Educational Analogy Dedicated for Didactical Process Simulation

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Abstract. The article presents a new method of modelling the didactical process using a developed educational network and the microsystems simulator. The didactical process can be represented in the intuitive form of a network of connected elements in a similar way to the electrical circuits. The network represents the differential equations describing a dynamic system which models the information flows as well as learning and forgetting phenomena. The solutions of the equations are more adequate than the direct formulas used in modelling ie. the learning and forgetting curves known from the literature. The network variables and their meaning are relative to generalized variables defined in the generalized environment. This enables using any of the microsystems simulators and gives access to many advanced simulation algorithms. The paper can be interesting for those who deal with modelling of the systems which incorporates the learning and forgetting process, in particular, in production processes or learning platforms.

Keywords: Didactical process simulation · Didactical process modelling · Educational analogy · Learning and forgetting curves · Educational environment

1 Introduction

The paper presents a new approach to modelling of the didactical process using the educational network defined by the author. The main reason for dealing with the subject is the apparent lack of use of modern methods of description and simulation in the didactics [37]. However, the mathematical models using direct mathematical formulas were previously created [28].

The initiation of the work was a series of works from various fields of science [12,28,25,19] and knowledge in the field of numerical methods and microsystems simulation methods [31,29]. There are many scientific works in each area. The most important of them, from the point of view of the article, are cited. Detailed issues raised in the article can also be found in the articles [36,32,34,33].

The analysis of the learning and forgetting process is based on forgetting curves [12,46,19] represented by the direct formulas. It was studied in the nineteenth century by Hermann Ebbinghaus [12] who first proposed forgetting curve (FC). The forgetting curve is very fast and after about 5 days about 25% of knowledge remains, then the fall is slower. After 30 days about 20% of the knowledge remains. This curve is still the subject of research. FCs are the solution to the respective differential equations. Their form and coefficients allow matching values to measured data obtained during the experiments. FC describe a dynamic model of brain activity in the sphere of learning. Different functions are used to describe the forgetting curves such as power or exponential [19]. Superpositions of functions (e.g. superposition of exponential functions [49,48]) and the more complicated models such as *Memory Chain Model* [25]) are also used. The learning curves are implemented in repetitive algorithms in many programs i.e. SuperMemo [4], Anki [2]. The learning platforms also support the didactical process (*Smart Learning Platforms*) [14]. The main problem is collecting the user activity data [45,6]. Currently, the analysis of the didactical process is more often based on the large amounts of data (*BigData*) [13,10]. Although, the modelling of forgetting is very important not only in the didactical process. The models are used to describe the efficiency of repetitive operations on production lines [21,7,20]: hyperbolic and exponential models [23,44,5], multiparameter and multidimensional models [8,24,47]. The mentioned above models are mainly based on the analytical equations. Their values of parameters are very sensitive and not intuitive. Even small changes in parameters values strongly affect the result. The better approach to the problem is to find a model based on a differential equation. However, differential equations are difficult to arrange and solve. One should look for methods of more intuitive representation of equations and methods of their effective solution. Example of the model of the brain activity at the level of neurons described as the electrical circuit can be found in [16].

The universal educational environment presented in this work allows the modelling of information flows and their collection in the learning and forgetting processes. It enables the representation of network equations in the form of a schematic (diagram) of connected elements at different levels of abstraction. The network is created by transforming the equations of the generalized network into the educational environment by using analogy [40].

The aim of the article is to present the developed educational environment and the educational network and its applications to modelling and simulating (monitoring) of the didactical process.

The educational environment allows a relatively simple description of various complex phenomena by using elements described by their mathematical models. The network can be represented in the form of a block diagram or connection diagram of network elements (schematic). The network is an intuitive representation of the set of differential equations describing the didactical process, here in a similar way to the electrical schematics. The solutions of the network equations are, in particular, equations describing the forgetting curves known from the literature [12,46,19]. The network can be generated manually (simple didactical process) or automatically (complex didactical process) and can be easily analysed and optimized by using microsystems simulator. It enables the analysis of very complicated didactical processes.

The network equations are formulated automatically thanks to the use of templates [18,27,30] discussed below. It is possible to select the values of the elements parameters and/or change the structure of the network in terms of design constraints. Behavioural modelling enables the use of direct formulas in the element models. In the paper, developed by the author *Model Definition Language* (*MDL*) implemented in the *Dero* simulator was used [30].

