An Agent-Based Model on Scale-Free Networks for Personal Finance Decisions

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***Abstract*—Personal finance decisions emerge from a complex network of human connections, where the nodes or agents — banks, investors, financial advisors — take their choices on the basis of a variety of factors. All these agents form a society, which we modeled as an Agent-Based Model (ABM) on a scale-free network. In this paper, we will consider: honest agents, regular agents, insincere agents, stubborn agents and skilled (or unskilled) agents. Honest agents report truthfully their opinion while insincere agents state an opinion which is different from their internal belief. Regular agents are characterized by the same propensity to listen, contrary to what stubborn agents do because these agents evaluate the counterpart's opinion but never approaches to it. Skilled and unskilled agents are the result of influence of the competence in the evolution of decisions in multi-agent systems. We perform a social simulation to show that, in particular, consensus, polarization, extremism or the emergence of a disordered regime are possible outcomes, even without explicit introduction of stubborn agents.**

***Index Terms*—Agent-based modeling, Multi-agent systems, Opinion dynamics, Scale free networks**

I. Introduction

Personal finance decisions are taken by individuals on the basis of a variety of factors, emerging from a complex network of human connections. All of these human connections involve several agents, many of them clustered into fixed categories: banks, financial advisors, investors. They form a society. Investors usually resort to financial advisors to improve their investment process. The latter are paid by the banks, whose aim is to steer the investors towards a particular investment decision and it is the reason why they ask the collaboration of financial advisors.

When we look at the *connections*, we realize that: i) the interaction is not of the any-to-any kind [1], since an agent will be typically connected with some other agents rather than all of them (e.g., we assume that investors connect with advisors but not with banks); ii) some agents have a large number of connections to other individuals, whereas most of them just have a handful (e.g., an advisor may have many customers, but customers usually have only one advisor). Societies satisfying such rules are the popular "scale-free" networks [2].

The society is modeled as an Agent-Based Model (ABM). Agent-based simulation is most commonly used to model individual decision-making and social-organizational behavior [1], [3]–[10]. It allows to investigate mutual and causal influences of the micro-elements on the complex system development [11], by involving research areas seemingly distant such as game theory and control theory (see e.g. [12] and [13]).

In this paper, to explain the diverse opinion structures within that kind of society, we extend the bounded confidence model of continuous opinion formation introduced in [14], by introducing Gaussian updating functions [15]. According to classical bounded confidence models, the agents interact with each other only when their opinions are close enough. But in many real world situations, the strength of this interaction usually depends on the distance between opinions (the lower the distance, the higher the strength). For this reason we considered a Gaussian updating function.

We will consider several categories of agents: honest agents, regular agents, insincere agents, stubborn agents and skilled (or unskilled) agents. Honest agents truthfully report their opinion while insincere agents state an opinion that may be different from their internal belief. Regular agents are characterized by a common propensity to listen, contrary to what stubborn agents do because these agents evaluate the counterpart's opinion but never approaches it. Skilled and unskilled agents behave as described in in [4], [5], where the influence of the competence in the evolution of decisions in multi-agent systems has been considered.

After describing the model in Section II, in Sections III and IV we consider some special cases, where the composition of this artificial society is made of the following classes of agents:

- honest, regular agents vs one class of stubborn agents;
- honest, regular agents vs two classes of stubborn agents;
- the presence of insincere agents in a population of regular honest agents;
- skilled regular agents vs unskilled regular ones.

For the sake of simplicity, we will assume that all these subsets have empty intersections i.e., they form a partition of the set of all agents. Moreover, we will focus only on scale free networks.

In this paper we wish to extend the simulation experiments performed in [14] by considering different compositions of the society. Then we perform a social simulation to show that, in particular, consensus, polarization, extremism or the emergence of a disordered regime are possible outcomes, even without explicit introduction of stubborn agents.

II. Agent-Based Model

We have examined the Bounded Confidence Model studied in [14] which we are going to describe below.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a graph that consists of a finite set of agents $i \in \mathcal{V} = \{1, 2, \ldots, n\}$ who are defined as nodes on a network and connected to each other with a finite set of links $\mathcal{E}$. Links between the agents indicate the communication channels through which opinions are exchanged and the influence is imposed. Communication requires a direct link between the agents $(i, j) \in \mathcal{E}$. In this model two agents always influence each other mutually, and hence we talk about a bilateral interaction, i.e. $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$.

