-Power WiFi, which typically implement unlicensed frequency bands, and operate on low power consumption and short transmission range, are designed to support M2M applications. The disadvantage of using such technologies are excessive interference between devices in the coverage of the unlicensed spectrum, which reduces the reliability of these systems, and complicating of initiating access to the radio environment increases the connection delays [2].

To address these issues in the development of IoT, the application of low-power technologies, such as Low Power Wide Area (LPWA), Sigfox, Long Range (LoRa), Weightless and Long Term Evolution (LTE), is recommended, and LTE cellular technology is the most suitable solution due to the wide coverage in the existing infrastructure, security, licensed spectrum and easier maintenance [3]. One possible solution to the LTE network scalability problem is based on an analysis of the RACH connection initiation procedure [4, 6, 7]. For a number of scenarios of M2M-interconnection the access delay for user equipment (UE) dominates, exerting a significant load on the channel even before the actual data transfer begins [8]. This problem appears at peak times, in the case of simultaneous activation of a large group of devices, for example, when sensors are reconnected after a power outage [9]. This burst arrivals can initiate RACH' overload for a long period of time.

In paper [4] modelling of session initiation procedure provided the opportunity to implement the RACH parameters to increase the probability of a successful connection (access success probability) and to reduce the average access delay [5]. In [6] the dependence of the collision probability on the number of M2M-devices was investigated in the conditions of rapidly growing M2M traffic and high demand of UEs to a single base station (BS). Here the approach with state-dependent arrival and service rates can be used [12].

The purpose of the current work is to continue an analytical model's development for evaluating performance measures, provided possible retransmission of three messages (Msg1, Msg3, Msg4), which we describe in Section 3. To verify the obtain results we build the simulation model of session initiation procedure, and in Section 4 present the part of simulated programme code. Section 5 is provided with numerical analysis of the dependence of different preambles processing time on average RA procedure delay and comparison of analytical and simulation methods.

2. Random Access procedure

The basic Random Access (RA) procedure, which initiate the connection between UE and eNB, consists of four steps and can be divided into two stages: a link synchronization step (Msg1, Msg2) and a service transfer (Msg3, Msg4).

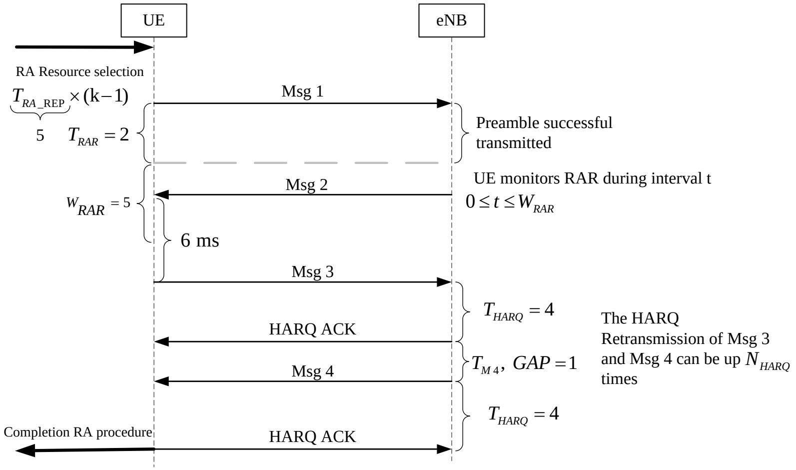

Figure 1. Sequence of messages' transmission in RA procedure.

It begins with transfer from UE to eNB the Msg1 (Preamble Transmission), and selection one from the set of 64 preambles [10, 13]. Chosen index of preamble request differentiates multiple devices. When two or more UEs select a same RA preamble, a collision occurs and all UEs should retransmit Msg1. Further the UE receives response – a message RAR (Random Access Response, Msg2) – from the eNB. If UE does not receive the response Msg2, user's transmitter increases the power and repeats the preamble transmission over the time interval, following which UE answers Msg3 (Connection Request). Then, the automatic acknowledgement HARQ ACK (Hybrid Automatic Repeat Request Acknowledgment) allows to protect the signaling message transmission. If the Msg3 is successfully transmitted and processed, the eNB responds with a Msg4 (Connection Response). If the UE does not receive from the eNB the Msg4, the Msg4 message will be sent again in specified time interval.