The educational environment and models described below were implemented on the *Quela* [42] platform. The platform was design based on the *DIKW* [9,38,22] model (Data, Information, Knowledge, Wisdom). The presented approach models the didactical process at the first, second and third level of the *DIKW* model. The 4th level can be also modelled by user profiling what is not discussed here.

An example of network simulation will be shown later in this work. The didactical process mathematically modelled by using direct mathematical formulas [28] can also be implemented using described below network as well.

2 The Theoretical Backgrounds

The circuit simulators are specialized programs that solve differential equations. The equations are described in various forms, in particular, as mentioned above the network of connected elements. The simulator in the field of electronics implements the nodal approach to the network analysis [18]. Development of the hardware description languages [1] lead to the microsystems simulators. The microsystems simulators are able to analyse the systems which belong to different environments. Each environment has its own restrictions due to the simulation process i.e. the variables should be analysed with individual accuracy, the model inputs and outputs values differ significantly. The educational environment defined below has also its specification, i.e. long time simulation has to be taken into account. The way to overcome difficulties is to define a generalized environment that defines i.e. nodal (*effort*) and branch (*flow*) variables. In each new environment, you can define nodal and branch variables and their accuracy of analysis [41,11]. Equations that occur in different environments are often the same. Thanks to environments, it is possible to use analogies and simulations of systems from different environments using any microsystem simulator. This enables the analysis of e.g. electrical-mechanical systems. This approach was used to create an educational environment. The most important issue is to define network and branch variables and give them their meanings (Table 1). The educational environment variables correspond to the generalized variables and electrical variables as well. Three basic variables are *information*, *information flow* and *knowledge*. The variables can be shown as the vector (1).

Table 1. Generalized variables for the electrical and educational environment.

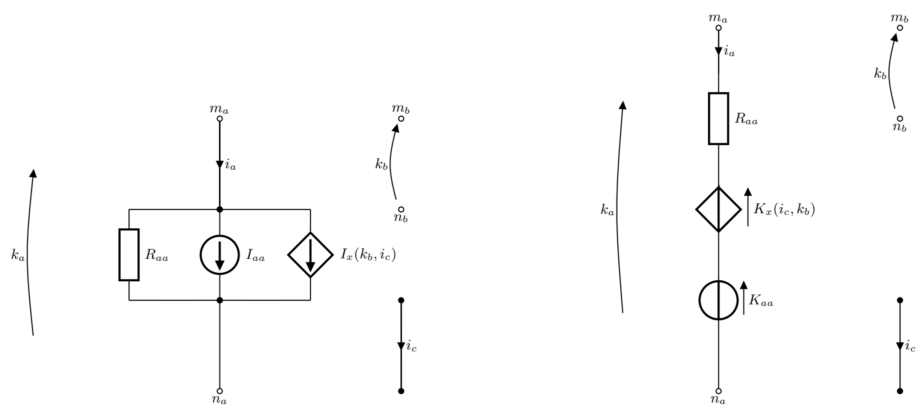
Generalized variables	Electrical env.	Educational env.
e *effort*	v *voltage*	k *knowledge*
f *flow*	i *current*	i *information flow*
p *state*	q *charge*	q *information*
W *energy*	E *energy*	E *workload*
W *work*	W *work*	W *work*

$$x = [k, i, q]^T \tag{1}$$

where: k - variables related to *knowledge*, i - *information flows*, q - variables describing *unit information*. According to the *Modified Nodal Equations* [18], let us define the basic branches of the network and its equations:

1. branch describing *information flow* $i = f_i(x, \dot{x}, t)$,
2. branch describing the level of *knowledge* with the *flow of information* as unknown $k = f_k(x, \dot{x}, t)$,
3. branch describing the level of *knowledge* with the *flow of information* as unknown $q = f_q(x, \dot{x}, t)$,

where $\dot{x}$ is a time derivative of variable x. Let us use the electrical schematics to represent the elements equations. Other graphical representation is also possible but is not so intuitive and well known. Formulation of network equations is accomplished by applying so-called templates [18,27,30] described below.



(a) branch of information (b) branch of knowledge

Fig. 1. The network branches.