In this paper each agent is characterized by the following triple: a couple of opinions, threshold level and a set of connections. Then: $i = \left\{\left(x_i(t), x_i^R(t)\right), \epsilon_i, \mathcal{N}_i\right\}$. All the opinions fall in the range $[0, 1]$ and are related to the decisions to buy a security rather than a different security or other financial instruments. At each time t, agent i selects a random counterpart j from his neighborhood $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ and the two share their opinions $x_i(t)$ and $x_j(t)$. If $x_i^R(t)$ is the opinion that agent i reports to the selected counterpart, then we have a first distinction between agents:

- $x_i^R(t) \neq x_i(t)$ in the case of *insincere* agents;
- $x_i^R(t) = x_i(t)$ in the case of *honest* agents.

Hence, according to the notation introduced in [1], this model is *continuous over a bounded interval* because

$$x_i(t) \in [0, 1] \quad \forall i \in \mathcal{V}, \ t > 0 . \tag{1}$$

It is also *bilateral* and *pairwise*.

Threshold levels are assigned to each agent at $t = 0$, with $\epsilon_i \in [0, 1]$.

Moreover, agents adjust their opinion upon the principle of bounded confidence. If $x_j^R(t)$ is the opinion that agent j reports to i, and

$$\begin{aligned} \Delta x_i(t) &= x_i(t+1) - x_i(t) \\ \Delta x_j(t) &= x_j(t+1) - x_j(t) \end{aligned} \tag{2}$$

are the changes of i and j' opinions, then

$$\begin{aligned} \Delta x_i(t) &= \mu \, \chi_{(-\epsilon_i, \epsilon_i)} \left(d_{i,j}(t)\right) \left(x_j^R(t) - x_i(t)\right) \\ \Delta x_j(t) &= \mu \, \chi_{(-\epsilon_j, \epsilon_j)} \left(d_{j,i}(t)\right) \left(x_i^R(t) - x_j(t)\right) \end{aligned} \tag{3}$$

where $\mu \in [0, 1]$ is the *adoption rate*, representing the proportion of counterpart's opinion an agent integrates into his prior, $d_{i,j}(t) = x_i(t) - x_j^R(t)$ and $\chi_{(-\epsilon_i, \epsilon_i)}(x)$ is the characteristic function of the interval $(-\epsilon_i, \epsilon_i)$. Then, according to the notation in [1], this model adopts a *non linear* updating function (because of threshold) and the interaction is in general *non symmetric* (because ϵ_i and ϵ_j could differ).

At this point two other agents classifications naturally arise. While the first classification concerns the distinction between $x_i^R(t)$ and $x_i(t)$, the second and the third concern the parameters ϵ_i and μ. The second classification is as follows:

- A *stubborn* agent i has parameter values of

$$\epsilon_i = 0 \vee \mu = 0 \, ; \tag{4}$$

- A *non-stubborn* agent i has parameter values of

$$\epsilon_i > 0 \wedge \mu > 0 \, . \tag{5}$$

The third classification concerns the definition of *regular* agents. The model studied in [14] assumes that threshold levels are equal across the population of regular agents, i.e. $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n$.

Lastly, according to the taxonomy introduced in [1], the updating frequency of this model is *periodic* because all the agents change their opinion at each time step.

The results enclosed in [14] concern the following cases:

- honest, regular agents;
- honest, regular agents vs one class of stubborn agents (the latter with the same opinion $x^S = 0$);
- honest, regular agents vs two classes of stubborn agents ($x^{S_0} = 0$ for the first class, $x^{S_1} = 1$ for the second);
- honest, regular agents vs insincere, regular agents.

Lastly, the authors of [14] compare the results on different network topologies: complete network, small world network and the scale free network.

In this paper we wish to extend the simulation experiments performed in [14] by replacing the updating function $\chi_{(-\epsilon_j, \epsilon_j)}\left(d_{i,j}(t)\right)$ with

$$e^{-\left(x_i(t) - x_j^R(t)\right)^2} \chi_{(-\epsilon_i, \epsilon_i)}\left(d_{i,j}(t)\right) \, ; \tag{6}$$

Then we examine different compositions of the society:

- we consider honest, regular agents vs one class of stubborn agents at varying α (the latter with the same opinion $x^S = \alpha \in [0, 1]$);
- we consider honest, regular agents vs two classes of stubborn agents at varying α and β ($x^{S_0} = \alpha$ for the first class, $x^{S_1} = \beta$ for the second);
- we consider the presence of insincere agents in a population of regular honest agents;
- we consider skilled regular agents vs unskilled regular ones.

The latter point is inspired by the results obtained in [4], [5], where the influence of competence in the evolution of decisions in multi-agent systems has been considered. Moreover, since we are interested in complex societies in which some individuals have a large number of connections to other people — whereas most individuals have just a handful — in this paper we will focus only on scale free network [2].

