Using Entropy Function for Definition States of Information System

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Abstract

The concept of a state of L. Zadeh of the theory of systems is in detail studied. The yielded concept educes with reference to information systems. It is offered to define a state and quantitatively to estimate, as well as a yield of information system, the entropy distribution function. Transferring from L. Zadeh theory to use of the equation of Kolmogorov`s–Chepmen`s is offered. As the initial data construction of distribution functions of entropy is recommended.

1 Introduction

The concept of state is often used in science and technology. It is most simply defined in the theory of the operation of systems. It is simply a set of values of the parameters of the elements of the systems. However, in the theory of systems, the concept of state considered more precisely and strictly depending on the type of system. Researchers especially associate this concept with dynamic systems, more precisely with continuous and discrete systems.

Currently, there is a particular interest in the study of information systems. For example, in [Mar14] the value of entropy considered in the study of the state of information systems, the essence of informational entropy analyzed. However, the concept of state in information systems not formally defined. This article solves two problems: first, to connect the concept of state with the classical results of systems theory and second, to connect this concept with the achievements of modern information theory. The verbal formulation of the concept of the state of the information system follows from the results of the article.

2 Formal Analogue of the State of the Dynamic Systems Theory

A dynamical system, according to [Nem49], is a group of transformations *{R_i}*, defined on a separable metric space *R* and having the properties:

1. R_i Defined for all t on $-\infty < t < \infty$.

2. The function $q = f(p,t)$, where q – the image of a point p from in R in accordance with R_i, has a group property:

$$f(p,t_0+t) = f(f(p,t_0),t).$$

3. The group R_i is continuous in the sense that for all t_0 and p_0, and sequences $\{t_n\}$ and $\{p_n\}$, converging to t_0 and p_0, the relation is true

$$\lim_{n\to\infty} f(p_n,t_n) = f(p_0,t_0). \quad (2)$$

The element p of R is the state of the dynamic system, and $q=f(p,t)$, describes the state of the system at the moment *t* provided that at the moment *t*=0 the system was in the state *p*.

It is formulated on the basis of the analysis of problems of celestial mechanics or problems of dynamics of a solid body. Therefore, the system inputs and outputs are not explicitly highlighted in the definition. This definition requires a slight change.

3 Formal Analogue of the State from Information Theory

An information system is a group of transformations $\{H_i\}$ defined on the probabilistic space *H* and possessing properties:

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1. The H_i transformations defined for all t on $[0, \infty)$.

2. The function $g=f(h,t)$, where g is the image of a point from H according to H_i, has a group property

$$f(h,t_0+t)=f(f(h,t_0),t). \quad (3)$$

3. The group Hi is continuous in the sense that for all t_0 and h_0 and all sequences $\{t_n\}$ and $\{h_n\}$ converging to t_0 and h_0, the relation is true

$$\lim_{n\to\infty} f(h_n,t_n)=f(h_0,t_0). \quad (4)$$

Element h of H is the state of the information system, and $g=f(h,t)$, describes the state of the information system at the time t, provided that at the time $t=0$ the system was in the state of h. This definition needs to specified and clarified.

As a function $f(t)$, in our opinion, we can take the density or entropy distribution function supplied to the input of the system. For example, we consider the function of differential entropy for normal distribution with probability density $f(t)=dnorm(t, m, \sigma)$, m=100 units, σ=20 units. It has the form:

$$h_1(t)=-\int_0^t f(z)\ln(f(z))dz. \quad (5)$$

Expression (5) is the first initial moment of the random entropy. Second initial entropy moment:

$$h_2(t)=-\int_0^t f(z)(\ln(f(z)))^2dz. \quad (6)$$

Similarly, we can find the higher initial moments of entropy. In practical applications, it is enough to limit you by two points. At the Figure 1 a graph of the $h_i(t)$ function is shown.

