

# Context Graphs for Argumentation Logics

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**Abstract.** Representing human reasoning and argumentation requires robust formalisms that go beyond classical, deductive logics in dealing with aspects of human reasoning such as inconsistency or defeasibility. The field of Formal Argumentation comprises a plethora of formalisms that address a subset of these problem while occupying intersecting regions of the space parametrised by the factors implementation, formality and expressivity. This multiplicity complicates the compatibility, comparability and reuse of argument formalisms and argumentative content. The modular approach to knowledge representation is designed to address these problems. This paper proposes that modularity is best achieved by refactoring argumentation formalisms into a representation language and an on-top graph structure – the **context graph**. We claim that argumentativeness arises from conflict- and support-relations between internally consistent contexts and conjecture that *every argumentation formalism is isomorphic to some combination of a classical logic and a context graph*.

## 1 Introduction

In complex decision situations, individuals and organizations face a multitude of options and alternatives. To reach a well-balanced and justified decision, it is essential to systematically weigh arguments for and against an option and to take into account different perspectives and assumptions. Support by information technologies is indispensable for finding relevant facts and arguments and to analyze and aggregate them in a given context – but currently, the requisite technologies are still in their infancy [CV18a]. Search engines and decision-supporting information systems like e.g. IBM Watson operate on single facts as information unit. They can extract large collections of facts from documents but cannot consolidate them into arguments, much less contextualize or validate arguments. Without an embedding into an argumentative frame, isolated facts cannot yield deeper insights or justifications, so mere big-data-style correlation analyses do not suffice for decision support in complex situations. To just name one example: in the context of human/machine dialogues, generating targeted explanations is indispensable to e.g. make erroneous behavior of a machine understandable [TM07].

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Meeting this challenge requires a mix of technologies: *i*) information retrieval and computational linguistics to extract facts, arguments, and contexts, *ii*) knowledge representation technology to model arguments and their contexts, *iii*) inference techniques to analyze, aggregate, and plausibilize them, and finally *iv*) human/computer interaction technologies to make the ensuing information systems amenable to humans. In recent years these challenges have inspired an increasing amount of research: argumentation mining [CV18b] seeks to extract argument(ation)s from text; argumentation theories [EV18] attempt to (empirically or normatively) model human argumentative reasoning; dialogue systems [Pra06] address the interactive aspect of argumentation. The speed of development in this area is exemplified by the recent emergence of IBM’s Project Debater [Pro] as the first system that implements the full argumentation pipeline – as demonstrated in its much publicized debate with professional debater Harish Natarajan.

The work presented in this paper is situated in the DFG Special Research Action (SPP) 1999 “RATIO: Robust Argumentation Machines” [RATIO]; concretely the ALMANAC (Argumentation Logics Manager & Argument Context Graph) [AL] project in RATIO. It makes a contribution to the two middle aspects: knowledge representation and reasoning.

Concretely, we show how to systematically and modularly extend classical logics by on-top graph structures yielding full-blown argumentation frameworks in a knowledge representation system (MMT) that allows for rapid prototyping and high intercompability of frameworks.

To this end in Section 2 we review previous attempts to unify and modularize the plethora of argumentation formalisms and compare them to our approach. In Section 3 we summarize the MMT system and language. In Section 4 we present context graphs in MMT and how they can be used to capture argumentation formalisms. We conjecture that context graphs enable us to characterize all of the existing argumentation formalisms. Section 5 provides a first example of how classical logics can be rendered argumentative through context graphs and Section 6 carries out a first test of our conjecture. Concluding, in Section 7 we summarize our findings and outline our future research plans.

## 2 State of the Art in Modularizing Formal Argumentation

Argumentation formalisms, for our purposes, include those frameworks that include an automated reasoning component. In particular, we don’t consider here:

- **argument structure theories** [Ree+17; Tou03] that investigate empirically which components make up (good) arguments;
- **argument schemes** [MWR17] that provide a taxonomy of argument-types that frequently occur “in the wild”;
- **representational frameworks** [Che+06] that aim at maximal coverage of argumentative content;
- **dialogue systems** [Pra06] that focus on the dynamics of argumentation and the associated computational problems.

This does not mean that these fields do not pose interesting research questions in our setting: especially argument schemes and argument dynamics are highly interesting from an MMT perspective. However, we leave it to future work to investigate these issues properly.

For a full review of the existing argumentation formalisms we defer to [Koh18] and the excellent survey provided by [Pra17]. Here we will only summarize the previous attempts to introduce greater unity and modularity to the zoo of existing argumentation formalisms.