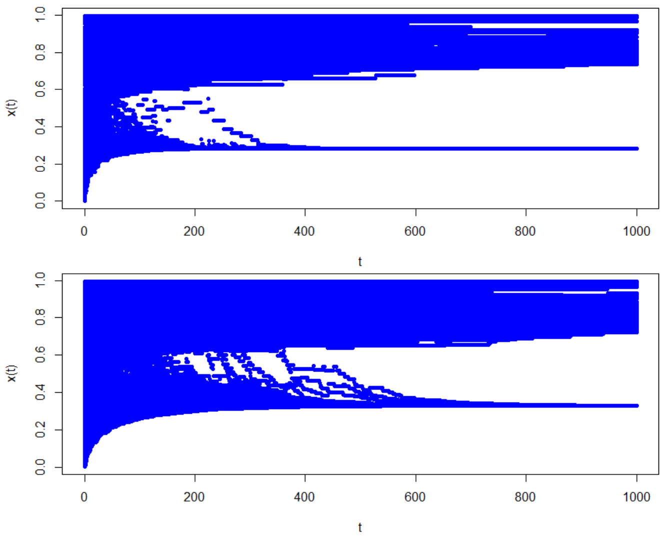

Fig. 1: Time evolution of the opinion dynamics in two selected runs. (Top plot): $\mu = 0.3$; (bottom plot): $\mu = 0.1$. Here $\epsilon = 0.3$ and $n = 500$ agents.

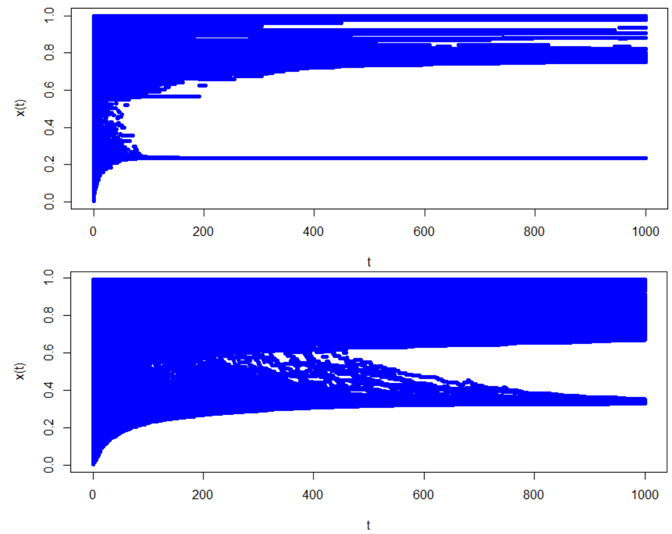

Fig. 2: Time evolution of the opinion dynamics in two selected runs. (Top plot): $\mu = 0.5$; (bottom plot): $\mu = 0.05$. Here $\epsilon = 0.3$ and $n = 500$ agents.

III. Opinion Dynamics

After defining the model and formulating it in an easily computable way through the paradigm of array programming, in this section we apply it to examine the resulting dynamics of agents, i.e. how their opinion changes over time. We use a simulation approach to examine the impact of the interaction coefficients. We developed an R code to perform these calculations, employed on a Windows machine equipped with a 2.80GHz Intel(R) Core(TM) i7 CPU and 16.0 GB RAM. For $T = 1000$ iterations and $n = 500$ agents, the simulations run for around 5 minutes.

Let us define the opinion vector as:

$$\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))^T \ .$$

A. Opinion Formation with Regular, Honest Agents

We start with the simplest case, in which only regular agents are present. The initial opinion vector $\mathbf{x}(0)$ has been drawn from a standard uniform distribution with $[0, 1]$ support. All agents have the same threshold level $\epsilon_i = \epsilon$, the same adoption rate and $x_i^R(t) = x_i(t)$ for every i (i.e., alla agents are honest).

The plots in Figs. 1 and 2 were obtained by considering the Gaussian updating function defined in (6). They show that the process of opinion formation within an integrated society (i.e. a society in which agents integrate opinions of others into their own) tends to self-organization and that the outcomes depend upon the parameter values. In this situation, those agents that have an initial starting opinion below a certain threshold (from the top plots of Figs. 1 and 2 it seems to be between 0.5 and 0.6) are rapidly drawn to a low central consensus — and μ speeds up the convergence to the low central consensus. Moreover, those agents that instead fall outside the attractor defined by this threshold, quickly settle down to a larger number of extreme opinions in which they are isolated from the low central consensus. In such a way, the model settles on a steady pattern and, as μ increases, high extreme opinions become more and more distinguishable (see top plots of Figs. 1 and 2). Actually, μ represents the proportion of counterpart's opinion an agent integrates into its own and the greater its value, the greater the number of opinions emerging after the transient.