The branch of information The branch of information is described by the (2) and its template by the (3).

$$i_a = k_a/R_{aa} + I_{xb}\, k_b + I_{xc}\, i_c + I_{aa} \tag{2}$$

$$\begin{bmatrix} +1/R_{aa} & -1/R_{aa} & +I_{xb} & -I_{xb} & +I_{xc} \\ -1/R_{aa} & +1/R_{aa} & -I_{xb} & +I_{xb} & -I_{xc} \end{bmatrix} \begin{bmatrix} k_{ma} \\ k_{na} \\ k_{mb} \\ k_{nb} \\ i_c \end{bmatrix} = \begin{bmatrix} -I_{aa} \\ +I_{aa} \end{bmatrix} \tag{3}$$

The equation can be represented in the form of the schematic (Fig. 1a). The meaning of the elements results from their equations. The R_{aa} element models losses in the transmission of information between nodes m_a and n_a. The I_{aa} is the source of information. The I_{x*} is the controlled source of information which is used in the modelling of the didactical process.

The branch of knowledge The branch of knowledge is described by the (4) and its template by the (5).

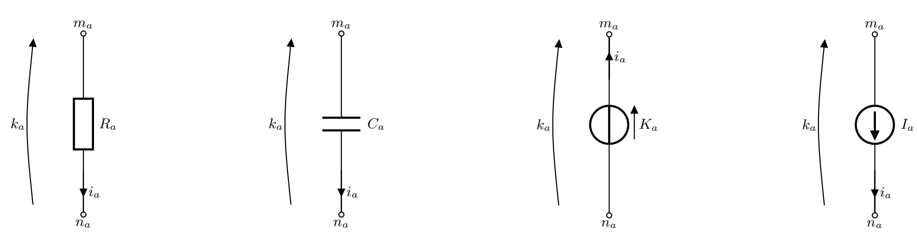
$$k_a = R_{aa}\, i_a + K_{xb}\, k_b + K_{xc}\, i_c + K_{aa} \tag{4}$$

$$\begin{bmatrix} & & & & & +1 \\ & & & & & -1 \\ -1 & +1 & +K_{xb} & -K_{xb} & K_{xc} & R_{aa} \end{bmatrix} \begin{bmatrix} k_{ma} \\ k_{na} \\ k_{mb} \\ k_{nb} \\ i_c \\ i_a \end{bmatrix} = \begin{bmatrix} \\ \\ -K_{aa} \end{bmatrix} \tag{5}$$

The equation (4) can be represented in the form of the schematic (Fig. 1b). The meaning of the elements results from their equations. The R_{aa} element models losses in the transmission of information as described above. The K_{aa} is the source of knowledge. The K_{x*} is the controlled source of knowledge which is used in the modelling of the didactical process.

2.1 The Basic Network Elements and Their Meaning

The use of predefined element models allows to easily create a network description using graphical symbols. Network equations can be formulated automatically by using the *Modified Nodal Equations* [18] as shown above. Let us use again the electrical schematics to represent the elements equations. The basic elements of the network and its equations are shown in Fig. 2. The elements equations can be represented according to the



$i_a = \frac{k_a}{R_a}$ (a) R | $i = C_a \cdot \frac{dk_a}{dt}$ (b) C | $K_a = f(t)$ (c) K | $I_a = f(t)$ (d) I

Fig. 2. Basic elements of the network.

branch equations described above. The template for R element using the information branch is represented by (6).

$$\begin{bmatrix} & & +1 \\ & & -1 \\ -1 & +1 & R_a \end{bmatrix} \begin{bmatrix} k_{ma} \\ k_{na} \\ i_a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

Template for element C (Fig. 2b) depends on the analysis. In the transient time analysis, the derivative $\dot{k_a} = \frac{dk_a}{dt}$ is calculated using differential formula (7) [27,31,30] - trapezoidal method [26,27] or better Gear formulae [15,27,31].

$$\dot{x} = \gamma\, x + d_x \tag{7}$$

where: d_x is a vector of the past of x. Substituting to the equation in Fig. 2b and performing the transformations we get (8).

$$\gamma C_a k_a = -C_a d_{k_a} \tag{8}$$

where $d_{k_a} = d_{k_{ma}} - d_{k_{na}}$ is a vector of the past of k_a. By saving the matrix equation, we get a template of the form (9).

$$\begin{bmatrix} +\gamma C_a & -\gamma C_a \\ -\gamma C_a & +\gamma C_a \end{bmatrix} \begin{bmatrix} k_{ma} \\ k_{na} \end{bmatrix} = \begin{bmatrix} -C_a d_{k_{ma}} \\ +C_a d_{k_{na}} \end{bmatrix} \tag{9}$$

