Solution for the problem of the parameters identification for autoregressions with multiple roots of characteristic equations

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Abstract. When describing a real image using a mathematical model, the problem of model parameters identification is of importance. In this case the identification itself is easier to perform when a particular type of model is known. In other words, if there is a number of models characterized by different properties, then if there is a correspondence with the type of suitable images, then the model to be used can be determined in advance. Therefore, in this paper, we do not consider the criteria for model selection, but perform the identification of parameters for autoregressive models, including those with multiple roots of characteristic equations. This is due to the fact that the effectiveness of identification is verified by the images generated by this model. However, even using this approach where the model is known, one must first determine the order of the model. In this regard, on the basis of Yule-Walker equations, an algorithm for determining the order of the model is investigated, and the optimal parameters of the model are also found. In this case the proposed algorithm can be used when processing real images.

1. Introduction

Mathematical modeling is used in many areas of science and technology, including image processing. In particular, methods focused on the description of images using models of random fields (RF), allow the development of algorithms for parameter estimation, filtering, detection of anomalies in the background of images and analyze them for a large number of simulated images.

There exist many images that are characterized by a smooth change in brightness [1,2]. Usually such images have a slowly decreasing correlation function (CF) in a certain given neighborhood. The first-order autoregressive (AR) image models [3,4] do not provide a strong correlation between pixels, since its CF decreases exponentially. Meanwhile, separable RFs that can be generated by one-dimensional autoregression with multiple roots of characteristic equations [5-6] are known. An important feature of ARs with multiple roots is their considerable simplicity compared to arbitrary high-order ARs. Also doubly stochastic models are used to describe real images [7-10]. However, if the structure of the described image is not so complicated, then it is better to use simpler models. This is due to the fact that the computational complexity of a double-stochastic model is much higher than, for example, a model with multiple roots, for which the AR of any order can be described using a single parameter.

At the same time, when describing images, for example, satellite images using mathematical models, it is necessary to identify parameters of the model. At the same time, the model should be selected in such a way that a compromise is found between its complexity and the similarity of the simulated image with the real one. The paper discusses the solution of the problem of the parameters identification for AR models with multiple roots of characteristic equations, which, owing to the separability of the CF, can significantly simplify this task. It should be noted that the application of the parameter identification approach based on Yule-Walker equations to determining the order and correlation parameters of AR models with multiple roots has a scientific novelty.

2. Brief overview of the parameters identification methods for a random processes

At present, the identification task for AR processes, the moving average (MA) and autoregression with moving average model (ARMA) by single-channel observations without noise is the most well-studied. Especially many methods have been developed for estimating the parameters of such processes. The estimation methods are based either on the direct use of observations, or on the initial calculation of the sample statistical characteristics (autocorrelations, spectral densities) from these observations, and then using them to determine parameter estimates. The first group of methods includes the least squares method (LSM) for the AR process [10,11], the maximum likelihood method (ML) for the AR, MA, and ARMA processes, and various types and modifications of these methods [12-14]. The second group is the Yule-Walker method for the AR process [13], the Box and Jenkins correlation methods for the ARMA, MA [13] processes, the Lindberger method for the AR, MA, and ARMA processes [15], the Durbin method for the MA process [16] , the Cleveland inverse autocorrelation method for the ARMA process [17] and other methods [18]. In both groups, there are methods in which parameter estimates are calculated by linear algorithms by solving a certain system of linear equations for example, LSM and Yule-Walker method for the AR process, as well as methods that use nonlinear methods for calculating estimates, which are reduced to numerical minimization algorithms for a certain function of the parameters, for example, the ML method and the Lindberger method for the APMA processes. The methods for determining the class of processes that can be AR, MA, and ARMA are much less developed [19]. In the work of Kitler and Whitehead [20], the problem of class determination is reduced to the problem of determining the order of the ARMA process. Therefore, the solution of the determination order and class of the model task is of interest. In the simplest case, such a task is reduced to a problem when it is necessary to determine its order using the selected model of AR. So optimal identification is very important for satellite image processing [21,22].