*Logic-Based Argumentation* The logic-based approach (LBA) [BH18] constitutes the paradigm that is most closely related to our work. In logic-based argumentation a base logic is extended by definitions of arguments, counterarguments (that is, attack between arguments) and argument graphs (composed from arguments and counterarguments). An argument is a pair of premises  $\Gamma$  and a conclusion  $\alpha$  such that  $\alpha$  can be derived from  $\Gamma$  in the proof theory of the base logic. One or several counterargument relations are usually defined depending on the choice of base logic. Usually these relate the claim of an argument to the premises (undercut, defeat) or the claim of another (rebuttal). Argument graphs can be created either through modelling of a given natural language argumentation or through automatic generation from a given knowledge base in the base logic.

*ASPIC+* On a high level, the structure of ASPIC+ [MP18] frameworks is similar to LBA in that it consists of a base logic (called the *logical language*) and an argumentative superstructure (called an *argumentation system*). However there are three crucial differences: firstly, in ASPIC+ the proof theory is part of the superstructure and may include **defeasible** rules. Secondly, knowledge-bases may not only include fixed knowledge (axioms) but also defeasible content (ordinary premises). Thirdly, the notions of argument, counterargument and argument graphs are not freely definable but fixed with respect to the argumentation system and the knowledge base.

As such ASPIC+ and LBA not only intersect in terms of expressivity but also have functional differences: LBA is best viewed as a manual on how to extend a given classical logic to an argumentative one. In contrast, ASPIC+ is a fixed framework that can be instantiated by some combination of a logical language, proof calculus and knowledge base but does not allow the modification of the constituents of the framework itself.

*Assumption-Based Argumentation* Assumption-based Argumentation (ABA) [Cyr+17] occupies a middle position between LBA and ASPIC+. Namely, it shares the basic structure of a base logic with an argumentative superstructure with the former two. In ABA the superstructure consists of sets of assumptions and a notion of contrariness for sentences of the language. Like in LBA the base logic is usually classical – that is there are no defeasible inference rules. On the other hand there is only one version of attack between sets of assumptions which is defined in terms of derivability and contrariness.

Historically, ABA is closely tied to the analysis of non-monotonic logics in terms of abstract argumentation. In essence, by choosing the correct classical base logic and a suitable notion of contrariness ABA allows the characterization of many non-monotonic logics and their semantics. In spite of this similarity to the LBA approach, ABA is not merely the fragment of LBA that deals with non-monotonic logics: there is no equivalent to the notion of contrariness in LBA. Neither is it a superset of ASPIC+ as it does not admit preference relations<sup>1</sup> over arguments (however the fragment of ASPIC+ without preferences can be translated into ABA [Dun16]).

Summarizing, our approach differs from the existing modular frameworks in several ways:

- Instead of defining counterarguments ad-hoc with respect to the given base logic as in LBA or fixing their definitions as in ASPIC+ and ABA, we seek to define them as logic- and even foundation-independently as possible. This enables greater modularity and is achieved by constructing them from very primitive notions such as theory-morphisms.
- Instead of being a mere superstructure on the base logic, context graphs enable the embedding of argumentative content into a powerful knowledge representation framework (OMDoc/MMT). OMDoc/MMT provides much more extensive ontology and context management capabilities than implementations of argumentation frameworks usually do. For example, content and in principle also arguments can travel across the context graph through theory morphisms and there is a priori no reason that different contexts could not have different representation languages.
- Instead of developing frameworks on paper we implement them from the start in the OMDoc/MMT knowledge representation system enabling features such as proof checking, semantic computation and visualization.
- Finally, instead of fixing the semantics of arguments in terms of deductions in the base logic, arguments in our approach can in principle take almost arbitrary form; we do not exercise this here, but leave it to future work.

We therefore conjecture that classical logics plus context graphs subsume ASPIC+, LBA, and ABA in terms of expressibility while at the same time being superior from a design perspective. In terms of expressibility we submit the ALMANAC-conjecture (see Section 4) that any argumentation formalism can be captured by a combination of a classical representation language and a context graph. From a design perspective our modular approach enables reuse, comparability and compatibility of formalisms and argumentative content in a powerful knowledge representation framework. This allows the rapid prototyping of new argumentation frameworks at a high level of implementation including capabilities such as automated proof checking, argumentation semantic computation and argument graph visualization.

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<sup>1</sup> However ABA extensions with preferences exist, see e.g. [Wak17]

### 3 Modular Knowledge Representation in the OMDoc/MMT system

OMDoc [Koh06] is a wide-coverage representation language for mathematical knowledge (formal) and documents (informal/narrative). In the last decade development has focused on the formal aspect leading to the OMDoc/MMT instance (Meta-Meta-Theories or Meta-Meta-Tool [RK13], henceforth simply MMT), which increases expressivity, clarifies the representational primitives and formally defines the semantics of this fragment.