By exceeding the Msg1 transmission counter the connection initiation procedure is considered unsuccessful. In case of exceeding number of Msg3/Msg4 transmission, the procedure starts from the new Msg1 preamble transmission, in case the maximum number of preamble transmission *preambleTransMax* is not reached. The example of complete successful connection initiation is presented on Fig. 1.

3. Mathematical model

In this work we extend the previous results, presented in [11]. We build the mathematical model of RACH procedure, taking into account the possibility of retransmission of messages (Msg1 or preamble, Msg3 and Msg4) between UE and BS with limited number of retransmissions.

Let us introduce probabilistic events A_i={Msg(i) is successfully transmitted}, inverse events $\overline{A_i}$={Msg(i) is unsuccessfully transmitted}, and denote the corresponding probabilities $\mathbf{P}(\{A_i\})=1-p_i$ and $\mathbf{P}(\{\overline{A_i}\})=p_i$, $i \in \{1,3,4\}$. We consider discrete-time

Markov chain $\{\xi_i, i = 0, 1, \ldots, (1 + N_3 + N_4)N_1\}$ over the state space $\mathcal{X} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbb{Z}^3 \bigcup (\mathbf{0}, \mathbf{0}, \mathbf{0})\}$, so that: $\{(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \left(\begin{smallmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ y_1 & y_2 & y_3 & & \\ z_1 & z_2 & & & \end{smallmatrix}\right) : x_1 = \overline{0, N_1}, y_1 = \overline{0, N_3}, z_1 = \overline{0, N_4}, x_4 \le y_3 \le N_3 x_4, x_5 \in \{0, 1\}, y_2 \in \{0, 1\}, z_2 \in \{0, 1\}, \}$, where:

x_1 is total number of transmitted Msg1,

x_2 is the number of successful transmitted Msg1, $(0 \le x_2 \le x_1)$,

x_3 is the number of times the counter N_3 is reached when transmitting Msg3,

x_4 is the number of times the counter N_4 is reached when transmitting Msg4 $(0 \le x_3, x_4 \le x_2)$,

x_5 is an indicator denoting the current state of the last transmitted Msg1, which equals to 1 in case of successful current transmission, 0 – in case of collision,

y_1 is total number of transmitted Msg3 after last successful Msg1's transmission,

y_2 is the number of successful transmitted Msg3 after last successful Msg1's transmission,

y_3 is the number of transmitted Msg3 (both successful and unsuccessful) followed with blocking of all Msg4 transmitted N_4 times,

z_1 is total number of transmitted Msg4 after last successful Msg3's transmission,

z_2 is an indicator denoting the current state of the last transmitted Msg4, which equals to 1 in case of successful transmission, 0 – in case of unsuccessful transmission.

Statement 1. The probability $P(\mathbf{x}, \mathbf{y}, \mathbf{z})$ of visiting the state $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ from the initial state $(\mathbf{0}, \mathbf{0}, \mathbf{0})$ is determined with (1):

$$\begin{aligned} P(\mathbf{x}, \mathbf{y}, \mathbf{z}) = p_1^{x_1 - x_2} (1 - p_1)^{x_3 + x_4 + x_5 u(z_2)} {p_3}^{N_3 x_3 - x_4 + y_3 + (y_1 - y_2) u(y_2 z_2)} \times \\ (1 - p_3)^{x_4 + y_2 u(y_2 z_2)} {p_4}^{N_4 x_4 + (z_1 - z_2) u(z_2)} (1 - p_4)^{z_2} C_{x_1 - 1}^{x_3 + x_4} C_{x_3 + x_4}^{x_3} \times \\ \times \left(\sum_{i=0}^{\frac{y_3 - x_4}{N_4}} (-1)^i C_{x_4}^i C_{y_3 - i N_3 - 1}^{x_4 - 1} \right)^{u(x_4 - 1)}, \end{aligned} \quad (1)$$

where $u(x)$ is Heaviside function. The multipliers with the Heaviside function in the exponents allow to ignore redundant retransmissions that arise if the connection initiation procedure is not successful.

We denote the set of states $\mathcal{X}_s = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) : x_5 = y_2 = z_2 = 1\}$, leading to successful session initiation and the set $\mathcal{X}_f = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) : x_1 = N_1, x_3 + x_4 = x_2, z_2 = 0\}$ of failed initiation procedure states.