3. Autoregressions with multiple roots of characteristic equations

Using AR models with multiple roots of characteristic equations, it is possible to obtain realizations of RFs that will be close in their properties to real images. In this case an important property of the generated RF will be its quasi-isotropy. The general formula for models of different multiplicities can be written as follows

$$x_{i,j} = \beta\xi_{i,j} - \sum_{i_1=0}^{N_1}\sum_{j_1=0}^{N_2}\alpha_{i_1,j_1}x_{i-i_1,j-j_1}\ , \quad (1)$$

where N_1 and N_2 characterize the multiplicity of the model; coefficients $\alpha_{i_1,j_1}(\alpha_{0,0}=0)$ are the products of the corresponding coefficients of one-dimensional AR along the axes x and y

$$\alpha_{i_1,j_1} = \alpha_{xi_1}\alpha_{yj_1}\ . \quad (2)$$

The coefficients of one-dimensional AR (2) can be obtained using expressions

$$\alpha_{xi_1}(\rho_x, N_1) = (-1)^{i_1+1}C_{N_1}^{i_1}\rho_x^{i_1}, \alpha_{yj_1}(\rho_y, N_2) = (-1)^{j_1+1}C_{N_2}^{j_1}\rho_y^{j_1}, \quad (3)$$

where $C_n^m = \dfrac{n!}{m!(n-m)!}$. Two-dimensional model's coefficient β is the normalized product of the corresponding coefficients of one-dimensional AR along the axes x and y

$$\beta = \frac{\sigma_x}{\sigma_\xi}\beta_x\beta_y\ , \quad (4)$$

where $\beta_x = \left((1-\rho_x^2)^{2N_1-1} \Big/ \sum_{l=0}^{N_1-1} (C_{N_1-l}^l \rho_x^l)^2 \right)^{-\frac{1}{2}}$, $\beta_y = \left((1-\rho_y^2)^{2N_2-1} \Big/ \sum_{l=0}^{N_2-1} (C_{N_2-l}^l \rho_y^l)^2 \right)^{-\frac{1}{2}}$.

When identifying parameters, let's approximate the CF of the initial data by the most appropriate model. In order to obtain the CF models of arbitrary orders, it is necessary to use expressions for one-dimensional CF of AR with multiple roots of characteristic equations [22]

$$B_x(k) = \sigma_x^2 \sum_{l=0}^{m-1} g(m,l,k) \frac{\rho^{2(m-l-1)}}{(1-\rho^2)^{2k-l-1}}, \tag{5}$$

where $g(m,l,k) = \dfrac{(m+k-1)!(2m-l-2)!}{l!(m-1)!(m-l-1)!(m+k-l-1)!}$. The variance of independent random values ξ_i, $i = 1,2,...,n$, can be found for a given variance of simulated RF $B_x(0) = \sigma_x^2$

$$\sigma_\xi^2 = \frac{\sigma_x^2 (1-\rho^2)^{2m-1}}{\sum_{l=0}^{m-1} \left(C_{m-1}^l \rho^l \right)^2}. \tag{6}$$

Thus, for spatial AR equation with characteristic roots of multiplicities *(m_1,m_2)* the expression for CF can be written as

$$B_{xy}(k_1,k_2) = \sigma_x^2 \sum_{l=0}^{m_1-1} g(m_1,l,k_1) \frac{\rho_x^{2(m_1-l-1)}}{(1-\rho_x^2)^{2k_1-l-1}} \sum_{l=0}^{m_2-1} g(m_2,l,k_2) \frac{\rho_y^{2(m_2-l-1)}}{(1-\rho_y^2)^{2k_2-l-1}}. \tag{7}$$

However, in the case of a model with multiple roots it is possible to separately carry out the identification of parameters by row and column, using formula (5).

4. Identification of parameters based on theoretical values of correlation functions

To solve the problem of identification, we will use AR models of arbitrary order

$$x_i = \rho_1 x_{i-1} + \rho_2 x_{i-2} + ... + \rho_m x_{i-m} + \xi_i, \quad i = 1,2,...,M, \tag{8}$$

where *m* is order of AR model.

Choosing parameters $\rho_1, \rho_2, ..., \rho_m$ it is possible to get a Gaussian RF $\{x_i\}$, $i = 1,2,...,M$ with a variety of correlation properties. In this case to write values of CF you can use the following expression

$$R_x(k) = \rho_1 R_x(k-1) + \rho_2 R_x(k-2) + ... + \rho_m R_x(k-m), \quad k > 0. \tag{9}$$

