Software-Algorithmic Tool for Analyzing the Processes of Messages Distribution in Social Networks

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Abstract. In this article is considered specific model of the message dissemination in the social network that was recently proposed by the authors. Every person in this model is considered as a neuron which activation function is found as a decision of the specific systems of differential equations which describe the information distribution in the chain of the network graph . This model allows to take into account the specific mechanisms for transmitting messages, where it is considered information graph in which each vertices are individuals who, receiving a message, initially form their attitude towards it, and then decide on the further transmission of this message, provided that the corresponding potential of the interaction of two individuals exceeds a certain threshold level. The authors developed the original algorithm for calculating the time moments of message distribution in the corresponding chain, which comes to the solution of a series of Cauchy problems for systems of ordinary nonlinear differential equations and propose realisation of this algorithm in Maple.

Keywords: Mathematic modelling, Social network, Graph of message flow, Excitation, Activity threshold, Cauchy problem.

1 Introduction

In recent years appear new technologies for data and knowledge organization in order to rationalize social life. In this context it is necessity for the combination of methodological approaches to praxeology and modern information technology. Tadeusz Kotarbinski [1] denote praxeology as the field of scientific research, which studies the general conditions and methods of correct, efficient and rational human activity This area of scientific research analyzes collective actions, considering them as complexes containing a plurality of actions and a plurality of subjects. The main task of praxeology can be formulated as:

- analysis of technology and analytical description of elements and forms of effective activity.
- creation of "rules of action" for the development of general norms of maximum expediency of actions.

In recent articles [34-38] we argue about a powerful public request for the construction of a highly effective systemic interdisciplinary platform that integrates methods and means of modern praxeology, personality psychology and social networks in order to solve technological problems in the field of people communication and create new modelling approach to the process communication of social groups .

Recently, many models of information dissemination in social networks have been considered. This models are based on the results of the researchers from different scientific fields, sociologists, psychologists. D. Iston [2] was formed the hypothesis that political life forms a certain "system immersed in the environment" and for the survival of the system should have the ability to respond effectively to external influences, while maintaining a constant connection with the external environment. K. Doych [3] proposed an information and cybernetic model that reflects the political system as a complex set of information flows and communication links of different levels which are formed by political agents. Mihalo Kozinsky [5] with a team of researchers at the Cambridge Center of Psychology conducted a study cycle, the results of which allowed the creation of an application for Facebook, called MyPersonality (http://mypersonality.org). The user, who was asked to identify the personality profile, responded to the question of the researcher, the creators of the program received data about the person. In 2012, an application was improved, the result of his work testified that by analyzing 68 likes on Facebook fashionable determine the color respondents (95% probability), a commitment to a particular Party USA (85% probability). The constant improvement of the application developers ensured its effective work after ten likes. This technology allows a respondent to receive psychological profile, which describes him quite clearly, are usually more accurate than they could make it work colleagues, 70 likes - better than the second, 150 likes - better than the parents. After 300 likes - it's better than a partner. With more and more analyzed actions of the respondent you can learn more than he can tell about himself [6].

The most important processes that are implemented in social networks is the process of message dissemination and process of public opinion formation. We are talking about models for distributing messages and models for forming the thought of both individuals and communities in general [8]. In [26] we divide this models according to the level of refinement on the corpuscular, in which it is possible to identify an individual for certain multiple characteristics and generalized models which describe the characteristics of groups of individuals or the community as a whole. The generalized models, for example, include the so-called epidemic models [17], the models of innovation diffusion [7], the Delay-Kendall model [15], the message distribution model in society [14], models based on the concept of message density [18]. Corpuscular models include a number of models that use cellular automata [19, 22], cascading models of various types [20], models of network autocorrelation [9], adaptive and imitation behavior model [18], “Game Name” model [21], quantum models are similar to Ising models [18].

Probabilistic approaches, in particular, the Markov chains [15], are widely used in simulation of social and communication processes, in particular various stochastic influences. The classes of tasks of forming and managing public opinion are important to solve problems that arise when it is necessary to change the opinion of individuals or target groups in a certain way due to the influence of certain agents [16].