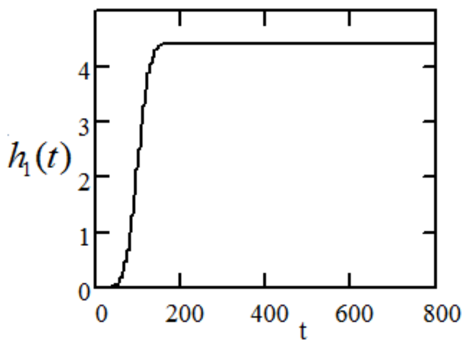

Figure 1: Graph of the $h_i(t)$ function

For our example, the minimum entropy value is 0 nat. and a maximum of 4, 415 nat. Median value h_1 (100)=2,207 nat. For the first case, the initial moments are: v_1(800)=4,415 nat. v_2(800)=19,989 nat.^{2}, the standard deviation δ(800)=0.707 nat., coefficient of variation η(800)=0.16. For the second case, the corresponding values: v_1(100)=2.207 nat., v_2 (100)=9.905 nat.^{2}, δ (100)=2,263 nat., η(100)=1.0125. For the given data, the probability density of random entropy values determined by the relations:

$$g_1(x)=\frac{1}{\sqrt{2\pi}\times 0.707}\times e^{-\frac{(x-4.415)^2}{2\times 0.707^2}},$$
$$g_2(x)=\frac{1}{\sqrt{2\pi}\times 2.263}\times e^{-\frac{(x-2.207)^2}{2\times 2.263^2}}. \quad (7)$$

Graphics densities $g_1(x)$ and $g_2(x)$ are provided below in Figure 2.

4 The Concept of Oriented Abstract Object L. Zadeh

According to [Zad64, Zad63] under the oriented abstract object (OAO) understand a certain system associated with some input signal (cause) u and output signal (consequence). Both signals are understood as vector functions of time. The relationship between them is not straightforward. A specific function u can correspond to several output functions y, and, conversely, a specific output signal can correspond to several input functions.

To formalize the OAO, the segment of the function u defined on the observation interval $[t_0, t_1]$, is denoted $u[t_0, t_1]$ on the closed or $u(t_0, t_1]$ on semi–open interval, depending on the context- simply u. As a result of the experimental study, a set of input–output pairs $(u(t_0, t_1], y(t_0, t_1])$ is usually obtained.

If the same signal is applied to the input of another sample of the test device, the output signal it does not have to be the same as in the first case, since the initial conditions for the second sample may be different. Therefore, this definition [Nem49] reflects the fact that more than one $y(t_0, t_1]$ can correspond to a given $u(t_0, t_1]$.

The set of ordered pairs of time functions on the specified interval denoted as

$$R(t_0,t_1]=\{u(t_0,t_1],y(t_0,t_1]\}. \quad (8)$$

Based on this concept, the following definition proposed in [Zad64]. OAO a is a family $R(t_0, t_1]$ = $\{u(t_0, t_1], y(t_0, t_1]\}$, $t_0, t_1 \in (0,\infty)$ of sets of ordered pairs (u, t) of time functions. Here the first element in (8) called the segment of the input signal or simply the input signal, and the second – the segment of the output signal or simply the output signal. Thus, the OAO identified with a set of input–output pairs that belong to the A. In addition, any segment of the pair for which $t_0 \le \tau_0 \le t_1$, $\tau_0 \le \tau_1 \le t_1$ must belong to the A.

The set of all segments u on the interval $(t_0, t_1]$, such that $(u,y)\in A$, called the space of input signals A and denote $R[u]$. Similarly, the set of all segments y, such that $(u,y)\in A$, called the output signal space and denote $R[y]$. It follows that the set $R(t_0, t_1]$ of all pairs $(u(t_0, t_1], y(t_0, t_1]) \in A$, there is some subset of the product $R[u]\times R[y]$. In the "list" of ordered pairs (u, y) each fixed u corresponds, generally speaking, to a set of different y and, conversely, to each fixed y – a set of different pairs.