CF of models with multiple roots for different roots z_ν, $\nu = 1,2,...,m,$ of characteristic equations described by expression

$$z^m - \rho_1 z^{m-1} - \rho_2 z^{m-2} - ... - \rho_m = 0, \tag{10}$$

under the condition of stability $|z_\nu| < 1$, $\nu = 1,2,...,m$, are represented by the following sum

$$R_x(k) = A_1 z_1^{|k|} + A_2 z_2^{|k|} + ... + A_m z_m^{|k|}. \tag{11}$$

Substitution in (9) values *k* = 1, 2, … , *m* leads to the well-known Yule-Walker system of equations, which, for example, for second-order systems takes the form

$$\begin{aligned} \rho_1 + \rho_2 R(1) &= R(1), \\ \rho_1 R(1) + \rho_2 &= R(2). \end{aligned} \tag{12}$$

The solution of this system allows you to find the coefficients $\rho_1, \rho_2, ..., \rho_m$ of equation (8) based on predetermined or estimated values of CF $R_x(1)$, $R_x(2)$, …, $R_x(m)$.

We will perform parameter identification for models with multiple roots of characteristic equations of *1-4*th orders. In this case the order can be identified if we take into account only the coefficients that make some contribution to the model. Table 1 presents the results of identification of the correlation parameters for the values of CF of AR with multiple roots. By rows, the actual multiplicity is presented, by columns — estimated parameters are presented. We assume $\rho = 0.8$ for models of all orders. In the left column – the values found, in the right column - the real values.

Table 1. Identification of parameters based on the theoretical values of CF

	m*=1		m*=2		m*=3		m*=4	
m=1	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=0.8,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$
m=2	$\rho_1=0.976,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=1.6,$ $\rho_2=-0.64,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=1.6,$ $\rho_2=-0.64,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=1.6,$ $\rho_2=-0.64,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=1.6,$ $\rho_2=-0.64,$ $\rho_3=3\times10^{-14},$ $\rho_4=0$	$\rho_1=1.6,$ $\rho_2=-0.64,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=1.6,$ $\rho_2=-0.64,$ $\rho_3=3\times10^{-14},$ $\rho_4=0$	$\rho_1=1.6,$ $\rho_2=-0.64,$ $\rho_3=0,$ $\rho_4=0$
m=3	$\rho_1=0.992,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=2.4,$ $\rho_2=-1.92,$ $\rho_3=0.512,$ $\rho_4=0$	$\rho_1=1.92,$ $\rho_2=-0.937,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=2.4,$ $\rho_2=-1.92,$ $\rho_3=0.512,$ $\rho_4=0$	$\rho_1=2.4,$ $\rho_2=-1.92,$ $\rho_3=0.512,$ $\rho_4=0$	$\rho_1=2.4,$ $\rho_2=-1.92,$ $\rho_3=0.512,$ $\rho_4=0$	$\rho_1=2.4,$ $\rho_2=-1.92,$ $\rho_3=0.512,$ $\rho_4=-1\times10^{-13}$	$\rho_1=2.4,$ $\rho_2=-1.92,$ $\rho_3=0.512,$ $\rho_4=0$
m=4	$\rho_1=0.995,$ $\rho_2=0,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=3.2,$ $\rho_2=-3.84,$ $\rho_3=2.048,$ $\rho_4=-0.41$	$\rho_1=1.969,$ $\rho_2=-0.979,$ $\rho_3=0,$ $\rho_4=0$	$\rho_1=3.2,$ $\rho_2=-3.84,$ $\rho_3=2.048,$ $\rho_4=-0.41$	$\rho_1=2.837,$ $\rho_2=-2.724,$ $\rho_3=0.886,$ $\rho_4=0$	$\rho_1=3.2,$ $\rho_2=-3.84,$ $\rho_3=2.048,$ $\rho_4=-0.41$	$\rho_1=3.2,$ $\rho_2=-3.84,$ $\rho_3=2.048,$ $\rho_4=-0.41$	$\rho_1=3.2,$ $\rho_2=-3.84,$ $\rho_3=2.048,$ $\rho_4=-0.41$

Analysis of table 1 shows that the values of the correlation coefficients are estimated the more accurately, the higher the multiplicity is. If the estimated multiplicity exceeds the real value, then the additional coefficients are either 0 or very close to it. Thus, the process of the model order identification can be carried out first for some large multiplicity. If the resulting coefficients do not have zero coefficients, then the calculation should be carried out at a higher multiplicity until we get zero coefficients. If there are zero coefficients, the order corresponds to the number of the last significant coefficient.