2.2 Solving Network Equations - Classical Time Analysis

The network equations presented above are algebraic differential equations (10).

$$f(x, \dot{x}, t) = 0 \tag{10}$$

where x is an unknown, $\dot{x} = \frac{dx}{dt}$ is the time derivative of x. The classical time analysis makes it possible to determine the time response of the network. The solving process begins with calculating the derivative $\dot{x}$ (discretization) using a differential scheme (11).

$$\dot{x}_n = \gamma\, x_n + d_{xn} \tag{11}$$

where: n is the identifier of the variable, d_x is a vector of the past, γ a coefficient depending on the type of differential scheme. In this way, we will obtain a system of non-linear (possibly linear) algebraic equations (12).

$$f(x_n, \gamma\, x_n + d_{xn}, t_n) = 0 \tag{12}$$

It can be solved, for example, by the Newton-Raphson method [18,27,31] by linearizing equations, i.e. expanding into the Taylor [43] series to the first order.

$$\underbrace{\left[\frac{\delta f(x_n^{p-1}, \gamma\, x_n^{p-1} - d_{xn}, t_n)}{\delta x_{t_n}} + \gamma \frac{\delta f(x_{t_n}^{p-1}, \gamma\, x_n^{p-1} - d_{xn}, t_n)}{\delta \dot{x}_{t_n}}\right]}_{\frac{\delta f^{p-1}}{\delta x_n}} (x_n^p - x_n^{p-1}) =$$
$$= -\underbrace{f(x_n^{p-1}, \gamma\, x_n^{p-1} - d_{xn}, t_n)}_{f^{p-1}} \tag{13}$$

where p is the number of Newton's iteration. After grouping, it takes the form of equations solved iteratively (14).

$$\underbrace{\frac{\delta f^{p-1}}{\delta x_n}}_{Y} \underbrace{{x_n}^p}_{x} = \underbrace{-f^{p-1} + \frac{\delta f^{p-1}}{\delta x_n} x_n^{p-1}}_{B} \tag{14}$$

The system of linear equations $Yx = B$ can be solved by using i.e. the LU method [27]: $L(Ux) = B$.

Further information on the methods and algorithms of network analysis can be found in the literature [18,27,31,30]. The process of solving network equations can be found in [27]. Implementation in the *Dero* simulator is described in details in [30,31].

3 Didactical Process Modelling

As mentioned above, the educational environment enables describing the didactical process in the form of the network of connected elements. Each element is described by its model (equation or set of equations). The didactical process needs several models, in particular, model of the didactical unit, learning and forgetting, exam, and evaluation. The models were developed due to the *DIKW* [9,22] describing relationships between parts of the educational process.

Didactic process in the context of learning objectives The information can be classified in terms of the didactical objectives. According to Bloom's Taxonomy [3], the main categories of learning objectives (o) can be distinguished: knowledge, comprehension, application, analysis, synthesis, evaluation (Table 2). A simplified model can use for

Table 2. Categories of learning objectives.

K	knowledge
C	comprehension
P	application
A	analysis
S	synthesis
E	evaluation

example concepts, procedures, achievements. Thus the learning and forgetting can be simulated in relation to the categories of learning objectives (*LO*). The LO can be treated as a vector of coefficients (15) used to profile student knowledge.

$$o = [o_K, o_C, o_P, o_A, o_S, o_E]^T \tag{15}$$

The values of network variables for each category of learning objectives can be used to monitor every part of the didactical process. The LO coefficients can also be used in the evaluation or optimization process. Let's rewrite the vector (1) including objectives (16).

$$x_o = [k_o, i_o, q_o]^T \tag{16}$$

where: k_o, i_o, q_o are the corresponding vectors of variables respect to the objectives categories (Table 2). This changes elements models and makes models more complicated. Every learning objective has to be modelled separately.

Model for knowledge management and data value extraction The *DIKW* [9,22] model shows the relationship between parts of the educational process. It connects data with information, knowledge, and wisdom. The basis of the model is data that come, for example, through research or discovery. Data is converted into information by presenting it. Information is converted into knowledge. Knowledge changes dynamically and allows to look at a given issue from different perspectives. Knowledge is difficult to transfer to another person. Wisdom is the ultimate level of understanding where main rule plays: analysis, synthesis, and evaluation (in terms of Bloom's taxonomy [3]). Experiences can be shared with other persons. Conversion between individual levels of the *DIKW* [9,22] model occurs in the system described here. Data, in general, are represented by didactical materials included in the didactical units. The amount of data and form determines the ease of data acquisition at the stage of conversion to information. Information presents data. The parameter here is the amount of information being transmitted per unit of time. Knowledge is the amount of processed information stored in the learning process. It can be described by appropriate numerical measures. Wisdom is experience in the use and processing of knowledge in practice. The numerical values of network variables related to the relevant didactical objectives are the measure of knowledge. The interrelationships between values for the category of the learning objectives form the student's profile (describing wisdom).