*MMT* is designed to be foundation-independent and introduces several concepts to maximize modularity and to abstract from and mediate between different foundations. This enables the reuse of concepts, tools, and formalizations. The MMT language *integrates successful representational paradigms*

- the logics-as-theories representation from logical frameworks,
- theories and the reuse along theory morphisms from the heterogeneous method,
- the Curry-Howard correspondence from type theoretical foundations,
- URIs as globally unique logical identifiers from OpenMath,
- the standardized XML-based interchange syntax of OMDoc,

and makes them available in a single, coherent representational system for the first time. The combination of these features is based on a small set of carefully chosen, orthogonal primitives in order to obtain a simple and extensible language design. Using these primitives, logical frameworks, logics and theories *within* some logic are all uniformly represented as MMT theories, rendering all of those equally accessible, reusable and extendable. Constants, functions, symbols, theorems, axioms, proof rules etc. are all represented as constant declarations, and all terms which are built up from those are represented as objects.

*Theories* are named groups of declarations at the top-level of MMT's surface syntax. A theory in MMT is declared as in Fig. 1. A theory usually declares a metatheory that provides the language to be used in the theory's declarations. In Fig. 1 this is first-order logic (FOL). The body of the theory can include other theories (in this case FOL natural deduction). In addition this is where new constants are declared.

*Theory morphisms* represent truth-preserving maps between theories. Examples include theory inclusions, translations/isomorphisms between (sub-)theories and models/instantiations (by mapping axioms to theorems that hold within a model), as well as a particular theory inclusion called **meta-theory**, that relates a theory on some meta level to a theory on a higher level on which it depends. This includes the relation between some low level theory (such as the theory of groups) to its underlying foundation (such as first-order logic), and the latter's relation to the logical framework used to define it – e.g. LF; see [Pfe01] for an overview.

All of this naturally gives us the notion of a *theory graph*, which relates theories (represented as nodes) via vertices representing theory morphisms (as in Fig. 2), being right at the design core of the OMDoc/MMT language.

It is a central advantage of the OMDoc/MMT system that theory morphisms “transport axioms, definitions, theorems, ...” to new contexts and thus induce

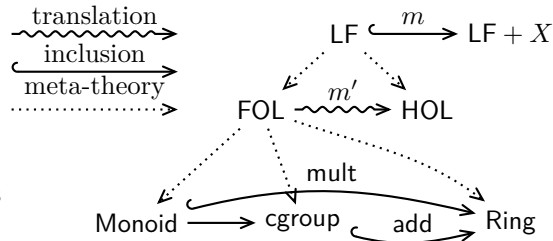
knowledge that is not explicitly represented in the graph. Therefore it is a central design invariant of the system that we can name all induced objects with canonical URIs, the MMT URIs, which contain enough information to reconstruct the induced objects themselves – given the graph.

```
theory background_knowledge : found:?Logic =
  include found:?NaturalDeduction |
  Tweety : type |
  bird : type → prop |
  axiom: ⊢ bird Tweety |
  |
```

**Fig. 1.** The head of the theory declaration contains the theory’s name and metalanguage (an MMT FOL implementation) and is delimited by “=”. The proof theory for FOL is included in the first line of the body. The other lines are constant declarations: read “Tweety is a bird”.

### 3.1 Derived Modules

Modules are the top-level declarations of the MMT language. The primitive modules are theories and theory morphisms (mainly views and structures). They have dedicated corresponding data structures in MMT’s abstract syntax that are known to other MMT components such as MMT’s type checker. In the MMT



**Fig. 2.** A Theory Graph with Meta-Theories

surface syntax a module declaration starts with a keyword and takes an optional definiens in terms of other modules. This is followed by the body of the module containing its declarations and a concluding module delimiter. Besides primitive modules, MMT offers users the ability to define **derived modules**. Derived modules elaborate into some combination of more primitive modules. This allows for the definition of a great variety of theory- or morphism-like modules while at the same time maintaining MMT features like proof-checking.

The conceptual advantage of derived declarations and modules in MMT is that they allow to combine the advantages of *deep* and *shallow encodings* – here from argumentation theories into MMT. Generally, an encoding models the meaning of (structures of) the source language using structures of the target language. We call an encoding **shallow** if the encoding preserves the structure

of the source language, otherwise **deep**. Derived modules allow us to give shallow embeddings by extending the “syntax” of MMT theory graphs with theory relations that structurally resemble the argumentation logics, while their elaboration generates the deep embedding into MMT primitives that can be managed and type-checked.