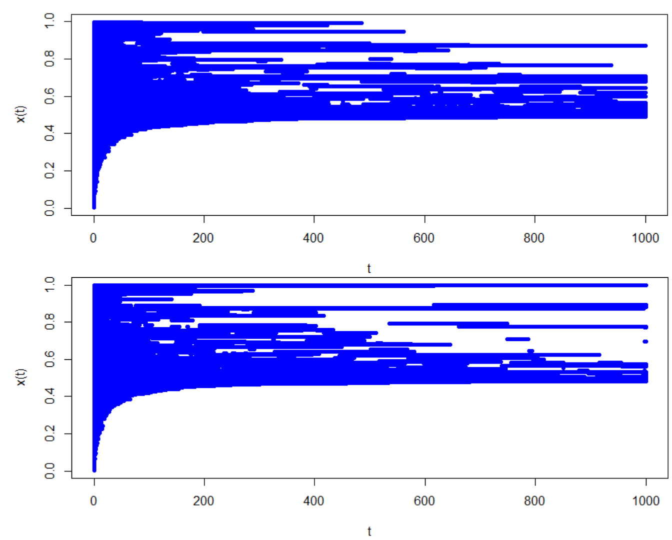

Fig. 3: Time evolution of the opinion dynamics in two selected runs. (Top plot): $\epsilon = 0.7$; (bottom plot): $\epsilon = 0.9$. Here $\mu = 0.3$ and $n = 500$ agents.

Fig. 3 underlines this effect by fixing μ and considering increasing, high values of threshold ϵ. As ϵ rises, high extreme opinions emerge.

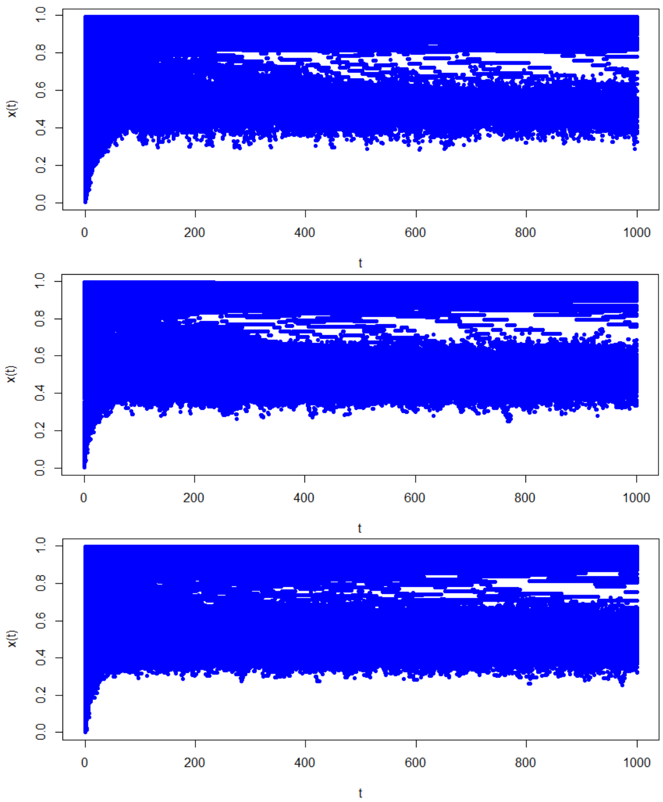

Fig. 4: The effect of insincere agents. Time evolution of the opinion dynamics in three selected runs: $m = 30$ (top plot); $m = 150$ (central plot); $m = 250$ (bottom plot). Here $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

B. *Opinion Formation with Insincere Agents*

In previous section we considered the case $x_i^R(t) = x_i(t)$, an assumption that has been relaxed by a number of authors in the last years (see, among others, [12], [16] in addition to the aforementioned [14]). In the following we assume that the i-th insincere agent states an opinion $x_i^R(t)$ drawn from a standard uniform distribution with $[0, 1]$ support, regardless the value of "true" or "internal" opinion $x_i(t)$.

With the inclusion of the insincere agents, the society $\mathcal{V}$ can be subdivided into two subsets, $\mathcal{H}$ (honest agents) and $\mathcal{I}$ (insincere agents), such that $\mathcal{V} = \mathcal{H} \cup \mathcal{I}$ and $\mathcal{H} \cap \mathcal{I} = \emptyset$. Anyway all the agents are regular, i.e. $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n$.

We assume a society $\mathcal{V}$ of $n = 500$ agents and that m of them are insincere. In Fig. 4 we examined the impact of the total number of insincere agents, m, on the opinion dynamics. As we can see, the low central consensus of Figs. 1 and 2 disappeared, and a disordered regime emerged in which opinions are in a constant state of change around a central opinion. Besides, increasing the willingness to listen (by increasing μ e.g.) does not seem to improve the picture (see Fig. 5): indeed, in the presence of insincere agents, a greater proportion of counterpart's opinion that an agent is willing to accept leads to a more pronounced disordered regime.