Didactical units and course The learning process creates engrams [49] or sets of engrams. However, the process of engrams creation still requires research. Taking into account the *DIKW* model, it was assumed that each engram is created by the set of information connected to the set of data, which may not be the case in general. The learning process is modelled as a collection of independent (learning) paths creating the sets of engrams [49]. Taking this into account, it was assumed that the didactical unit (*DU*) can be decomposed into the collection of pieces of information (parts) connected with the data (didactical material *DM*) and forming the appropriate set of engrams. The didactical unit usually uses many different didactical materials. The course is the collection of the didactical units. The knowledge level of the *DU* is the superposition of the knowledge level of its parts (*DM*). The presented modelling method corresponds to the method evaluation of knowledge for individual categories of learning objectives. All topics must be represented in the exam test to correct evaluation of the whole process.

The whole didactical process The result of the whole didactical process is the superposition of the knowledge levels for each course (DC) as shown in Fig. 3. In this way, the total level of knowledge over time is determined. The total level of knowledge changes over time due to material repetitions.

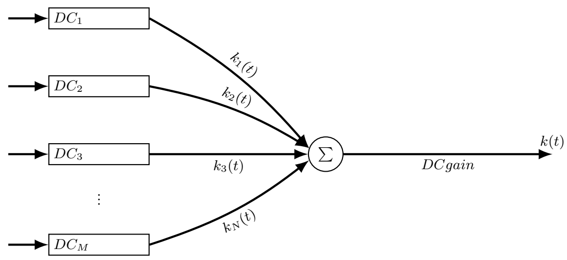


Fig. 3. Total level of knowledge of the didactical process.

Material retention Every repetition of the single *DM* is represented separately as shown in Fig. 4. The total knowledge level of the *DU* is the superposition of the knowledge level

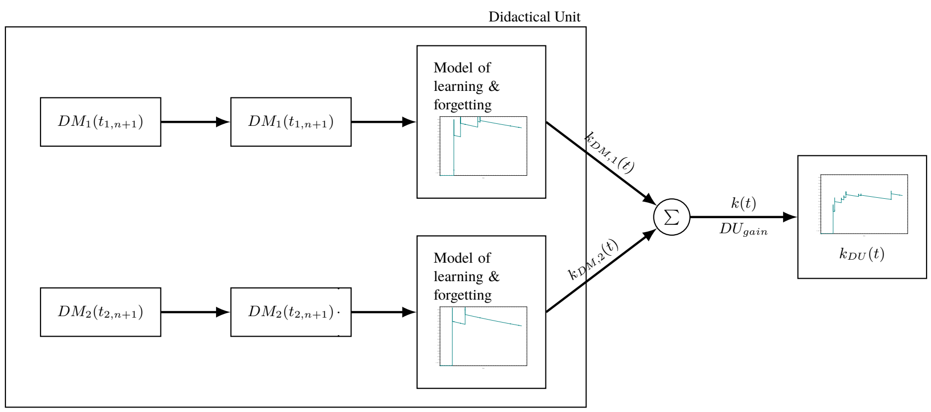
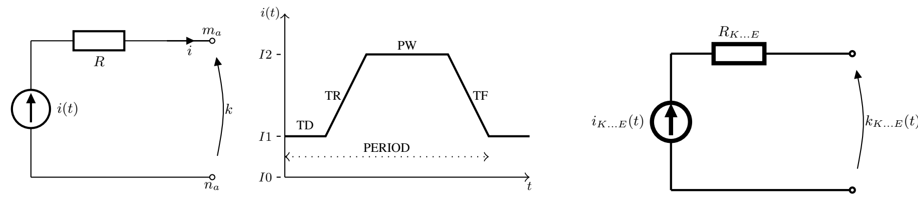


Fig. 4. Retention of the materials in the single didactical unit.

of its parts (*DM*). The total knowledge level of the didactical course is the superposition of the knowledge level of every *DU*.