## 4 Context Graphs and the ALMANAC-Conjecture

The idea first put forward in [Koh18] is to enrich theory graphs with argumentative relations to capture existing argumentation theories and offer increased context management capabilities. In this paper this idea is elaborated in terms of derived **theory relations**. In context graphs, theories are interpreted as potentially mutually exclusive yet internally consistent contexts. The intuition is that an argument is a **locally valid** “deduction” in that it holds with respect to its context – a set of assumptions, axioms, inference rules or even global theories that can be interpreted, for example, as the persuasions of an agent, the tenets of a certain field or the contents of a knowledge base.

We argue that argumentation theories are particular instances of such context graphs where the base logic and the admissible theory relations are fixed. MMT already comprises an immense archive of possible base logics: the LATIN Logic Atlas [Cod+11; LATIN]. In this paper we use derived relations to enrich it with argumentative theory relations. Generally we submit

*Conjecture 1 (ALMANAC Conjecture).* Each argumentation formalism  $\mathcal{A}$  can be captured by a classical object-language  $\mathcal{L}$  and a context graph scheme  $\mathcal{G}$  such that  $\mathcal{A}$  is isomorphic to  $\langle \mathcal{L}, \mathcal{G} \rangle$ .

Due to the prominence of argumentation in human thought, if true, the ALMANAC-conjecture would imply that much of human dialectical capability can be constructed from classical logics plus extremely simple ingredients such as theory morphisms. This reduction of an important human faculty to simple ingredients would constitute a first step towards a *grammar of thought* that would identify the basic components of human reasoning and the rules of their composition.

## 5 Example 1: Inconsistency Graphs

The simplest way to extend a classical logic with argumentative capabilities is to make use of the logic’s built-in inconsistency symbol  $\perp$ .  $\perp$  is either part of the language or can be defined in almost all classical logics. An inconsistency relation based on  $\perp$  is thus of special interest as classical logics – in particular first-order logic and higher order logics – traditionally form the metatheories used to approximate natural mathematical language. Therefore any theory relation defined only in terms of  $\perp$  could be used in almost arbitrary mathematical domains via MMT’s inheritance mechanisms.

We shall develop the intended joint-inconsistency relation in the context of propositional logic. As such we require that all theories in an inconsistency graph have a common meta-theory that includes at least propositional logic. Then we define inconsistency graphs as follows:

**Definition 1 (Inconsistency Relation).** *Let  $\{\mathcal{T}_1, \dots, \mathcal{T}_n\}$  be a set of theories,  $C$  a conflict theory declaring only  $\vdash \perp$  and  $\mathcal{M}$  a theory that includes propositional logic and is a meta-theory for all  $\mathcal{T}_i$  and  $C$ . Then  $\{\mathcal{T}_1, \dots, \mathcal{T}_n\}$  are jointly inconsistent iff there exists a total view  $C \mapsto \bigcup_{i=1}^n \mathcal{T}_i$ .*

In other words a collection of theories is jointly inconsistent iff there exists a homomorphism from the inconsistent theory into their union. Procedurally, MMT requires the user to declare an inconsistency relation by specifying the conflict theory  $C$  and an assignment for  $\vdash \perp$  either in the form of a declaration of the target theory or in the form of a proof  $\phi$  that  $\bigcup_{i=1}^n \mathcal{T}_i \vdash \perp$ . It then elaborates the declaration by implicitly constructing  $\bigcup_{i=1}^n \mathcal{T}_i$  through the use of includes and a view from  $C$  to  $\bigcup_{i=1}^n \mathcal{T}_i$  that maps  $\vdash \perp$  to  $\phi$ . Next it checks whether  $\phi$  is a valid  $\bigcup_{i=1}^n \mathcal{T}_i$  proof.

Inconsistency graphs are then defined like so:

**Definition 2 (Inconsistency Graph).** *Let  $\mathfrak{G}$  be a theory graph with vertices  $V$  and edges  $E$ .  $\mathfrak{G}$  is an inconsistency graph if*

- $V$  is a set of theories;
- $E = Views \cup Metas \cup Includes \cup Inconsistencies$  where *Views*, *Metas*, *Includes* are the theory morphisms described earlier and *Inconsistencies* is an inconsistency relation
- there is a meta-theory  $\mathcal{M} \in V$  that includes at least propositional logic such that  $(\mathcal{M}, \mathcal{T}) \in Metas$  for all  $\mathcal{T} \in V$

Figure 3 displays an example of such an inconsistency graph. Here the meta-theory is first-order logic and the theories declare an assortment of facts involving some common ground. The theory “Common Ground” corresponds to the theory declaration in Fig. 1. Two jointly inconsistent extensions of “Common Ground” arise on the question whether Tweety can fly or not. The double-arrow denoting joint inconsistency is elaborated by MMT into the part of the graph color-coded in red. The view  $\varphi$  maps  $d$  to a proof of  $\perp$  using conjunction introduction (*CI*) and falsum introduction (*FI*).