From a mathematical point of view, this essentially boils down to defining the system as a relationship rather than, as usual, some function or operator. The difference can explained by the example of the integrator. The values of the input and output signals at the same time t are related to each other by a differential equation

$$\frac{dy(t)}{dt}=u(t). \tag{9}$$

The statement that the integrator is OAO can described by a set of ordered pairs of functions of time of the form

$$(u(t),\alpha+\int_{t_0}^{t_1}u(\xi)d\xi, t_0\leq t\leq t_1\in(0,\infty),$$

where the parameter α belongs to the space of real numbers, and the function u – to the class of time functions, integrable on any finite interval. In this case, each fixed value u $(t_0, t_1]$ corresponds to a set $y(t_0, t_1]$, each element of which corresponds to different values of the parameter α:

$$y(t)=\alpha+\int_{t_0}^{t_1}u(\xi)d\xi, t_0\leq t\leq t_1. \tag{10}$$

Any mathematical relation between u and y, that defining the set of pairs of input–output that form A is called the characteristic input–output for A. In this sense, (10) is a characteristic input / output for A. More generally, if the input and output signals of the system A satisfy differential equation of the form

$$\begin{aligned} a_n\frac{d^n y}{dt^n}+\cdots+a_0(t)y= \\ =b_m(t)\frac{d^m u}{dt^m}+\cdots+b_0(t). \end{aligned} \tag{11}$$

Then this equation is the input–output characteristic for A, since it defines the set of all input–output pairs belonging to A.

It is useful to parameterize (or move) many input–output pairs R $(t_0, t_1]$ so that each segment of the input signal u $(t_0, t_1]$ and each parameter value corresponds to a single segment of the output signal $y(t_0, t_1]$. Such a parameterization would correspond, roughly speaking, to the page numbering of the "list" of input–output pairs, on each page of which pairs with the same output signals are written out. A States are essentially the values of such a parameter. From this point of view, the main role of the concept of state is to provide the ability to associate a single output signal with each input signal, using the state of the system as a parameter.

5 Concept of State

We present an approach to the construction of the concept of the state of L. Zadeh [Zad64]. *Statement*: based on the content of section 4, it can be assumed that parameter α parametrizes A if there is some function $\underline{A}$ defined on the product $\Sigma x R[u]$ and such that for all pairs (u, y) belonging to A and all t_0 and t_1 can be chosen from Σ such α that

$$y=\underline{A}(\alpha;u). \tag{12}$$

For each α of Σ and for each u of $R[u]$ in this case, the pair $(u, \underline{A}(\alpha;u))$ is an input-output pair, which belongs to the A. To call α by the state of the system, it is necessary for the function $\underline{A}$ to have the property of conjugating reactions, which formulated as follows. We agree that uv denotes a signal in which a segment $v=v$ $(t, t_1]$ follows a segment $u=u(t, t_1]$. This is one of the reasons for choosing to use half-open observation intervals. Otherwise, there would be a difficulty with the definition uv at the point t, provided that $u(t)\neq v(t)$. In particular, if by definition $u=u(t_0, t_1]$ and $u=u(t, t_1]$, then $uu=u(t_0, t_1]$.

Definition 1. A function $\underline{A}$ $(\alpha; u)$ has the property of conjugating reactions: if for each α from Σ and each uu of $R[uu]$ there is an element α * from Σ, uniquely defined by α and u, that

$$\underline{A}(\alpha;uu^{`})=\underline{A}(\alpha;u)\underline{A}(\alpha^{*};u). \tag{13}$$

Condition (13) means that the output signal (the response of the system corresponding to the value of the parameter α and the segment uu of the output signal) coincides with the response segment corresponding to the parameter α and the input signal u, followed by the response segment corresponding to the parameter α* and the input signal u.

Definition 2. If α is used to parameterize A, and the function $\underline{A}(\alpha;u)$ has the property of conjugation of reactions, then the elements Σ represent the state A, the space Σ is called the state space A, and the input-output characteristic is the state of the system A. If $u=u(t_0, t_1]$, then α of $\underline{A}(\alpha;u)$ is called the initial state of the system A at time t_0 and is denoted by s (t_0). In this regard, the characteristic input-output-state of the system A can be represented in a more explicit form as

$$y(t_0,t]=\underline{A}(s(t_0);u(t_0,t]), \tag{14}$$

Where $u(t_0, t_1]$ is the segment of the input signal, $s(t_0)$ – the initial state of the system, and y $(t_0, t]$ – the corresponding output signal. Thus, equation (14) States that the initial state of the system A at the time t_0 and the interval $u(t_0, t_1]$ of the input signal uniquely determines the interval of reactions y $(t_0, t]$.