In [24] was proposed a new approach to modeling the process of opinion dissemination in social group based on the procedures resembles the process of transferring excitation in the nerve cell: if an input signal exceeds a certain threshold, a cell forms a certain signal at the output. When the cell becomes active, its threshold of excitability changes or disappears whith time. Using this approuch we create a new algorithm for calculating the time moments of message distribution in the corresponding chain, which comes to the solution of a series of Cauchy problems for systems of ordinary nonlinear differential equations and propose a realisation of this algorithm in Maple.

2 Some Features of the Base Model

In [24-25] was considered graph $G=(V,U)$ which denote a social group. The process of information interchange was described as follows. Every person can receive a message and generate a new message of the same context if it's social and communication potential exceeds a certain threshold. The message raises the growth of the social and communication potential of an individual according to a certain law (excitation equation of the axon), depending on the mass of the message. Receiving a re-message may also increase the social and communication potential and further re-transmit messages or participate in discussions, forums, etc. In [26] where proposed a certain function $u(x,i,t)$ that describes the level of opinion for each person, its deviation from the state of equilibrium caused by the information $i' \in I$ at some point in time. The force of interaction of two person x_2 and x_1 can be defined as

$$F(u(x_2,i,t)-u(x_1,i,t)) = \begin{cases} f(u(x_2,i,t)-u(x_1,i,t)), t \geq \tau_1, u(x_1,i,t) \neq 0, \\ f(-u(x_1,i,t)), t < \tau_1, u(x_1,i,t) \neq 0, u(x_2,i,t)=0 \\ 0, u(x_1,i,t)=0, \end{cases} \quad (1)$$

$$\tau_k = \min\{t: f(0-u(x_{k-1},i,t)) = \delta(x_{k-1},x_k,i)\}$$

If consider some analogue of the concept of an individual "mass" in the context of the opinion distribution we can build the analogue of the second law of Newton and write the equation system of the dissemination of communication excitation:

$$\begin{aligned} m_k u''(x_k,i,t) &= F(u(x_{k+1},i,t)-u(x_k,i,t)) - \\ &- F(u(x_k,i,t)-u(x_{k-1},i,t)), \end{aligned} \quad (2)$$

$$k=\overline{1,n},\ (x_k,x_{k+1}) \in G(i)$$

where $u(x_k,i,t)=0$, $u'(x_k,i,t)=0$, $u(x_0,i,t)$ is a given function defining the initial perturbation. It is the time of activation:

$$\tau_k = \min\{t: f(0-u(x_{k-1},i,t)) = \delta(x_{k-1},x_k,i)\}. \quad (3)$$

Similarly, we get the general Cauchy problem for the single chaine $(x_1, x_2, ..., x_n)$ in graph $G(i)$ at $\tau_k \le t \le \tau_{k+1}$:

$$m_1 u''(x_1, i, t) = f(u(x_2, i, t) - u(x_1, i, t)) - f(u(x_1, i, t) - u(x_0, i, t)),$$

$$\begin{cases} m_2 u''(x_2, i, t) = f(u(x_3, i, t) - u(x_2, i, t)) - f(u(x_2, i, t) - u(x_1, i, t)), \\ ... \\ m_{k-1} u''(x_{k-1}, i, t) = f(u(x_k, i, t) - u(x_{k-1}, i, t)) - f(u(x_{k-1}, i, t) - u(x_{k-2}, i, t)), \\ m_k u''(x_k, i, t) = f(0 - u(x_k, i, t)) - f(u(x_k, i, t) - u(x_{k-1}, i, t)). \end{cases} \quad (4)$$

Initial conditions:

$u(x_k, i, \tau_k) = 0, u'(x_k, i, \tau_k) = 0$, $u(x_{r-1}, i, \tau_r), u'(x_{r-1}, i, \tau_r)$ are known, $r = \overline{1, k}$.