## 6 Example 2: ABA and Asymmetric Conflict Relations

As we have noted in Section 2 ABA is one of the major unifying frameworks in argumentation and non-monotonic logic. We will carry out a first test of the ALMANAC-hypothesis by characterizing ABA in terms of a context graph. Before we tackle this problem we need to formally define ABA-frameworks in the following section.



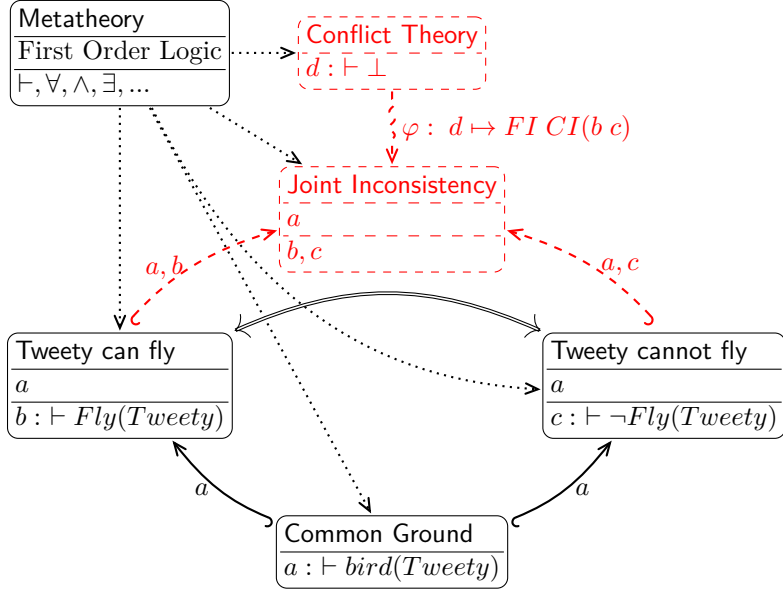


Fig. 3. A context graph scheme characterizing the binary joint inconsistency relation.

### 6.1 Assumption-Based Argumentation

ABA presupposes a classical base logic together with a proof calculus:<sup>2</sup>

**Definition 3 (Deductive System).** A deductive system is a pair  $\langle \mathcal{L}, \mathcal{R} \rangle$  where  $\mathcal{L}$  is any classical logic that contains at least the falsum symbol  $\perp$  and  $\mathcal{R}$  is a proof calculus on  $\mathcal{L}$  that has at least the falsum introduction rule  $\vdash \alpha, \vdash \neg\alpha \longrightarrow \vdash \perp$ .

Such classical deductive systems are then rendered argumentative through the addition of set of assumptions and a contrary-operator:

**Definition 4.** (Assumption-Based Framework) An Assumption-Based Framework with respect to a deductive system  $\langle \mathcal{L}, \mathcal{R} \rangle$  is a triple  $\langle T, ABox, \bar{\ } \rangle$  where  $T, ABox \subseteq \mathcal{L}$  are a Tbox and a set of assumptions respectively.  $\bar{\ }$  is a function  $\mathcal{L} \longrightarrow \mathcal{L}$

The contrary operator  $\bar{\ }$  is freely definable together with the base logic determines the the characterized non-monotonic logic. Such characterization results exist among other logics for logic programming, default logic and autoepistemic logic and non-monotonic modal logic [Bon+97].

While the notion of argument is not explicitly defined in ABA, their role is essentially played by derivations. A formula  $\alpha$  over  $\mathcal{L}$  is **derivable** from a set of formulas  $T$  (put  $T \vdash \alpha$ ) if there exists a sequence of formulas  $\beta_1, \dots, \beta_n$

<sup>2</sup> Our exposition here follows [Bon+97].

s.t.  $\beta_n = \alpha$  and for each  $i = 1, \dots, n$ ,  $\beta_i \in T$  or there is  $\alpha_1, \dots, \alpha_m \vdash \beta_i$  with  $a_j \in \{\beta_1, \dots, \beta_{i-1}\}$  for all  $j=1, \dots, m$ .