3.1 Didactical Process As a Network

The information (2nd) level of the *DIKW* [9,38,22] model is responsible for the presentation of data and is related to the flow of information. The information flow is modelled by the elements of the previously discussed educational environment. The parts of the *DU* studied in the paper are modelled as the network of information sources. The information can be collected by the student who transforms them into knowledge. The model is composed of an information source and an information resistance corresponding to difficulties in providing information (Fig. 5). The source of information is related to data. The efficiency of the information source is related to the speed of information transfer per unit of time. Information sources may be related to different categories of



(a) one objective category (b) $A \ldots K$ objective categories

Fig. 5. Model of didactical material.

learning objectives (Table 2). Models for one category of didactical objectives is shown in Fig. 5a. More complicated model for $A \ldots K$ categories (Table 2) is shown in Fig. 5b (bolded). It means that the system has many variables describing information flow and levels of knowledge for particular categories of learning objectives. The didactical unit described above can be modelled using *MDL* language of the Dero [30] simulator.

Model of learning and forgetting As mentioned above, different functions are used to describe the forgetting curve [19], in particular power, exponential, and superposition of functions. As shown in the [49] the superposition of the exponential function probably the better describe the forgetting curve than the power function. As shown in [35], none of such relatively simple functions can fit the forgetting curve in all its points. In the work, the piecewise linear model (Fig. 6) was used. The model is relatively simple

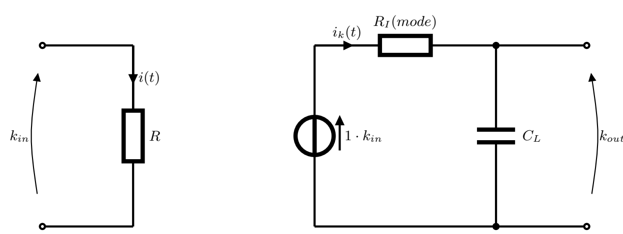


Fig. 6. Piecewise linear model of learning and forgetting.

and in this form do not fit all points on the forgetting curve. The more complex model is required. The model takes into account every category of learning objectives. Parameters of elements can be dependent, for example, on time and/or mode. The model enables flexible modelling of the learning and forgetting by changing the time constant $\tau = R_I C_L$. It is realized by changing the R_I parameter. The mode denotes learning or forgetting. The repetition of the material is also taken into. The initial values of τ are as follow [34]: about $20..31 minutes$ for learning, about $9..12 hours$ for short time forgetting, about $31..43.4 days$ for a long time forgetting. The values changes in time due to i.e. the material repetition. The model describes the learning process for the isolated part of the material which creates single engram or set of engrams. The model is similar to the *Memory Chain Model* [17,39].

Evaluation The evaluation of the courses in term of knowledge level for each objective category can be modelled as a comparator. The results of tests or examinations for individual learning objectives and results of simulations should be provided at the input of the model. The appropriate differences will be obtained at the output of the model. Depending on the results, there may be three cases:

- results of the simulation overlap with test results,
- the results of the simulation are worse,
- the results of the simulation are better than the results obtained.

Evaluation of the didactical process can also be carried out by determining the appropriate user profile for each category of learning objectives. This is possible by setting the appropriate parameters. When comparing exam results and/or values obtained as a result of the simulation, it is possible to determine whether a given user profile is preserved or not.

Network representation of material repetitions In the example, there are 3 repetitions of the same didactical material as shown in Fig. 7. The network describes the flow of infor-

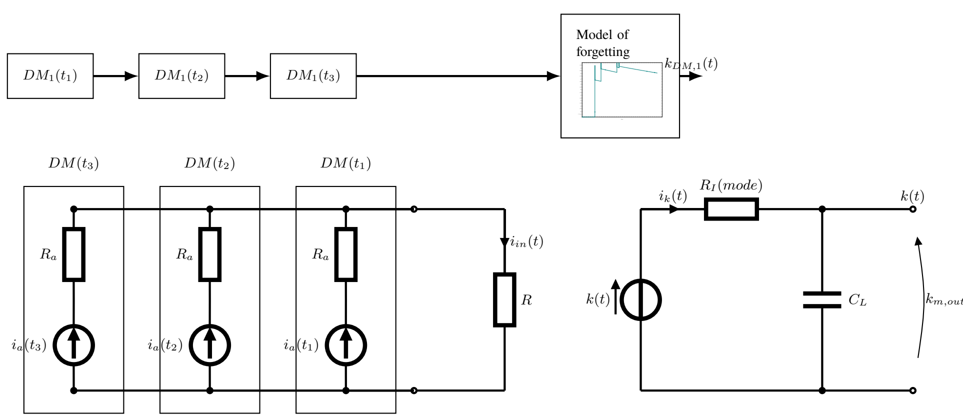


Fig. 7. Network representation of the retention of the didactical material.

mation at the specific time points represented by the information sources. The learning and forgetting are modelled using the model described earlier (Fig. 6). The input file for *Dero* simulator is presented on the Listing 1.1.