Then access success probability and access failure probability are derived with (2) and (3) respectively:

$$\pi_s = \sum_{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathcal{X}_s} P(\mathbf{x}, \mathbf{y}, \mathbf{z}), \quad (2)$$

$$\pi_f = \sum_{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathcal{X}_f} P(\mathbf{x}, \mathbf{y}, \mathbf{z}). \quad (3)$$

Statement 2. For $N_1 \in \{1, 2, 3\}$ expression (2) can be obtained in closed form (4):

$$\begin{aligned} \pi_s = 1 - \frac{4N_1 - 6}{N_1(1 - p_3)} \Bigg[\Big(p_1 + (1 - p_1)\big(p_3^{N_3} + (1 - p_3) p_4^{N_4}\big)\Big)^{N_1} - \\ - p_3 \Big(p_1 + p_3^{N_3 - 1} (1 - p_1) \big(p_3 + (1 - p_3) p_4^{N_4}\big)\Big)^{N_1} \Bigg], \end{aligned} \quad (4)$$

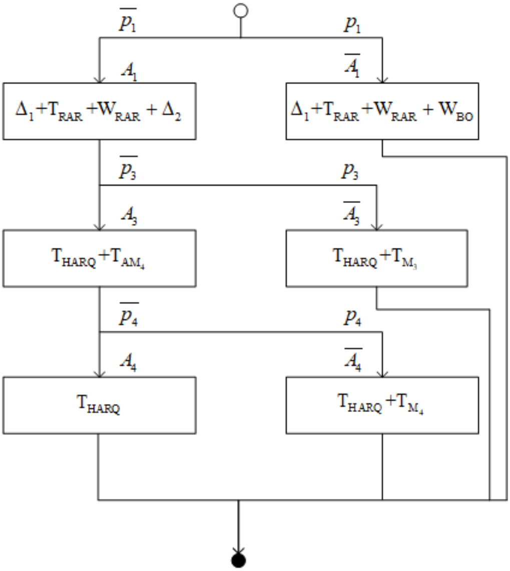

Figure 2. Time intervals for message transmission depending on probabilistic events.

Since the normalization condition $\sum_{(\mathbf{x},\mathbf{y},\mathbf{z})\in\mathcal{X}} P(\mathbf{x},\mathbf{y},\mathbf{z}) = 1$, the access failure probability could be obtained as $\pi_f = 1 - \pi_s$.

On Fig. 2 the scheme of transitions between events of procedure with instructions of corresponding time intervals is presented. Total delay of the transition from the initial state $(\mathbf{0},\mathbf{0},\mathbf{0})$ to the state $(\mathbf{x},\mathbf{y},\mathbf{z})$ is the sum of the time intervals of the transmission corresponding messages, and described by formula (5):

$$\begin{aligned} d = d(\mathbf{x},\mathbf{y},\mathbf{z}) = x_3(\triangle_1 + T_{RAR} + W_{RAR} + \triangle_2 + N_3(T_{HARQ} + T_{M_3}) + \\ + x_4(\triangle_1 + T_{RAR} + W_{RAR} + \triangle_2 + T_{HARQ} + T_{AM_4} + N_4(T_{HARQ} + T_{M_4})) \\ + (y_3 - x_4)(T_{HARQ} + T_{M_3}) + x_5 u(z_2)(\triangle_1 + T_{RAR} + W_{RAR} + \triangle_2) + (x_1 - x_2) \times \\ \times (\triangle_1 + T_{RAR} + W_{RAR} + W_{BO}) + y_2 u(y_2 z_2)(T_{HARQ} + T_{AM_4}) + (y_1 - y_2)u(y_2 z_2) \times \\ \times (T_{HARQ} + T_{M_3}) + (z_1 - z_2)u(z_2)(T_{HARQ} + T_{M_4}) + z_2 T_{HARQ}. \quad (5) \end{aligned}$$

To find average access delay $\overline{d}$ the formula (6) is presented.

$$\overline{d} = \frac{\sum_{(\mathbf{x},\mathbf{y},\mathbf{z})\in X_s} P(\mathbf{x},\mathbf{y},\mathbf{z})\, d(\mathbf{x},\mathbf{y},\mathbf{z})}{\pi_s}. \quad (6)$$

4. Simulation model

This section presents the simulation model of RA procedure. To verify the results obtained using the formulas from Section 2, the code for the simulation model was written, which is a simulated attempt of initiation connection between UE and the BS.