Then

$$\tau_{k+1} = \min\{t : f(0 - u(x_k, i, t)) = \delta(x_k, x_{k+1}, i)\}. \quad (5)$$

Taking into account “space” distribution of social group we get the system of equation:

$$\begin{aligned} & m_k u''(x_k, i, t) = \Xi_{out}\{F(u(x_p, i, t) - u(x_k, i, t)), \\ & (x_k, x_p) \in G(i)\} - \Xi_{inp}\{F(u(x_k, i, t) - u(x_p, i, t)), (x_p, x_k) \in G(i)\}, x_k \in U, \end{aligned} \quad (6)$$

where operator Ξ_{out} decscribe the summary influence of the object x_k for all partners x_p, $(x_k, x_p) \in G(i)$, Ξ_{inp} decscribe the summary influence of all partners x_p for the object x_k $(x_p, x_k) \in G(i)$.

Here we consider such operators:

$$\Xi_{out}\{F(u(x_p, i, t) - u(x_k, i, t)), (x_k, x_p) \in G(i)\} =$$
$$= F(u(x_{p_0}, i, t) - u(x_k, i, t), \tau(x_{p_0}) = \min_p \tau(x_p),$$
$$\Xi_{inp}\{F(u(x_k, i, t) - u(x_p, i, t)), (x_p, x_k) \in G(i)\} =$$
$$= F(u(x_k, i, t) - u(x_{p_0}, i, t)), \tau(x_{p_0}) = \min_p \tau(x_p).$$

Thus, in this approach it is necessary to solve a series of Cauchy tasks, the solution of which will help to find a sequence of time moments $\tau_1, \tau_2, ...$, that describe the times of activation of the relevant individuals.

3 General Algorithm, Data Structures, Programming and Numerical results

Let $V = \{1, 2, ..., n\}$, $f(x) = \alpha x + \beta x^2$. Then we can describe the structure of corresponding relationships in the social group as a matrix of incydence $R = (r_{ij})_{i,j=1}^{n}$,

$$r_{ij} = \begin{cases} 0, (i, j) \notin U, \\ \delta(i, j), (i, j) \in U, \end{cases}$$

where $\delta(i, j)$ - corresponding thresholds.

We can correspond to every person object $p(j)$ which can be denoted as an array $p(j) = (p_1^j, p_2^j, ..., p_{N_j}^j)$,where p_1^j - indicator of activation, p_2^j - number of the object, that activated the object j , p_3^j - time of the object j activation, p_4^j -number of differencial equations in system (3), which is necessary for finding the force of influense for the object j , $p_5^j, p_6^j ..., p_{p_4^j}^j$ -the initial condition for the Cauchy problem (4).

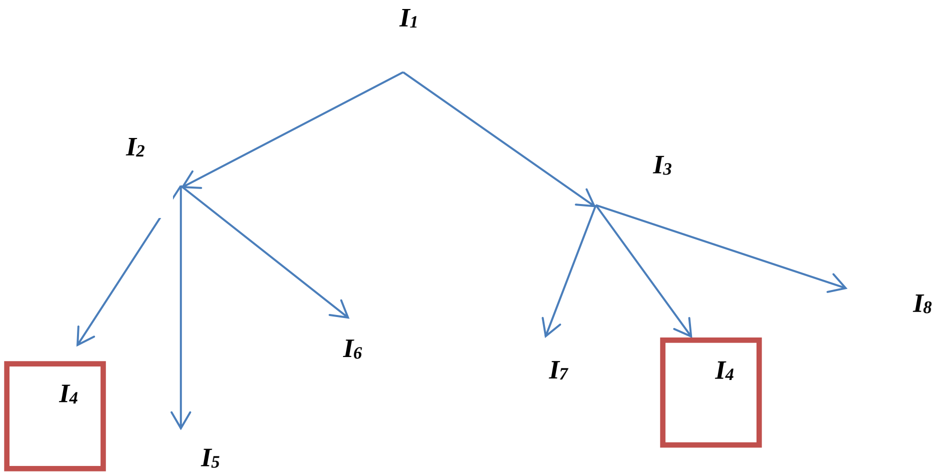

Fig. 1. Illustration of message dissemination process

Then we can propose the following algorithm Ω:

1. Consider first person and find the "temporary" times of activation for all corresponding partners according to the matrix of incydence R using (3). Corresponding times we can write into the array temp [1..n].
2. Find the number of an object j whith minimum of nonzero "temporary" time of activation $\tau(j)$ and activate them.
3. Form all elements of vector $p(j) = (p_1^j, p_2^j, ..., p_{N_j}^j), p_1^j = 1$.
4. Form the Cauchy problem for all corresponding to j partners according to the matrix of incydence R, solve them and find "temporary" times of activation according to the formula (4).
5. Repeat p.2-4 while there are non-activated objects.