Definition 3. Let system A be in the state s $(t_0)=\alpha$ and at its input a signal $u=u$ $(t_0, t_1]$ is given.

Thanks to conjugation of the reactions $\underline{A}$ ($\alpha; u$), there is an element $\alpha^* \in \Sigma$ such that the equation (13) holds for any $u = u\ (t, t_1]$.

The element α^*, which is uniquely determined by the values s (t_0) and $u = u$ (t_0, t_1], is called the state of system A at time t and is denoted by s (t). Thus, the state of the system at time t uniquely determined by the state of the system in time t_0 and the value of the signal at its input in the interval between these points in time. Symbolically

$$s(t) = s(s(t_0), u(t_0, t_1), \qquad (15)$$

and the resulting equation is called the state equation A. Therefore, the conjugation property of reactions (13) can expressed as:

$$\underline{A}(s(t_0); uu^{`}) = \underline{A}(s(t_0); u)\underline{A}(s(t); u). \qquad (16)$$

The reaction of system A, which is in the state s (t_0), to the input signal uu must be identical to the response of system A, which is in the state s (t_0), to the input signal u and the subsequent reaction of the same system, which is in the state s (t), at the input signal u.

In [Zad64] it is shown that the function $\underline{A}$ (α;u) has the property of conjugation of reactions defined by equations (13) and (16), it follows that the function from equation (15) has the property of conjugation of states

$$s(s(t_0); uu^{`}) = s(s(t_0); u); u^{`}). \qquad (17)$$

This property is equivalent to the group property 2 in the definition of the dynamic system. Consider a simple example with the input output characteristic:

$$\frac{dy}{dt} + y = u. \qquad (18)$$

In this case, the input–output pairs defined on have the form

$$(u(t)), \alpha e^{-(t-t_0)} + \int_{t_0}^{t} e^{-(t-\xi)} u(\xi) d\xi, \quad t_0 < t \le t_1. \qquad (19)$$

If we identify Σ with the axis of real numbers (0, ∞) then the parameter α from equation (19) can be used to parameterize A. Moreover, writing the equation

$$y(t) = \alpha e^{-(t-t_0)} + \int_{t_0}^{t} e^{-(t-\xi)} u(\xi) d\xi, \quad t_0 < t \le t_1, \qquad (20)$$

it is easy to verify the validity of an identity:

$$\alpha e^{-(t-t_0)} + \int_{t_0}^{t} e^{-(t-\xi)} u(\xi) d\xi = = \alpha e^{-(t-\tau)} + \int_{t_0}^{t} e^{-(\tau-\xi)} u(\xi) d\xi, \qquad (21)$$

Where $t_0 \le \tau_0 \le t$ and

$$\alpha^* = \alpha e^{-(\tau-t_0)} + \int_{t_0}^{t} e^{-(\tau-\xi)} u(\xi) d\xi. \qquad (22)$$

Equation (20) is equivalent to the relation of the form (13) y = $\underline{A}$ (α; u), since it determines the values of y for $t > t_0$. Moreover, equations (20) and (22) indicate that the function on the right side of equation (20) has the property of conjugation of reactions. Therefore, equation (20) can be called the input–output–state characteristic for system A, where α is the state of the system at time t_0 and Σ = (0, ∞), we also note that putting $t = t_0$ (which is valid if it does not contain delta functions with a singularity at the point t_0), we obtain

$$s(t_0) = \alpha = y(t_0). \qquad (23)$$

It follows that the state of system A at time t_0 can identified with the output signal of this system at time t_0. This concludes the state definition and an example illustrating the definition.

As result of the study of the concept of "state" L. Zadeh note the following.

1. The result is the introduction of the concept of an abstract object, defined as a family of ordered pairs of time functions. An abstract object is defined by itself, regardless of how the concept of state is introduced to it.