The main part of the code is the `imfn` function. The input values for this function are as follows:

1. the maximum number of retransmissons N_1, N_3, N_4;
2. collision probability p_1;
3. probability of unsuccessful transmissions of Msg3 and Msg4;
4. time intervals vector.

After setting the value to variables using given time interval vector, we build the matrix, which contain the final state that the system transit to after the connection attempt is completed, and the time it takes for one attempt to initiate a connection.

```
imfn=function(N1,N3,N4,p,p34,app,time){
 del1=time[1]; del2=time[2]
 Trar=time[3]; Wrar=time[4]
 Wbo=time[5]
 Tm3=time[6]; Tm4=time[7]
 Tam4=time[8]; Tharq=time[9]
 m=matrix(0,app,14)
 colnames(m)=c("x1","x2","x3","x4","x5","y1","y2",
 "y3","z1","z2","t","c1","c3","c4")
```

Next is the `for` loop, which execute set number of iterations. The more iterations are set, the better simulation is obtained. The body of the `for` loop starts with another `while` loop, which iterates until the connection is initiated or the counter N_1 is exceeded. The body of the `for` loop begins with the adding to the matrix element value $\triangle_1$, which corresponds to the duration, required for the current attempt – the time interval determined for RA Resource seletion before sending the message Msg1. Next, the `sample` function returns one value (0 is the collision of the Msg1, 1 is the successful transmission of the Msg1), which is then added to the value of the matrix element, corresponding to the given total number of Msg1 retransmission. Appropriate time intervals are added to the total duration (T_{RAR}, W_{RAR}).

```
for(i in 1:app){
while(m[i,1]<N1 && m[i,10]!=1){
m[i,11]=m[i,11]+del1
m[i,12]=m[i,12]+1
res1=sample(c(0,1), size=1, replace=T, prob=c(p,(1-p)))
m[i,1]=m[i,1]+1
m[i,11]=m[i,11]+Trar+Wrar
m[i,12]=m[i,12]+1
```

5. Numerical analysis

In this section probabilistic characteristics of the procedure are analyzed. The results of calculations using analytical and simulation models are compared.

We define three scenarios, which differs on the allowed numbers of retransmissions N_1, N_3 and N_4: first scenario stands for $N_1 = N_3 = N_4 = 2$ number of attempts, second presents $N_1 = 4$, $N_3 = N_4 = 2$ and third is $N_1 = 10$, $N_3 = N_4 = 5$. Furthermore, each scenario presents two different sets of time intervals, needed for preamble processing time: set `"a"` define $\triangle_1 = 1$ ms, $\triangle_2 = 1$ ms, $T_{RAR} = 1$ ms, $W_{RAR} = 1$ ms, and set `"b"` define $\triangle_1 = 5$ ms, $\triangle_2 = 2$ ms, $T_{RAR} = 2$ ms, $W_{RAR} = 5$ ms.

The results of simulation model are statistics, collected with 500000 iterations, and presented on Fig. 3 and Fig. 4. Fig. 3 depicts the number of successful and unsuccessful states of the system at the end of the procedure in dependence of collision probability.

The state at the end of procedure is considered final if the last message – Msg4 – is transmitted successfully or if the number of attempts N_1 is reached. The light gray block consists of all attempts, needed for system to obtain the state, which belongs to $\mathcal{X}_f$, and dark gray block counts all attempts, at the end of which the system falls into a state of the set $\mathcal{X}_s$. In Fig. 4 we obtain the total number of preamble retransmission due to reaching N_3 or N_4.

We use the same scenarios to obtain the analytical results using formula (6) for average access delay. As it could be seen on Fig. 5, with increasing of preamble' and HARQ's retransmission attempts, average access delay expectedly increases. Specifically, assuming conditions with long-term preamble processing time (all `b` scenarios), the longest delay is 40.4 ms, but in scenario `3-b` it increases more than 4 times and reaches value 172.3 ms. Significantly reducing average delay is possible by assuming scenario `a`, thereby characteristics becomes 25.9 ms and 121.9 ms respectively.

Another numerical result was obtained using analytical formulas (1) and (2) to find the access success probability in the dependence of preamble collision in case of limiting attempts $N_1 = 10$, $N_3 = N_4 = 5$. As it could be seen on Fig. 6 for different low values of Msg3/Msg4 retransmission probabilities $p_3 = p_4 = \{0.1, 0.3, 0.5\}$, graphs almost match: for $p_3 = p_4 = 0.1$ probability $\pi_s = 0.6513$, for $p_3 = p_4 = 0.3$ probability $\pi_s = 0.6494$, for $p_3 = p_4 = 0.5$ probability $\pi_s = 0.6267$. But for higher values of p_3 and p_4 even under condition of minimum value of collision probability $p_1 = 0.001$, access success probability does not exceed value 0.8402.