A similar relationship between correlation coefficients and CF values can also be obtained for the two-dimensional case, i.e. images. The relation for CF values corresponding to expression (9) for the three-point model, is written as

$$R(k_1,k_2) = \rho_{10}R(k_1-1,k_2) + \rho_{01}R(k_1,k_2-1) + \rho_{11}R(k_1-1,k_2-1), k_1>0, k_2>0. \quad (13)$$

It is easy to verify that solving the two-dimensional Yule-Walker system of equations (13) for the Habibi model CF will give correlation coefficients identical to the coefficients of the first-order two-dimensional AR model. To increase the order of the AR, as in the one-dimensional case it is necessary to increase the number of correlation coefficients. In this case, the RF model can be written as follows

$$x_{i,j} = \sum_{l=0}^{m_j}\sum_{k=0}^{m_i} \rho_{kl} x_{i-k,j-l} - \rho_{00}x_{i,j} + \xi_{i,j}\ ,\ i=\overline{1,M_1},\ j=\overline{1,M_2}\ , \quad (14)$$

where $\{x_{i,j}\}$ is RF implementation or simulated image; ρ_{kl} are correlation coefficients for elements lagging behind each other along the axes i and j by k and l pixels respectively; $\{\xi_{i,j}\}$ is two-dimensional RF of independent Gaussian random variables with zero mean $M\{\xi_{i,j}\}=0$ and variance $M\{\xi_{i,j}^2\} = \sigma_\xi^2 = [1-\sum_{l=0}^{mj}\sum_{k=0}^{mi}\rho_{kl}R(k,l) + \rho_{00}R(0,0)]\sigma_x^2$; m_i and m_j are orders of the model; M_1 and M_2 are the image size.

The number of components of the model, taking into account the random increment will be equal to $(m_i+1)\times(m_j+1)$. Using formulas (9) and (13), we can write the relation for calculating the CF values

$$R(k_1,k_2)=\sum_{l=0}^{m_j}\sum_{k=0}^{m_i}\rho_{kl}R(k_1-k,k_2-l)-\rho_{00}R(k_1,k_2)\,,\ k_1>0,k_2>0\,. \qquad (15)$$

The expression (15) can also be used for the case of non-separable CFs if the parameters of an arbitrary AR RF are identified and the order is a priori unknown.

5. Parameter identification on the basis of real images CF

Let us identify the parameters of the model based on the proximity of the CF model and the given data. In the first case, we will consider the AR with a separable CF and separately calculate the coefficients for the row and column. In the second case, we use equations based on the expression (15).

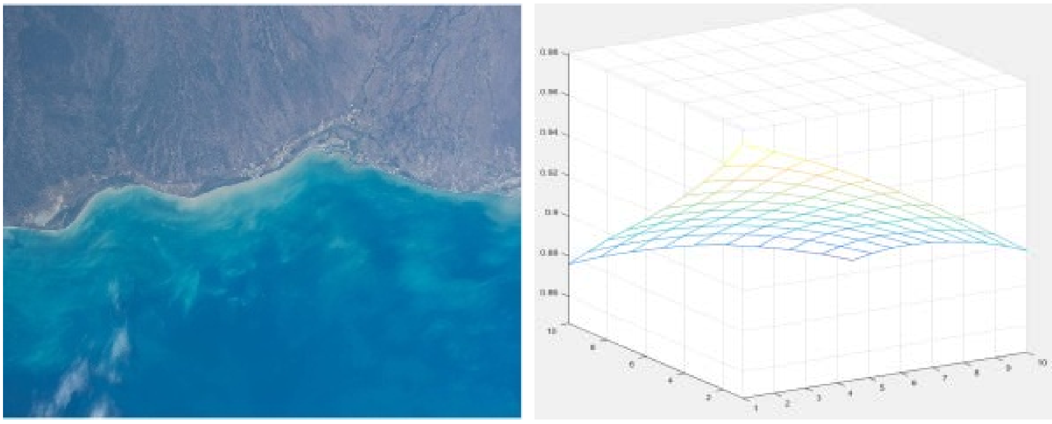
Let there be a real image represented as $I(i,j), i\in 1,...,M_1, j\in 1,...,M_2$. Then its CF can be expressed as follows

$$R_I(k_1,k_2)=\frac{1}{\sigma_I^2}\sum_{i=k_1+1}^{M_1}\sum_{j=k_2+1}^{M_2}(I(i,j)-m_I)(I(i-k_1,j-k_2)-m_I)\,, \qquad (16)$$

where m_I is average brightness over the entire image; σ_I^2 is brightness variance calculated over the entire image.

Figure 1a and Figure 1b show the image to be investigated and its CF, respectively. Identification is performed for the 4th order AR model.