Listing 1.1. Example of the input file describing material retention.

```
1  .TASK "EXAMPLE OF THE DIDACTICAL PROCESS SIMULATION"
2  .LIB "types.mdl"         # Data types
3  .LIB "units.mdl"         # Units (minuts,hours, etc)
4  .LIB "math/one.mdl"      # Math function one
5  .LIB "math/pulse.mdl"    # Math function pulse
6  .LIB "edu/vars.mdl"      # Types of net variables
7  .LIB "edu/bus.mdl"       # Bus variables definitions
8  .LIB "edu/inputs.mdl"    # Inputs definitions
9  .LIB "edu/outputs.mdl"   # Outputs definitions
10 .LIB "edu/exam.mdl"      # Exam model
11 .LIB "edu/didacticalunit.mdl" # Didactical unit model
12 .LIB "edu/exercise.mdl"# exercise model
13 .LIB "edu/student.mdl" # learning and forgetting model
14 .MODEL "DM" DIDACTICALUNIT;
15 .MODEL "student" "STUDENT";
16 # didactical material repetitions
17 "DM"."DM1" "OBJID."K 0 "OBJID."<EDUT> I0=0.0 I1=0.0 I2=1.0 W=1.0 TD=2017-01-01T14:00:00 TR=10 TF=10 PW=45min
       ↪ PERIOD=1000days;
18 "DM"."DM2" "OBJID."K 0 "OBJID."<EDUT> I0=0.0 I1=0.0 I2=1.0 W=1.0 TD=2017-02-01T14:00:00 TR=10 TF=10 PW=45min
       ↪ PERIOD=1000days;
19 "DM"."DM3" "OBJID."K 0 "OBJID."<EDUT> I0=0.0 I1=0.0 I2=1.0 W=1.0 TD=2017-05-01T14:00:00 TR=10 TF=10 PW=45min
       ↪ PERIOD=1000days;
20 # learning and forgetting model
21 "student"."ST" "OBJID."<EDU> 0 "OBJID."<EDUT> RIN=1 OKN10=0 OCO20=0 OAP30=0 OAN40=0 OSY50=0 OEV60=0 CO=100 RL
       ↪ =27 RLDIV=0.5 RF=4320 RF5=3116 RF30=96744;
22 .CMD
23 .PROBE ADD TR("*");
24 .PROBE ADD TR("*");
25 .OPTI ALLP=1, HMIN=1;
26 .OP; # initial values
27 .TR STEP=24:00:00 T0=2017-01-01T00:01:00 TMIN=2017-01-01T00:01:00 TMAX=2017-05-15T00:01:00 TFMT=REAL TITLE="
       ↪ DIDACTICAL PROCESS SIMULATION";
28 .END
```

The file can be automatically generated based on student activity. In line 1, the network simulation task starts and the title is set. Model libraries are loaded in lines 2..13. Model lines storing common parameters were placed on lines 14 and 15. Three didactical materials were included in the network description (*DM1*, *DM2* and *DM3* - lines 17 ... 19). The learning and forgetting process is described by the *ST* element (line 21). The command block starts on line 22. The commands for deriving the results of the analysis are on lines 23 and 24. Initial values for differential equations are calculated in line 26. The time analysis command is on line 27. The system description ends with the *.END* directive (line 28).