To simplify the algorithm of information chains formation we can propose a graph $\tilde{G}(i)$ in which the same vertex (that models an individual) can simultaneously be in several information chains (see Fig.1).

Let us consider a part of any social group containing 20 objects which numbers are 1,2,3,…,20 respectively. Obviously, for the presentation of the relevant information it is convenient to use classes and object-oriented paradigm. But in our case it is necessary to solve the systems of differential equations and algorithms on graphs. Therefore we will use programming in the system Maple. Maple-programming has flexible facilities for working with strings and lists and also has the capability to solve differential equations. This system has very interesting ability for automatically generating new identifiers in cycles, which is very convenient for defining mathematical objects. But in Maple we can not use object-oriented programming and must use array *istota:=Array*[1..20,1..20] for the representation of corresponding personal information . In this array each column contains information related to the corresponding person. So *istota[1,j]*– indicator of activation (1–is active, 0–not active), *istota [2,j]*–number of the object, that activated the object j, *istota [3,j]*– time of the object j activation, *istota [4,j]*-number of differencial equations in system (3), which is necessary for finding the force of influense for the object j, *istota [5,j]*– *istota [20,j]*–the initial condition for the Cauchy problem. Taking into account similarity of the equations (4) we can form this systems automatically in Maple program and don't use any additional information for the correspondent Cauchy problem presentation.

Let consider the more detailed steps of algorithm Ω (Fig.3) . Among the main stages we can distinguish the initialization block, the calculation of elements of the array, where the conditional activation moments are stored, the automatic formation of the corresponding Cauchy tasks and their solutions, the calculation of real moments of activation, correct inicialization of all necessary fields of array *istota*. In Fig.2 we can see fragment of program where we denote the activation of first person (number 1) and influence of this person for the partners.

```
for i from 1 to 20 do
if incyd[1,i]>0 then
ti:=fsolve(abs(f(-v[0](x)))-incyd[1,i],x,0..5,maxsols=2);
if whattype(ti)=exprseq and whattype(ti[1])=float
  then
   temp[i]:=ti[1]; istota[1,i]:=0;
   istota[2,i]:=1; istota[3,i]:=temp[i]; istota[4,i]:=1;
elif whattype(ti)=float then
   temp[i]:=ti; istota[1,i]:=0;
   istota [2,i]:=1;
   istota [3,i]:=temp[i];
   istota [4,i]:=1;
end if; end if; end do;
```

Fig. 2. Illustration of the process of person activation

The most interesting feature of our program realization is automatic formation and solving of the corresponding Cauchy problem (see Fig.3). The solution to this problem consists of three parts: the formation of the system of differential equations, initial conditions formation, the right part of which we obtain from the corresponding fields of an array istota and Cauchy problem solving (we use standart Maple-function dsolve).

```
sys_ode_[0] := [];
ics_[0] := [];
for i1 to n-1 do
sys_ode_[i1] := [op(sys_ode_[i1-1]), diff(v[i1](t), t,
t) = f(v[i1+1](t)-v[i1](t))-f(v[i1](t)-v[i1-1](t))];
ics_[i1] := [op(ics_[i1-1]), v[i1](tau[indmin]) =
istota[4+2*i1-1, indmin], (D(v[i1]))(tau[indmin]) =
istota[4+2*i1, indmin]] end do;
sys_od := op(sys_ode_[n-1]), diff(v[n](t), t, t) =
f(-v[n](t))-f(v[n](t)-v[n-1](t));
ct := op(ics_[n-1]), v[n](tau[indmin]) = 0,
(D(v[n]))(tau[indmin]) = 0;
F := dsolve([sys_od, ct], numeric, output =
 listprocedure);
ui := proc (x) options operator, arrow; rhs(F(x)[2*n])
end proc;
ti := fsolve(abs(f(-ui(x+ tau[indmin])))-incyd[indmin,
i], x, 0 .. 5, maxsols = 2);
 if whattype(ti) = exprseq and whattype(ti[1]) = float
then tt := ti[1] elif whattype(ti) = float then tt :=
ti end if;
```