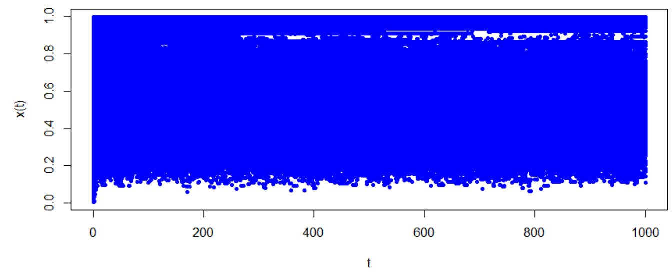

Fig. 5: The effect of insincere agents. Time evolution of the opinion dynamics for $\mu = 0.5$, $\epsilon = 0.3$, $m = 250$ and $n = 500$.

C. *Opinion Formation with Regular and Stubborn Agents*

We now relax the assumption on the regularity of agents. We assume that the set of agents $\mathcal{V}$ is divided into two distinct groups, $\mathcal{R}$ (regular agents) and $\mathcal{S}$ (stubborn agents), such that $\mathcal{V} = \mathcal{S} \cup \mathcal{R}$ and $\mathcal{S} \cap \mathcal{R} = \emptyset$.

As defined in Section II, an agent i is called stubborn if at least one of the two parameter values ϵ_i, μ is zero. Stubborn agents can be described as individuals that are biased towards their initial opinions. They have the ability to exert their influence onto others but cannot be influenced by the rest of society.

We assume a society $\mathcal{V}$ of $n = 500$ agents and that m of them are stubborn. Stubborn agents are assigned same initial opinion $x^S = \alpha \in [0, 1]$. In the following we assume that stubborn agents cannot be distinguished from other regular agents. Hence they fall in the neighborhood of regular agents, which cannot identify and avoid them. In this way stubborn and regular agents usually interact.

In Fig. 6 we examined the impact of the total number of stubborn agents, m, on the opinion dynamics for $\alpha = 0$. In the plots we can spot the presence of the $x_i = 0$ extreme opinion of the stubborn agents and, with respect to Figs. 1 and 2, we notice also that the low central consensus is vanished. More precisely, when m is not too big ($m = 30$), the low central consensus deviates toward the position of stubborn agents but it disappears when the number of stubborn agents increases. In Fig. 7, we examined the impact of α on the time evolution of opinion dynamics, by considering $\alpha = 0.3$. In this case we can spot the presence of a consensus on the position of the stubborn agents that persists with increasing number of stubborn agents. Then we can argue that a consensus can be stimulated by stubborn agents, but it resists to their excessive proliferation if α is sufficiently distant from 0. A similar consideration can be done for the opposite position, as we can see in Fig. 8. If α is sufficiently near to 1, the consensus cannot be reached.

D. *Opinion Formation with Two Groups of Stubborn Agents*

We now extend the previous section by introducing another group of stubborn agents. We assume a society $\mathcal{V}$ of $n = 1000$ agents and that $2m$ of them are stubborn. Two classes of stubborn agents are assigned to the initial opinions $x^{S_1} =$

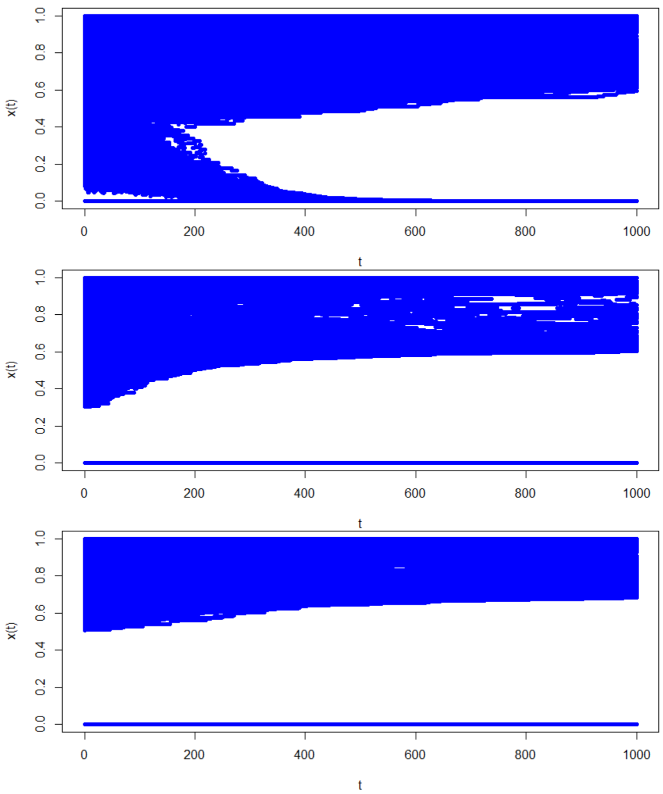

Fig. 6: The effect of stubborn agents ($\alpha = 0$). Time evolution of the opinion dynamics in three selected runs: $m = 30$ (top plot); $m = 150$ (central plot); $m = 250$ (bottom plot). Here $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