Attacks between sets of formulas are then defined as follows:

**Definition 5.** (*Attack*) A set of formulas  $F$  attacks another one  $F'$  iff there is  $f \in F'$  s.t.  $F \vdash \bar{f}$ . If  $F' = \{f\}$  we say that  $F$  attacks  $f$ .

The key difference to the notion of inconsistency discussed in Section 5 is that attack may be asymmetric. Consider the following example:

*Example 1.* Consider the assumption-based framework defined as follows:

- $\langle \mathcal{L}, \mathcal{R} \rangle =$  Propositional Language and Calculus.
- $T = \{a \rightarrow b\}$
- $ABox = Arg_1 \cup Arg_2$
- $Arg_1 = \{\neg c\}$
- $Arg_2 = \{a, b \rightarrow c\}$
- $\bar{\varphi} = \neg\varphi$

Clearly,  $Arg_2$  attacks  $Arg_1$  as we can derive  $c$  (the contrary of  $\neg c$  using Modus Ponens twice. However, there is no attack from  $Arg_1$  to  $Arg_2$  as  $Arg_1 \cup T$  is already logically closed and  $C \notin Arg_1$ .

## 6.2 Assumption-Based Attack Graphs

The challenge in characterizing ABA in terms of a context graph is to capture the asymmetric attack relation in terms of theory morphisms.

Towards this goal we modify the metalogic in which we are working. Instead of propositional logic we now allow for any logic with propositional letters, a proof calculus and an unary contrariness operator  $\bar{\phantom{x}}$ . Such a contrariness operator could be classical negation but we also allow for any other notion of contrariness. In order to define assumptions, we extend this metalanguage slightly: in addition to a contrary operator we equip it with an assumptionhood operator  $\vdash$  and a rule *assumptionhood implies derivability (aid)* that states  $\vdash\alpha \longrightarrow \vdash\alpha$  for any well-formed formula  $\alpha$ . We call the resulting language an **assumption-enriched logic**.

The assumptionhood operator is used to distinguish declarations that are assumptions from “ordinary” declarations. Contrary to an ordinary declaration, an assumption can become the target of an attack. In this way it introduces the asymmetry we are after:

**Definition 6 (Assumption-Based Attack Relation).** Let  $\mathcal{T}, \mathcal{T}_1, \mathcal{T}_2, \mathcal{C}, \mathcal{C}_1, \mathcal{C}_2$  be theories that share a common meta-theory  $\mathcal{M}$  that includes an assumption-enriched logic such that

- $\mathcal{T}_1$  includes  $\mathcal{T}$ ,
- $\mathcal{T}_2$  includes  $\mathcal{T}$ ,
- $\mathcal{C}$  declares only a constant  $c$ ,

- $\mathcal{C}_1$  includes  $\mathcal{C}$  and declares only  $\vdash c$ ,
- $\mathcal{C}_2$  includes  $\mathcal{C}$  and declares only  $\vdash \bar{c}$ .

Then  $\mathcal{T}_1$  **attacks**  $\mathcal{T}_2$  iff

- there exists a total view  $\delta : \mathcal{C} \mapsto \mathcal{T}$  such that  $\phi(c) = t$  for some expression  $t$  of  $\mathcal{T}$  (the **kernel** of the attack),
- there exists a total view  $\phi : \mathcal{C}_1 \mapsto \mathcal{T}_1$  such that  $\phi(\vdash c) = \vdash t$  (the **assumption wing** of the attack),
- there exists a total view  $\psi : \mathcal{C}_1 \mapsto \mathcal{T}_2$  such that  $\phi(\vdash \bar{c}) = \pi$  where  $\pi$  is a proof of  $\bar{t}$  (the **derived wing** of the attack).

The kernel of the attack pinpoints the conflict and enforces that the declarations from the wings are mapped to mutual contraries of the same expression. The derived wing designates the origin of the attack arrow whereas the assumption wing demarcates its target.

Analogously to the joint inconsistency case, given a declaration of an attack relation by the user, MMT will implicitly construct and proof-check the conflict theories and views. In this case the user has to provide three view declarations in the body of the attack module.