Fig. 3. Creation and solving the corresponding Cauchy problems

The important task is correct initialization of all necessary fields of array *istota* (see Fig.4).

4 Examples of the Software Using

Let consider such incidence matrix: $\delta_{1,2} = 0.015$, $\delta_{1,7} = 0.061$, $\delta_{1,8} = 0.049$, $\delta_{1,16} = 0.015$, $\delta_{1,17} = 0.037$, $\delta_{1,18} = 0.043$, $\delta_{2,13} = 0.052$, $\delta_{4,14} = 0.083$, $\delta_{6,11} = 0.0028$, $\delta_{6,20} = 0.0309$, $\delta_{7,10} = 0.084$, $\delta_{7,13} = 0.0905$, $\delta_{8,6} = 0.0349$, $\delta_{8,13} = 0.0928$, $\delta_{12,19} = 0.0767$, $\delta_{16,4} = 0.0732$, $\delta_{16,9} = 0.0162$, $\delta_{17,5} = 0.0629$, $\delta_{17,15} = 0.0812$, $\delta_{18,12} = 0.0482$.

Using algorithm, described above, we can find the moments of activation for every objects. The process of activation can be illustrated on the Fig. 5. Every vertex of graph, described in Fig.5, is marked by two numbers. First is the number of element and second is time of activation (in brackets).

```
if whattype(tt) = float then
   if istota[3, i] = 0 then
temp[i] := tt+tau[indmin];
istota[2, i] := indmin;
istota[3, i] := temp[i];
istota[4, i] := istota[4, indmin]+1;
for i1 to 2*n do
istota[4+i1, i] := rhs(F(temp[i])[1+i1])
end do;
for i1 to 4+2*n do
print(istota[i1, i])
end do
elif istota[3, i] > 0 and tt+tau[indmin] < istota[3, i]
then
temp[i] := tt+tau[indmin];
istota[2, i] := indmin;
istota[3, i] := temp[i];
istota[4, i] := istota[4, indmin]+1;
for i1 to 2*n do
istota[4+i1, i] := rhs(F(temp[i])[1+i1])
end do
end if
end if
```

Fig. 4. Inicialization of all necessary fields of array *istota*

We can assign each objects number to the activation time. Then we reseive the set of activation times: 0, 0.095, 0, 0.700, 0. 623, 0.536, 0.133, 0.126, 0.406, 0.973, 0.876, 0.637, 1.046, 1.940, 0.927, 0.096, 0.118, 0.123, 2.318, 1.537. We can investigate the process of information shock wave propagation in real-time mode.

As a next example of our approach we can consider Harward Dataverse and Twitter user timelines belonging to Representatives in the House of the 115th U.S. Congress. They were collected from the Twitter API using Social Feed Manager [28]. There are part of the series of timelines for the Senators Representatives tweets :

RepMattGaetz on May 3, 2017, 10:49:48 a.m.,
RepRonEstes on May 4, 2017, 9:31:05 a.m.,
RepRyanZink on May 4, 2017, 9:32:37 a.m.,
RepRonEstes, on May 4, 2017, 9:40:36,
RepMikeJohnson on May 5, 2017, 10:34:13 a.m.,
RepAnthonyBrown, on May 5, 2017, 10:37:51 a.m.,
RepRutherfordFL, on May 12, 2017, 10:38:02 a.m.