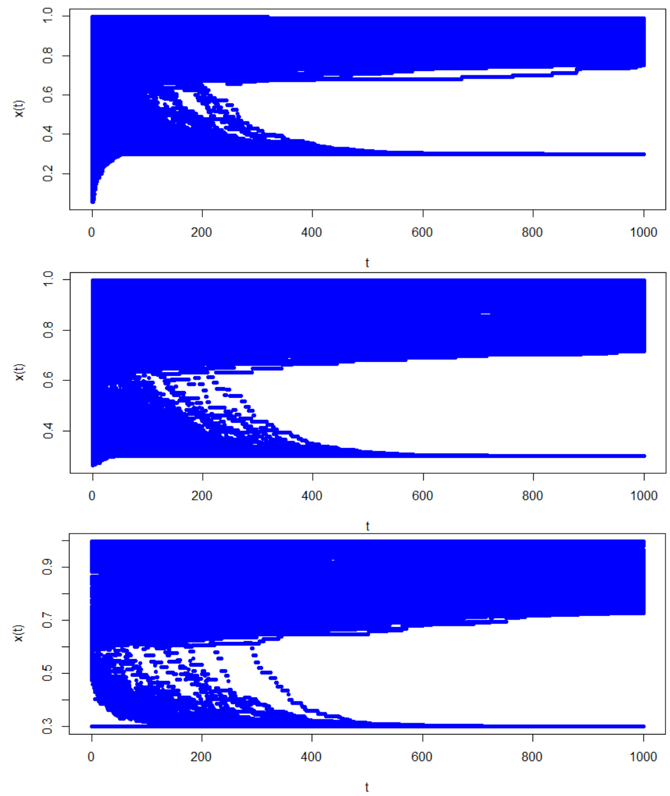

Fig. 7: The effect of stubborn agents ($\alpha = 0.3$). Time evolution of the opinion dynamics in three selected runs: $m = 30$ (top plot); $m = 150$ (central plot); $m = 250$ (bottom plot). Here $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

$\alpha \in [0, 1]$ and $x^{S_2} = \beta \in [0, 1]$, with $\alpha \neq \beta$. We assume that the groups of stubborn agents are equally sized, consisting of $m = 150$ stubborn agents each.

In Fig. 9 we fixed $\beta = 1$ while α varies from 0 to 0.5. When $\alpha = 0$ the extremism prevails and the society ends in a complete polarisation of the opinion space; see top plot of Fig. 9, where it is also possible to identify the isolated position of S_2 class of stubborn agents with $x^{S_2} = 1$. When α reaches 0.5 (bottom plot of Fig. 9) the majority of regular agents concentrate in the center, forming a large single opinion class. If we denote this class by $\mathcal{C}$, we have that for $T \to +\infty$, $x_i(T) \to 0.5\ \forall i \in \mathcal{C}$.

In Fig. 10, we examined the impact of ϵ on the opinion formation process with regular agents and two groups of stubborn agents, for which we assumed $x^{S_1} = 0$ and $x^{S_2} = 1$. When $\epsilon < 0.3$ the agents whose opinion is approximately in the range $[0, 0.6]$ move towards either a central consensus or the position of S_1 (however there is room for alternative opinions at the upper bound of opinion range). Anyway, as ϵ rises, the central consensus vanishes and only $x^{S_1} = 0$ remains in addition to the high extreme opinions.

IV. Skilled Regular, Honest Agents

Although it is not a strict rule, we have a tendency to think that more well-educated and competent people are also

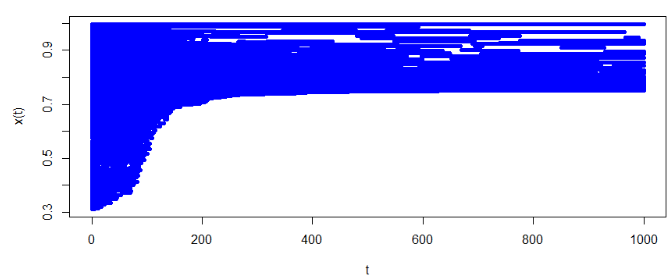

Fig. 8: The effect of stubborn agents ($\alpha = 0.75$). Number of stubborn agents: $m = 150$. Here $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