2. The concept of state introduced as a method of parameterization of a set of input–output pairs that provide providing a unique dependence of the output signal and the state of the system. There are countless ways to parameterize input-output pairs. Hence, we should conclude that any characterization of input–output can match many of the characteristics of the input–output–state are essentially equivalent. The input–output–state characteristic can considered as a description of an oriented abstract object with a specific choice of a system of parameters for a set of its input–output pairs.

3. Definition 3 extends to a broader class of systems than dynamic systems. In this regard, definitions 1 and 2 are more General definitions of the concept of state than the indirect definition of the concept contained implicitly in the definition of a dynamic system.

6 An Example of the Concept of State in the Information System

As an input u (t), we use the density functions of the distributions of the random variable of entropy (7) – g_1 (t), g_2 (t), shown in Figure 2.

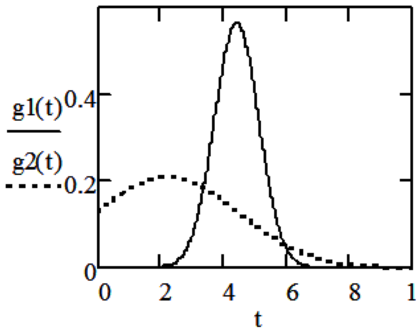

Figure 2: Density functions

We apply these functions to construct states and exit functions of information systems, applying the results of the theory of L. Zade. To illustrate the calculations, we use the integrator element. The dependence for it input–output is represented by a differential equation:

$$\frac{dy}{dt} + y = u. \quad (24)$$

In this equation, u_1 (t) = g_1 (t), u_2 (t) = g_2 (t). Since it is represented by OAO, the first dependence can be described by a set of ordered pairs of time functions of the following form (for example, g_1 (t)).

$$(g_1(t)), \alpha + \int_{t_0}^{t_1} g_1(\xi)d\xi, \quad t_0 \le t \le t_1 \in (0,\infty) \quad . \quad (25)$$

In this case, each fixed value g_1 (t_0, t_1) corresponds to a certain set y (t_0, t_1], each element of which corresponds to different values of the parameter α:

$$y(t) = \alpha + \int_{t_0}^{t_1} g_1(\xi)d\xi, \quad t_0 \le t \le t_1, t \in (0,\infty) \quad . \quad (26)$$

This relationship between g_1 (t) and y, which determines the set of input–output pairs that make up system A, is the input–output characteristic for A, and α, the state of the system. But for this, it is necessary to require that the function of system A, on the basis of parametrization, has the property of conjugating reactions and define a new function $y = \underline{A}$ (α; u), satisfying the property

$$\underline{A}(\alpha; uu^{`}) = \underline{A}(\alpha; u)\underline{A}(\alpha^*; u), \quad (27)$$

where $\alpha^* = \alpha e^{-(\tau - t_0)} + \int_{t_0}^{t} e^{-(\tau-\xi)} g_1(\xi)d\xi$, and uv is a signal in which the segment $v = g_1$ (t, t_1] follows the segment $u = g_1$ (t_0, t]. In this case, we can assert

$$s(t_0) = \alpha = y(t_0), \quad (28)$$

which means the state of the system at time t_0. It can identified with the output of this system at time t_0. Followed by

$$s(t) = s(s(t_0); g_1(t_0, t]), \quad (29)$$

$$s(s(t_0); u^{`}) = s(s(t_0); g_1(t_0, t]); g_1(t, t_1]^{`}) \quad . \quad (30)$$

The value of the output variable defined as

$$y_{(t_0,t]} = \overline{A}(s(t_0); g_1(t_0, t]). \quad (31)$$

Consider the numerical presentation of the example with the initial data for the ***maximum entropy point*** in Figure 1. The average entropy value and standard deviation will be equal to v_1 = 4.415 nat., Σ = 0.707 nat. The integrator input function is u_1 (t) = *dnorm* (*t, v_1, σ*). We take the initial values of time t_0 = 1; 3h. For them, the state values will be s (1) = 6.815 nat. and s (3) = 0.023 nat., output variables:

$$y_{11}(t) = s(1) + \int_1^t u_1(\xi)d\xi;$$

$$y_{12}(t) = s(3) + \int_3^t u_1(\xi)d\xi$$

and their integral components are

$$v_{11}(t) = \int_1^t u_1(\xi)d\xi; v12(t) = \int_3^t u_1(\xi)d\xi.$$

In Figure 3 and 4 are graphs of these functions. It follows from the figures that there is practically no difference between the graphs.