**Definition 7 (Assumption-based Attack Graph).** Let  $\mathfrak{G}$  be a theory graph with vertices  $V$  and edges  $E$ .  $\mathfrak{G}$  is an Assumption-based Attack graph if

- $V$  is a set of theories;
- $E = \text{Views} \cup \text{Metas} \cup \text{Includes} \cup \text{Attacks}$  where Views, Metas, Includes are the theory morphisms described earlier and Attacks is an attack relation;
- there is an assumption-enriched meta-theory  $\mathcal{M} \in V$  such that  $(\mathcal{M}, \mathcal{T}) \in \text{Metas}$  for all  $\mathcal{T} \in V$

*Example 1 (continued).* Consider again our earlier example only that now  $T = \mathcal{T}$ ,  $Arg_1 = \mathcal{T}_1$  and  $Arg_2 = \mathcal{T}_2$ . The metalanguage is propositional logic where contrary is just plain negation. It is extended by the assumptionhood operator  $\vdash$  and *aid*. We then need to construct the kernel and wings of the attack from the canonical theories  $\mathcal{C}$ ,  $\mathcal{C}_1$ ,  $\mathcal{C}_2$ . Fig. 4 showcases this in detail. The kernel  $\delta : \mathcal{C} \mapsto \mathcal{T}$  maps  $w$  to  $c$  thereby ensuring that both of the wings have to map to the same formula  $c$  (and its contrary  $\bar{c}$  respectively). The assumption wing  $\phi$  requires an assumption as its target, the derived wing  $\psi$  a derivation. We achieve this by setting the codomain of  $\phi$  to  $Arg_1$ , mapping  $\vdash w$  to  $\vdash \neg c$ , and the codomain of  $\psi$  to  $Arg_2$ , mapping  $\vdash \bar{w}$  to a derivation  $c$ . The derivation uses negation introduction and twice modus ponens to derive  $\neg \neg c$  which is the contrary of  $\neg c$ . Note that we cannot reverse the wings and map  $\vdash c$  to  $\vdash \bar{c}$  in  $Arg_1$  as  $\vdash \bar{c}$  is not declared in  $Arg_1$ .

Example 1 showcases the major result of this section: assumption-based attack graphs subsume assumption-based frameworks. We simply translate the assumption-based framework's contrary to our meta-language's contrary, each subset of assumptions to a theory with assumptions as  $\vdash$ -declarations and instantiate every attack as an assumption-based attack.

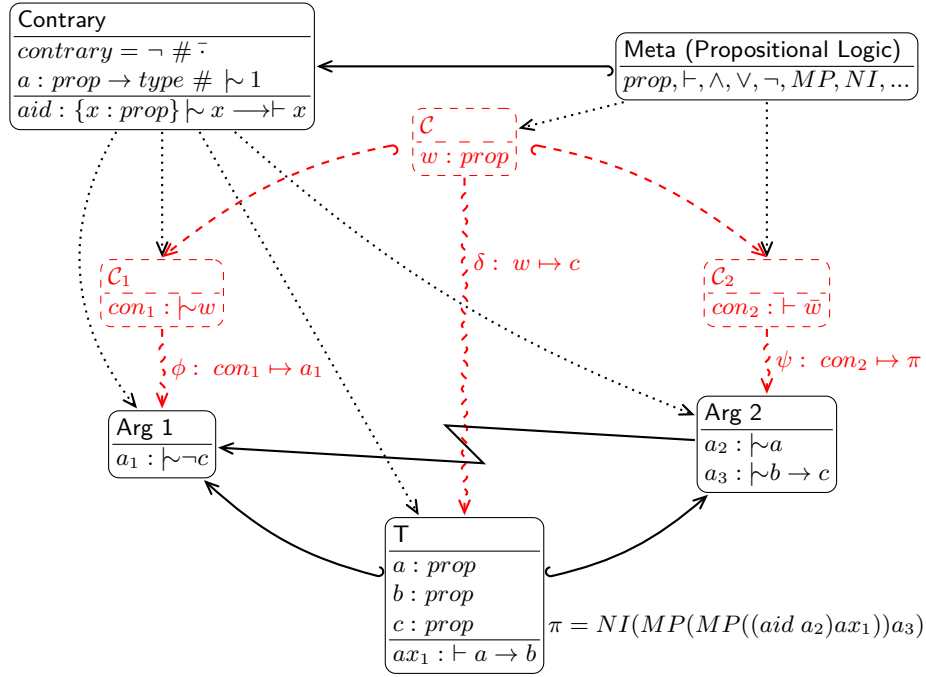


Fig. 4. ABA Example

## 7 Conclusion

We have shown how to render classical logics argumentative through context graphs using the examples of inconsistency graphs and assumption-based attack graphs. In this process we have confirmed the ALMANAC conjecture for the case of Assumption-Based Argumentation Frameworks – one of the major paradigms in formal argumentation. In future work we will demonstrate how this modular approach to argumentation formalism design enables the rapid prototyping of full-blown argumentation systems including proof-checking, semantic computation and visualization. In addition we want to investigate additional elaborations for attack and other argumentation relations. A first idea to be investigated for ABA would be an alternative way to facilitate non-symmetric attacks by insisting that the view  $\varphi$  in Figure 4 – i.e. the view into the target of the attack – be simple, i.e. maps constants to constants. Furthermore, we note that the elaborations have a distinct flavor of categorical constructions. This suggests that there is a connection of our work in institution theory. We want to extend [Rab08] to study this in more detail.