those best disposed to dialogue. According to this view of competence-opinion relation, an agent with an attitude to listen other people is characterized by a high competence, while an individual unwilling to listen and dialogue is usually marked by a lower level of the described trait. Hence we postulate that the threshold of Gaussian bounded confidence model depends on the degree of competence, e.g. replacing Eq. (6) with:

$$e^{-(x_i(t)-x_j(t))^2} \chi_{(-\epsilon_{i,j}, \epsilon_{i,j})} \left(d_{i,j}(t)\right), \tag{7}$$

where

$$\epsilon_{i,j} = \frac{\epsilon}{1 + e^{c(y_j - y_i)}}, c \gg 1 \tag{8}$$

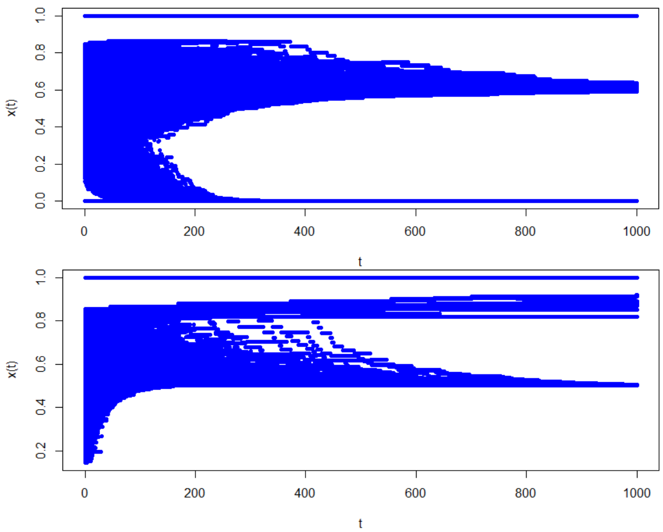

Fig. 9: The effect of two classes of stubborn agents (fixed $\beta = 1$). Time evolution of the opinion dynamics in two selected runs: $\alpha = 0$ (top plot); $\alpha = 0.5$ (bottom plot). Here $\mu = 0.1$, $\epsilon = 0.3$.

and

$$\mathbf{y} = (y_1, \ldots, y_n)^T$$

is the competence vector, which is supposed to be constant in time. In this way we are assuming that each agent i is characterized by two variables, $(x_i(t), y_i)$. Eq. (8) has been considered in [5] in order to model the so-called *equality bias effect* (see also [4]).

The competence vector $\mathbf{y}$ has been drawn from a standard uniform distribution with: $[0, 1]$ support for the first m agents, $[10, 15]$ support for the remaining ones. For simplicity, initial opinion vector $\mathbf{x}(0)$ has been arranged in such a way its elements are in ascending order, i.e. $x_1(0) < x_2(0) < \cdots < x_n(0)$.

In Fig. 11 the system evolves toward two clusters, characterizing two subpopulations with different decisions driven by the most competent agents (upper part of the plot) and the less skilled ones (lower part). We can spot the presence of a region in which regular skilled agents continuously change their opinions, in the upper part of the plot, and the presence of a lower consensus for the unskilled people.

V. Conclusions and Future Perspectives

We have built and simulated an Agent-Based Model (ABM) for opinion dynamics in personal finance decisions. We employed a Gaussian bounded confidence with pairwise random meetings to examine the role of different categories of agents in opinion formation. The model was simulated on a scale free network. Our findings can be summarized as follows.

- When only regular, honest agents are present those agents with an initial starting opinion that is below a certain threshold are rapidly drawn to a low central consensus; μ speeds up the convergence to the low central consensus. Moreover, as ϵ rises, high extreme opinions emerge.