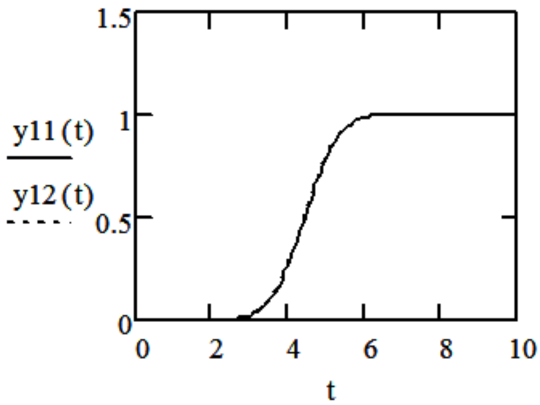

Figure 3: Charts $y_{11}(t)$ and y_{12}(t)

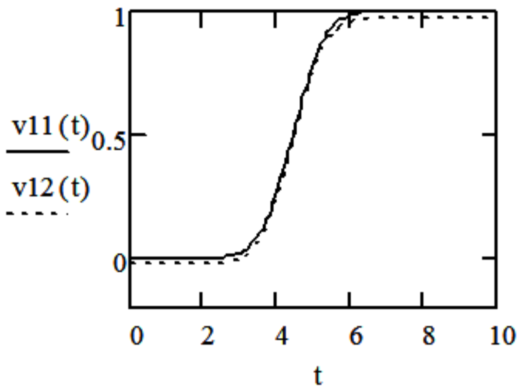

Figure 4: Charts v_{11}(t) and v_{12}(t)

Consider the presentation of the example with the initial data for the point of the ***average value of entropy*** in Figure 1. The mean value of entropy and the standard deviation are v_2 = 2,207 nat., Σ = 2.263 nat. The integrator input function is u_2 (t) = *dnorm* (*t, v_2, σ*). We take the initial values of time t_0 = 1; 3h. For them, the state values are *s* (1) = 0.158 nat., *s* (3) = 0.565 nat., and the output variables are:

$$y_{21}(t) = s(1) + \int_1^t u_2(\xi)d\xi;$$

$$y_{21}(t) = s(3) + \int_3^t u_2(\xi)d\xi.$$

And their integral components are:

$$v_{21}(t) = \int_1^t u_2(\xi)d\xi;$$

$$v_{22}(t) = \int_3^t u_2(\xi)d\xi.$$

Figure 3-6 show how, depending on and, the values of variables at the integrator output, measured by the value of the entropy distribution function. They can also act as the values of future states in the case of continuation in time of the process under consideration.

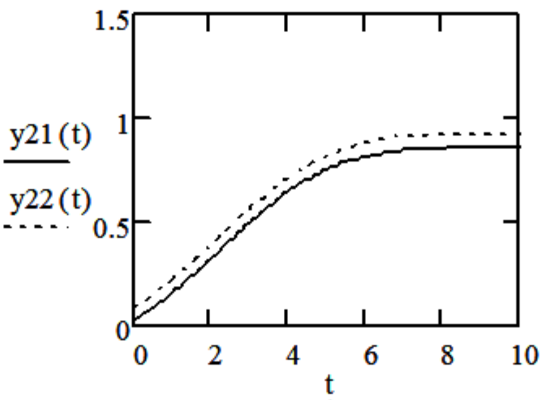

Figure 5: Charts $y_{21}(t)$ and y_{22}(t)