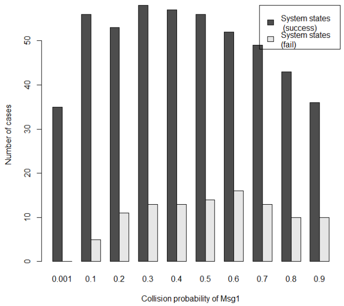

Figure 3. Number of states for (un)successful access procedure.

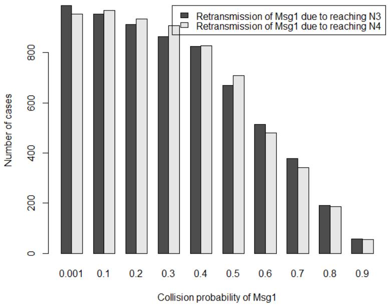

Figure 4. Number of preamble retransmissions.

To compare the results, obtained with analytical and simulation models, we use relative deviation formulas (7) and (8):

$$\epsilon_{prob} = \frac{|\pi_s^{anal} - \pi_s^{sim}|}{\pi_s^{anal}} \times 100\%, \tag{7}$$

$$\epsilon_{delay} = \frac{|\overline{d}^{anal} - \overline{d}^{sim}|}{\overline{d}^{anal}} \times 100\%. \tag{8}$$

Table 1 shows the comparison results. Considering such a small difference in deviations, it can be concluded that analytical formulas are correct.

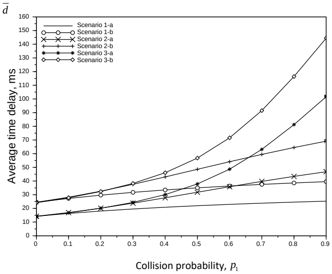

Figure 5. Average time delay in different scenario.

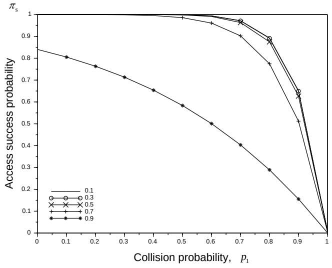

Figure 6. Access success probability for different retransmission's probability of Msg3/Msg4.

Table 1

Comparison of analytical and simulation models

p_1	π_s^{sim}	π_s^{anal} (2)	ϵ_{prob}	$\overline{d}^{sim}$	$\overline{d}^{anal}$ (6)	ϵ_{delay}
0.001	1	1	0.0000E+00	24.194	24.144	2.0431E-03d
0.1	1	1	0.0000E+00	27.865	27.778	3.1118E-03
0.2	1	1	1.0200E-07	32.413	32.362	1.5880E-03
0.3	1	0.99999	4.0920E-06	38.331	38.253	2.0550E-03
0.4	0.9999	0.9999	4.4894E-05	46.251	46.077	3.7682E-03
0.5	0.9991	0.99902	6.6823E-05	56.51	56.789	4.9141E-03
0.6	0.9939	0.99395	8.3078E-05	71.519	71.605	1.1963E-03
0.7	0.9722	0.97175	4.2186E-04	91.917	91.519	4.3487E-03
0.8	0.8918	0.89262	9.0794E-04	116.36	116.42	4.9418E-04
0.9	0.6503	0.65131	1.4951E-03	144.55	144.45	7.1568E-04

6. Conclusions

The main results obtained within this study can be implemented in the concepts of smart parking in big cities or flame detector in remote industrial. The average delay for transmitting data from UE is important in the performance of the technical conditions: information on the status of each sensors must be provided in real time. Our results indicate the exponentiate growth of access delay in case of longer longer preamble processing time' assignment, and according to the collected statistical data, the probability of successful connection with the increase in the probability of collision decreases slowly.

The model is planned to be used in simulating adaptive radio access schemes for LTE networks as a development of previous research [14].

Acknowledgments

The publication has been prepared with the support of the "RUDN University Program 5-100" and funded by RFBR according to the research projects No. 17-07-00845, 19-07-00933. The authors thank Elvira Zaripova for fruitful discussions.

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