- for RF having separable CF the results are as follows: $\rho_{10}=1.098; \rho_{20}=-0.39; \rho_{30}=0.364; \rho_{40}=-0.082;$ $\rho_{01}=0.828; \rho_{02}=0.0047; \rho_{03}=0.111; \rho_{04}=0.038$; $\varepsilon=(\hat{R}-R)^2=0.387$.
- for RF having non-separable CF the results are as follows: $\rho_{10}=0.998; \rho_{20}=-0.514; \rho_{30}=0.43; \rho_{40}=-0.133;$ $\rho_{01}=0.185; \rho_{11}=0.152; \rho_{21}=-0.194, \rho_{31}=0.142; \rho_{41}=-0.092;$ $\rho_{02}=-0.174; \rho_{12}=0.309; \rho_{22}=-0.265; \rho_{32}=0.217; \rho_{42}=-0.097; \rho_{03}=-0.048; \rho_{13}=0.186; \rho_{23}=-0.205;$ $\rho_{33}=0.153, \rho_{43}=-0.079; \rho_{04}=-0.122; \rho_{14}=0.267; \rho_{24}=-0.242; \rho_{34}=0.22; \rho_{44}=-0.102;$ $\varepsilon=(\hat{R}-R)^2=0.014$.



a) b)

Figure 1. The image (a) for which the adjustment of the parameters is carried out and its CF (b).

Analysis of the obtained values of the error variances shows that the use of the model with an non-separable CF provides a greater proximity between the modeled and the real CF. This is explained by the fact that for this model 24 correlation parameters were calculated, while for a model with a separable CF only 8 parameters. At the same time, a sufficient proximity of the CF is provided, especially in the neighborhood of zero. Therefore, it is advisable to use such models to reduce computational costs. Figure 2 shows the CF cross sections for the original image (presented by solid line), as well as for the RF with separable CF (presented by dash-dotted line) and the RF with non-separable CF (presented by dashed line).

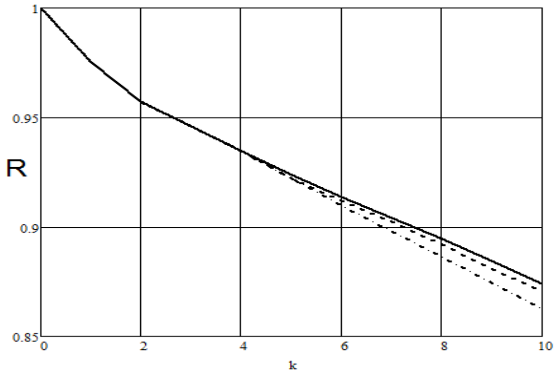


Figure 2. CF cross sections of the original image and identifiable models.

Analysis of the presented curves shows that as *k* increases the discrepancy between the real and simulated CF also increases. It is possible to achieve greater proximity by increasing the order of the AR, but this leads to higher computational costs. Similar research was conducted with a sample of 100 images. Analysis of the results shows that the use of non-separable CF models provides the proximity of CF to 10-15 times more than the use of separable CF models in 85% of cases. However, in 15% of cases separable models were successfully used to describe the real image, which made it possible to significantly reduce computational costs of image processing.

6. One dimensional example of calculating parameters

However, if a model with multiple roots is used, it is sufficient to calculate only the first correlation coefficient, then calculate the correlation parameter using the first correlation coefficient and use correlation parameter to find the remaining correlation coefficients according to the expression (3).

For example, if $\rho_1 = 3.6$ and *N*=4 then it is easy to calculate parameter of the AR with multiple roots of characteristic equations from the following equation

$$3.6 = (-1)^{1+1} C_4^1 \rho^1, \rightarrow \rho = 0.9 . \qquad (17)$$

After that we find the second, third and fourth correlation coefficients

$$\begin{aligned} \rho_2 &= (-1)^{1+2} C_4^2 \rho^2 = -4.86, \\ \rho_3 &= (-1)^{1+3} C_4^3 \rho^3 = 2.916, \\ \rho_4 &= (-1)^{1+4} C_4^4 \rho^4 = -0.6561. \end{aligned} \qquad (18)$$

So it is quite a simple task to perform identification of parameters in this case.