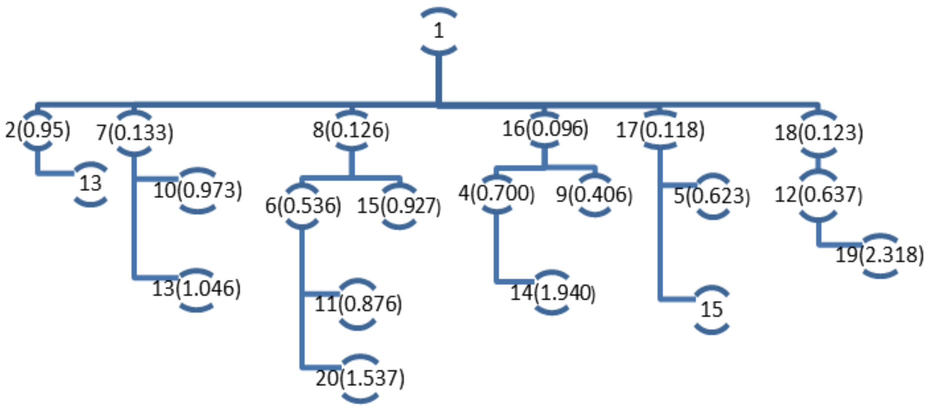

Fig. 5. Illustration of the process of person activation

So, we have normalized time series: 0, 1, 1.0015, 1.0140, 2.2196, 2.2242, 2.2258. Using approach described above for the parameters , we get the corresponding excitation levels: 1.38, 1.859e-5, 2.224e-7, 0.25, 8.88e-5, 1.427e-9.

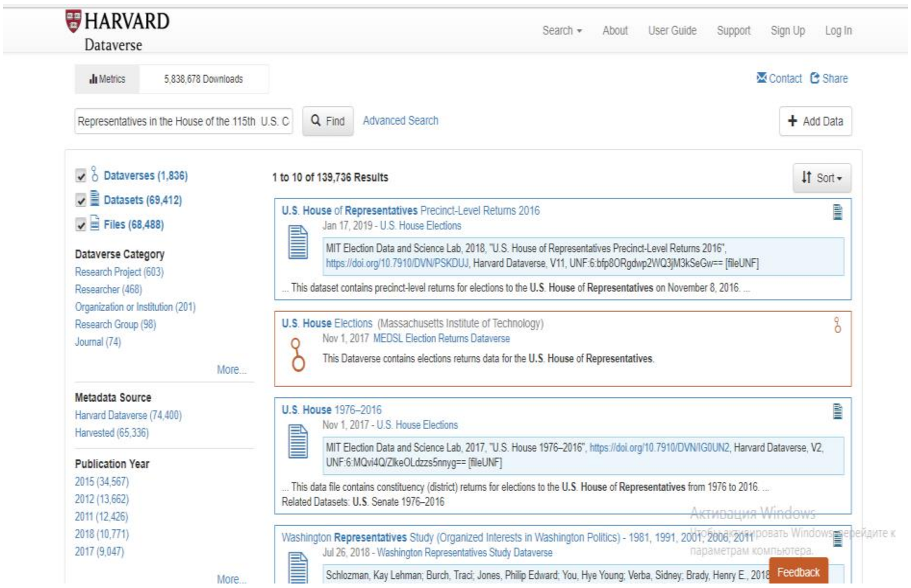

Fig. 6. Harward Dataverse and Twitter user timelines belonging to Representatives in the House of the 115th U.S. Congress.

5 Conclusions

Thus, we propose a program realisation of the models of people opinion forming the in social groups. In our realisation we solve the problem of multidimensional systems of differential equation. In our system, the maximum dimension of a nonlinear system of the form (4) in the canonical form is 20. Thus, we have a limit on the number of message propagation levels. We can offer several solutions of this problem. One of the ways is to use the continualization procedure and replace the corresponding system of equations with the Boussinesq equation. Then, if the number of equations exceeds 20, the first equations can be replaced by the Boussinesq equation and the rest can be solved using the standard procedure.

It should be noted that individuals in this model are actually considered as neurons that take qualified decisions regarding the further message distribution of a certain type. This approach, based on its essential grounds, is as close as possible to the real processes that occur in social networks, which allow you to generate statements about the high level of adequacy of the proposed class of models.

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