4 Results and Discussion

The system described above was used in practice. Several assumptions have been made. The output gain is a superposition of the gains for every part of the course. The courses were simulated individually for each student based on their activities. The results of the simulation are compared with the real results obtained during the exams. Questions cover all topics what means the uncertainty of the assessment will not occur. The minimal score of the test was set to 0.51 (51%). Final grades are within limits ($0.51..0.60$, $0.61..0.70$, $0.71..0.80$, $0.81..0.90$, $0.91..1.00$). Example of the student activities during the didactical process for Information Security (IS) course is shown in Fig. 8. The $BI - Knowledge$ represents simulated results of the designed course. The simulated level of student knowledge represents the variable of $Knowledge$. The real level of student knowledge obtained during the exam represents $Evaluation$. The initial level of knowledge was set to 0. The default parameters of the learning and forgetting process were set. The simulation does not include activities that took place outside the registered didactical process. Because of this, there are differences between the simulated and real process. The simulation results are all the more accurate the more activity data

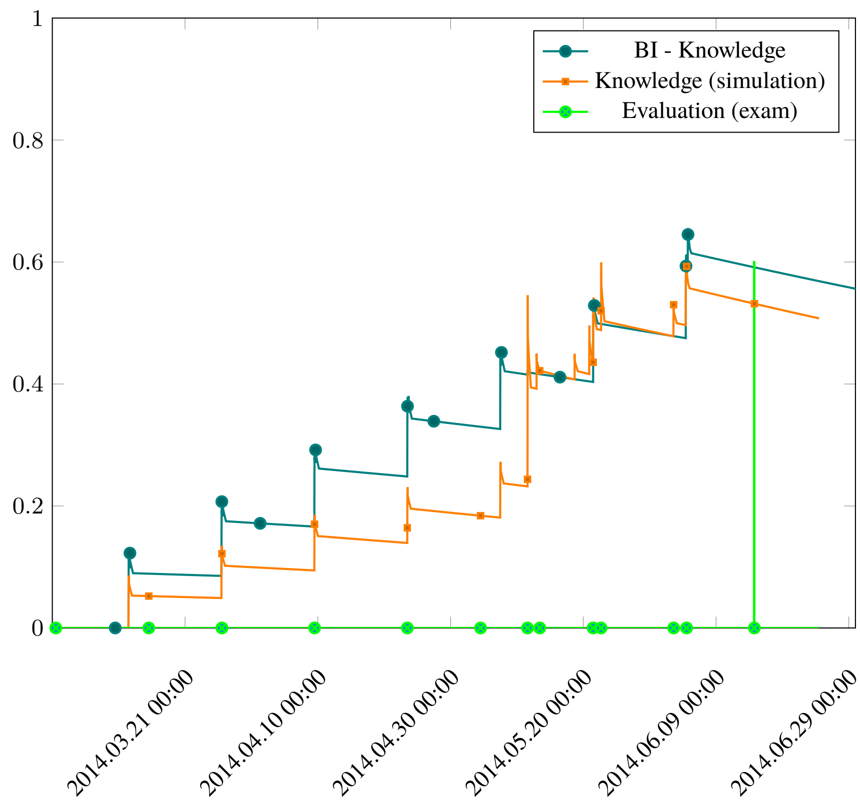


Fig. 8. Didactical process simulation for IS course based on the student's activity.

is available. The expected level of knowledge differs from the real level at around 10%. As can be seen, there are no visible activities before the exam. It means that the student did not use the system - worked off-line on his own materials. Further information and simulation results, including long-term simulations after completing the course, can be found in literature [32,34,35].

5 Conclusions and Prospects for Further Research

As shown in the article the didactical process can be described by the differential equations represented in the intuitive form of schematics (here electrical like). Other notations can also be used or developed (further work). The developed educational analogy enables defining basic types of equations as element models. Behavioural modelling allows creation models based on mathematical functions, including nonlinear ones. The model's parameters can be easily adjusted to the measurement data in the optimization process. The network describing the complicated didactical process should be generated automatically taking into account students activities. The learning and forgetting can be modified using both behavioural modelling or the circuit models (not discussed in the article). The elements models are very sensitive to the input parameters. The most important are τ constants in the model of learning and forgetting, which values have a real-life interpretation (not discussed here).

The presented approach allows the use of microsystems simulator and gives access to many advanced simulation and optimization methods and algorithms implemented in the simulators. It enables designing more ergonomic didactical processes. The tool gives the opportunity to reduce expenditure on the teaching and learning process through more effective management of the process structure, time spent on the learning and the number of material repetitions. It also allows for detecting critical parts of the process. The approach allows reducing the time and costs of designing the didactical process.

The practical implementation of the system on the *Quela* platform is still used in the research. The issues described above can be used in many areas, e.g.: on the learning e-learning platforms, in medical research, in psychology.

Further work will focus on modelling of the simulation process, using faster simulation techniques, the use of advanced techniques (i.e. *ODOS* [26]) to analyze the process.

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