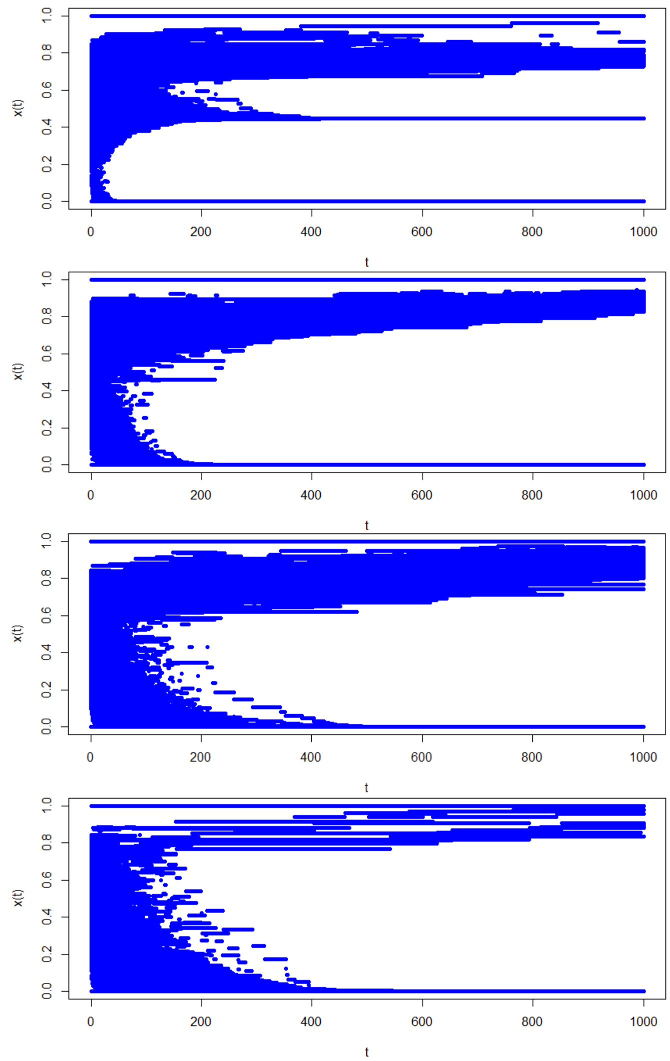

Fig. 10: Opinion dynamics with regular agents and two groups of stubborn agents ($\alpha = 0$ and $\beta = 1$). Selected single runs for the given parameter values are displayed. From the top to the bottom, respectively: $\epsilon = 0.2$, $\epsilon = 0.45$, $\epsilon = 0.55$ and $\epsilon = 0.7$. Parameter $\mu = 0.3$.

- With the inclusion of the insincere agents, the low central consensus disappeared, and a disordered regime in which opinions are in a constant state of change around a central opinion, emerged by varying the number of insincere agents. Anyway, a greater proportion of counterpart's opinion, that an agent integrates into his prior, leads to a more pronounced disordered regime.
- When we relax the assumption on the regularity of agents, in presence of stubborn agents, if the number of these agents is not too big, the low central consensus deviates toward the position of stubborn agents but it disappears with the increase of this number.
- When another population of stubborn agents is added, the extremism prevails and the society ends in a complete polarisation of the opinion space.

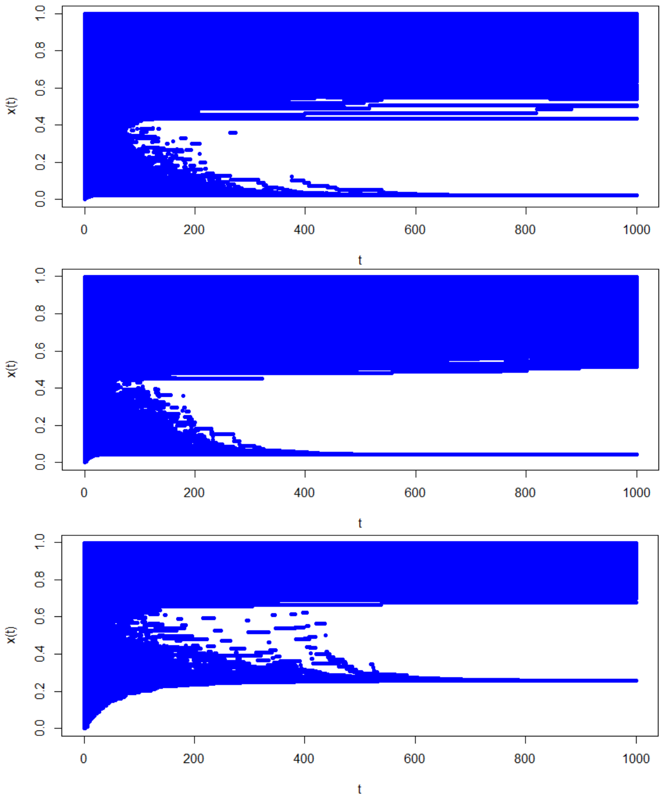

Fig. 11: Skilled regular agents vs unskilled ones. Time evolution of the opinion dynamics in three selected runs: $m = 50$ (top plot); $m = 100$ (central plot); $m = 500$ (bottom plot). Here $\mu = 0.2$, $\epsilon = 0.4$ and $n = 1000$ agents.

- The system of skilled and unskilled agents evolves toward two clusters; regular skilled agents continuously change their opinions, while the presence of a lower consensus is due to the unskilled people.

Since our ultimate goal is to achieve a better understanding of human decisions in personal finance in a real world context, we expect to validate our results concerning a stylized model of a real society. Future work therefore includes the collection of large amount of user interaction information from online social networks, e.g. Twitter, and the analysis of the dynamic sentiments of users to investigate realistic opinion evolution, as proposed in [17].

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