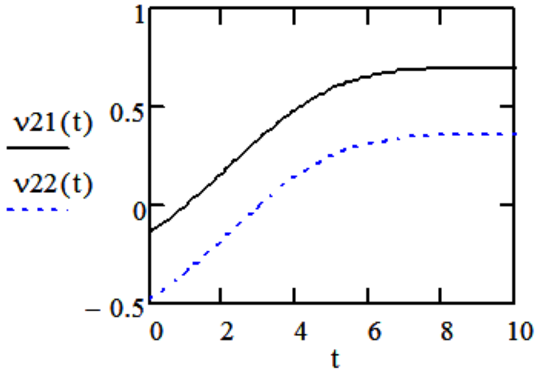

Figure 6: Charts v_{11}(t) and v_{12}(t)

We have considered an example of calculation provided, that the second phase of the process does not depend on the duration of the first phase. This is not fully consistent with equation (32) below. If we take into account this dependence, we will have to build two-dimensional graphs of calculations.

7 Analogy of the Theory of L. Zade and the Kolmogorov–Chapman Equations for Information Systems

Based on the study of the state model of L. Zadeh, a qualitative conclusion suggested: in the information system, the input state can be the value of the entropy distribution function at the initial moment of time before the process of information transformation in the system begins. For the values of the variable at the output of the system, take the values of the entropy distribution function obtained as result of the transformation in the system.

Heuristic statement. For a complex system, as a subject of future research of its informational property, try to apply the Kolmogorov–Chapman equation [Fel57]. This equation described using the theory of L. Zade, but using the entropy distribution functions to determine the states and output variables of the system [Sma10].

Consider an example that is simpler than the integrator, namely, a two-phase single-beam random process from the standpoint of solving the simplest Kolmogorov – Chapman equation. This allows us to show the process of solving the Kolmogorov – Chapman equation and compare the adequacy of the research with the theory of L. Zade. Let us present an example for the numerical illustration of the solution of the Kolmogorov–Chapman equation:

$$\begin{aligned} p_{02}(t_0, t+\Delta t) &= p_{01}(t_0, t) \times \\ &\times p_{12}(t, t+\Delta t), \quad t_0 < t < t+\Delta t \end{aligned} \quad (32)$$

Equation (32) reflects the presence of three discrete states and two phases with continuous distributions following each other. Moreover, the second phase is dependent on the first phase. It is required to calculate the output variable (state 02), if the initial state is determined by the delay in the first phase t_0, and the continuous distributions are independent.

The initial data: t_0 = 10 nat., the first phase f_{01} = *dnorm* (t, v_1, σ_1), v_1 = 50 nat., σ_1 = 12 nat., the second phase f_{12} = *dnorm* (t, v_2, σ_2), v_2 = 40 nat., σ_2 = 7 nat. The probabilities that the phases will be at least t represented as

$$p_{01}(t)=\int_t^{\infty} f_{01}(z)dz,\ p_{12}(t)=\int_t^{\infty} f_{12}(z)dz.$$

The variables t_0, t are measured by the measure nat.

Recall that we are investigating an information system defined by information states and exits, the densities and probabilities introduced above already measured in advance by entropy distributions. Perform the following numerical calculations:

A. Phases are independent.

$$p_{01}(t)=\int_{t_0+t}^{\infty} f_{01}(z)dz \Big/ \int_{t_0}^{\infty} f_{01}(z)dz,$$

$$p_{12}(t)=\int_{t}^{\infty} f_{12}(z)dz, \tag{33 a}$$

$$p_{02}(t)=p_{01}(t)p_{12}(t).$$

The results of the calculations are presented in Figure 7. For example, consider the values of the curves at the point t = 35 nat .: p_{01} (t) = 0.662 nat., p_{12} (t) = 0.762 nat., p_{02} (t) = 0.505 nat.

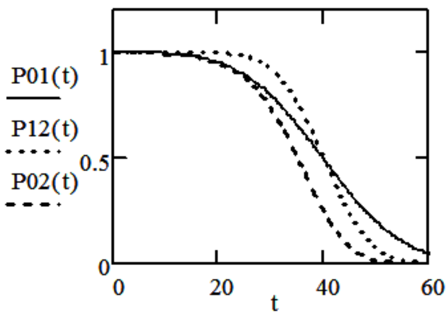

Figure 7: Plots p_{01} (t), p_{12} (t), p_{02} (t) for s (10)

The initial state

$$s(t_0)=\int_0^{t_0} f_{01}(z)dz$$

Is: s (10) = 4.136 × 10^{-4} nat., and s (30) = 0.048 nat.