7. Conclusion

In this paper a brief overview of the methods used to identify the parameters of the AR processes is presented. Models with multiple roots and a method for determining the order of the model based on the Yule-Walker equations are considered. It is shown that the proposed method enables to determine the order of the model for simulated images with sufficient accuracy. A comparative analysis of the identification of the parameters of models with a separable and non-separable CF of the real image is performed. The analysis shows that models with a non-separable CF require more computational costs at the same order of a model, however, they provide greater proximity of the CF in comparison with separable CF models. The identification results may be used to describe real satellite images and for the processing of such images.

8. References

[1] Soifer V A 2001 *Methods of computer image processing* (Fizmatlit) 784 p
[2] Gonzalez R, Woods R 2005 *Digital image processing* (Moscow: Technosphere) 1072 p

[3] Vasiliev K K, Omelchenko V A 1995 *Applied theory of random processes and fields* (Ulyanovsk: UlSTU) 256 p
[4] Glazov G N, Kostevich A G 2002 Modeling discrete Gaussian random fields *Intellectual systems in management, design and education* **2** 19-27
[5] Vasiliev K K, Andriyanov N A 2017 Analysis of autoregression with multiple roots of characteristic equations *Radiotekhnika* **6** 13-17
[6] Andriyanov N A, Vasiliev K K 2018 Application of autoregression with multiple roots of characteristic equations for image representation and filtering *Information technologies and nanotechnologies Proceedings of ITNT-2018* (Samara National Research University) 1023-1031
[7] Andriyanov N A, Vasiliev K K and Dement'ev V E 2018 Analysis of the efficiency of satellite image sequences filtering *Journal of Physics: Conference Series* **1096** 012036 DOI: 10.1088/1742-6596/1096/1/012036
[8] Danilov A N, Andriyanov N A and Azanov P T 2018 Ensuring the effectiveness of the taxi order service by mathematical modeling and machine learning *Journal of Physics: Conference Series* **1096** 012188 DOI: 10.1088/1742-6596/1096/1/012188
[9] Vasiliev K K, Andriyanov N A 2017 Synthesis and analysis of doubly stochastic models of images *CEUR Workshop Proceedings* **2005** 145-154
[10] Bolshakov A A, Karimov R N 2006 *Methods for processing multidimensional data and time series: a manual for universities* (Moscow: Hotline - Telecom) 522 p
[11] Zhdanov A I, Katsyuba O A 1982 Identification by the method of least squares of the parameters of the autoregression equations with additive measurement errors *Automatics* **43(2)** 29-38
[12] Anderson T 1976 *Statistical analysis of time series* (Moscow: Mir) 755 p
[13] Box G, Jenkins G 1974 *Analysis of time series. Forecast and management* (Moscow: Mir) 406 p
[14] Koreisha S, Yoshimoto G 1991 A Comparison among Identification Procedures for Autoregressive Moving Average Models *International Statistical Review/ Revue Internationale de Statistique* **59(1)** 37-57
[15] Lindberger N A 1974 Computer identification of linear random process *Intern. J. Control* **19(6)** 1021-1045
[16] Durbin J 1959 Efficient estimation of parameters in moving average models *Biometrika* **46(3-4)** 306-316
[17] Cleveland W S 1972 The inverse autocorrelations of a time series and their applications *Technometrics* **14(2)** 277-293
[18] Lukashin Yu P 2003 *Adaptive methods for short-term forecasting of time series: a textbook for universities.* (Moscow: Finance and Statistics) 416 p
[19] Andrianov N A, Gavrilina Yu N 2017 The problem of identifying the parameters of auto-regressions with multiple roots *Modern problems of design, production and operation of radio systems* **1-2(10)** 95-98
[20] Kittler J, Whitehead P G 1976 Determination of the model structure using pattern recognition techniques *Proc. of 4th IFAC Symp. Ident. Syst. Parameter Estimation* **3** 22-30
[21] Plotnikov D E, Kolbudaev P A and Bartalyov S A 2018 Selection of seasonally homogeneous areas based on the analysis of time series of satellite images *Computer Optics* **42(3)** 447-456 DOI: 10.18287/2412-6179-2018-42-3-447-456
[22] Varlamova A A, Denisova A Yu and Sergeev V V 2018 Information technology for processing remote sensing data for assessing plant ranges *Computer Optics* **42(5)** 864-876 DOI: 10.18287/2412-6179-2018-42-5-864-876
[23] Vasiliev K K, Krasheninnikov V R 2015 *Statistical image analysis* (Ulyanovsk, UlSTU) 214 p

Acknowledgement
The work was supported by RFBR, Projects №17-01-00179, №18-31-00056.