In Figure 8 s (30) for comparison, an analogue of Figure 7 s (10) is shown. To estimate the uncertainty function at the output of the system based on the application of the Kolmogorov–Chapman equation, the indicator y (t) = 1 – p_{02} (t) should be used instead of the indicator p_{02}(t).

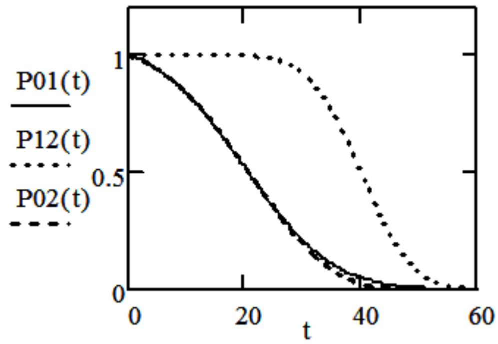

Figure 8: Plots p_{01} (t), p_{12} (t), p_{02} (t) for s(30)

The given example illustrates a method for determining the state and magnitude of a function at the output of a system based on the solution of the Kolmogorov–Chapman equation. Real information systems are more complex, they, as a rule, "multipath", can contain in each "information ray" more than two phases of random processes. The number of states (initial and intermediate) can be very large.

B. Phases are dependent. In this case, the formulas for the probability of phase implementation take the form:

$$p_{01}(t)=\int_{t_0+t}^{\infty} f_{01}(z)dz \Big/ \int_{t_0}^{\infty} f_{01}(z)dz,$$

$$p_{12}(t,t+\Delta t)=\int_{t+\Delta t}^{\infty} f_{12}(z)dz \Big/ \int_{t}^{\infty} f_{01}(z)dz,$$

$$p_{02}(t,t+\Delta t)=p_{01}(t)p_{12}(t,t+\Delta t). \tag{33 b}$$

Let t = *30h.*, Δt = *15h.*, Then p_{01} (30) = 0.798, p_{12} (30.45) = 0.249, p_{02} (30.45) = 0.199. Since p_{02} (t, Δ) is a function of two variables, for it we can to construct a two-dimensional graphical dependence.

Similarly, we can consider an example of the application of the Kolmogorov–Chapman equation covered by feedback. The use of systems of equations of the Kolmogorov–Chapman type for estimating and predicting the values of the indicators of entropic (informational) uncertainty, in our opinion, can be effective. It requires further study.

8 Conclusion

The essence of the proposed model is that the concept of state, the input and output of information systems should be measured by such indicators that measure entropy and information. Therefore, we propose an informational modification of the model L. Zade. To work with such a model, it is necessary, on the basis of the method of moments of a random variable of entropy, to approximately construct the necessary distribution functions of all the components constituting the information system. Then, using these distribution functions, you can apply the model L. Zade. An example of calculation for the integrator [Zad64] is given.

The essence of the method consists in modifying the method for solving the Kolmogorov – Chapman equation by applying in it the distribution functions of the random variable of entropy indicated in the preceding paragraph of the conclusion. The simplest examples for an equation with three discrete states and two random phases with normal distributions are considered. General conclusion: the state and output indicators in the information system should be measured by the entropy (information) associated with a certain probability.

We present a number of modern works related to current applied research areas in the application of information systems models, including those based on the use of entropy. In [Kud16], issues related to the concept of information and terminology in this area, as well as models of information, communication and info communication systems and their interconnection are considered. In [Liv17], an analysis of information security systems is conducted from the position of determining the total entropy of an information system. In [San08], clustering algorithms based on multilevel entropy sub graphs are proposed. In [Kho16], a cloud computing model in information systems with a Web interface based on a multi-channel queuing system with "cooling" and iterative solution of the Kolmogorov-Chapman equations. The concepts of informational entropy, coarse entropy, knowledge granulation and measures of granularity in incomplete information systems are considered in [Lia06].

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