## References

- [AL] *ALMANAC: Argumentation Logics Manager & Argument Context Graph*. Project Home Page. URL: <http://kwarc.info/projects/almanac/> (visited on 06/17/2018).
- [Bar+18] Pietro Baroni, Dov Gabbay, Massimiliano Giacomin, and Leendert van der Torre, eds. *Handbook of Formal Argumentation*. College Publications, 2018.
- [BH18] Philipp Besnard and Anthony Hunter. “A Review of Argumentation based on Deductive Arguments”. In: (2018). Ed. by Pietro Baroni, Dov Gabbay, Massimiliano Giacomin, and Leendert van der Torre, pp. 437–484.
- [Bon+97] A. Bondarenko, P.M. Dung, R.A. Kowalski, and F. Toni. “An abstract, argumentation-theoretic approach to default reasoning”. In: *Artificial Intelligence* 93.1 (1997), pp. 63–101. DOI: [https://doi.org/10.1016/S0004-3702\(97\)00015-5](https://doi.org/10.1016/S0004-3702(97)00015-5).
- [Che+06] Carlos Chesñevar et al. “Towards an Argument Interchange Format”. In: *The Knowledge Engineering Review* 21.04 (Dec. 2006), p. 293. DOI: [10.1017/s0269888906001044](https://doi.org/10.1017/s0269888906001044).
- [Cod+11] Mihai Codrescu et al. “Project Abstract: Logic Atlas and Integrator (LATIN)”. In: *Intelligent Computer Mathematics*. Ed. by James Davenport, William Farmer, Florian Rabe, and Josef Urban. LNAI 6824. Springer Verlag, 2011, pp. 289–291. URL: [https://kwarc.info/people/frabe/Research/CHKMR\\_latinabs\\_11.pdf](https://kwarc.info/people/frabe/Research/CHKMR_latinabs_11.pdf).
- [CV18a] Elena Cabrio and Serena Villata. “Five Years of Argument Mining: a Data-driven Analysis”. In: *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI-18*. International Joint Conferences on Artificial Intelligence Organization, July 2018, pp. 5427–5433. DOI: [10.24963/ijcai.2018/766](https://doi.org/10.24963/ijcai.2018/766).
- [CV18b] Elena Cabrio and Serena Villata. “Five Years of Argument Mining: a Data-driven Analysis”. In: *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI-18*. International Joint Conferences on Artificial Intelligence Organization, July 2018, pp. 5427–5433. DOI: [10.24963/ijcai.2018/766](https://doi.org/10.24963/ijcai.2018/766).
- [Cyr+17] Kristijonas Cyras, Xiuyi Fan, Claudia Schulz, and Francesca Toni. “Assumption-based argumentation: Disputes, explanations, preferences”. In: *IFCoLog Journal of Logics and Their Applications* 4.8 (2017), pp. 2407–2456.
- [Dun16] Phan Minh Dung. “An Axiomatic Analysis of Structured Argumentation with Priorities”. In: *Artif. Intell.* 231.C (Feb. 2016), pp. 107–150. DOI: [10.1016/j.artint.2015.10.005](https://doi.org/10.1016/j.artint.2015.10.005).
- [EV18] Frans H van Eemeren and Bart Verheij. “Argumentation Theory in Formal and Computational Perspective”. In: (2018). Ed. by Pietro Baroni, Dov Gabbay, Massimiliano Giacomin, and Leendert van der Torre, pp. 3–73.

- [Koh06] Michael Kohlhase. *OMDoc – An open markup format for mathematical documents [Version 1.2]*. LNAI 4180. Springer Verlag, Aug. 2006. URL: <http://omdoc.org/pubs/omdoc1.2.pdf>.
- [Koh18] Michael Kohlhase. “Towards Context Graphs for Argumentation Logics”. In: ed. by Rainer Gemulla et al. Vol. 2191. CEUR Workshop Proceedings. CEUR-WS.org, 2018, pp. 203–214. URL: <http://ceur-ws.org/Vol-2191/paper25.pdf>.
- [LATIN] *The LATIN Logic Atlas*. URL: <https://gl.mathhub.info/MMT/LATIN> (visited on 06/02/2017).
- [MP18] Sanjay Modgil and Henry Prakken. “Abstract Rule-Based Argumentation”. In: (2018). Ed. by Pietro Baroni, Dov Gabbay, Massimiliano Giacomin, and Leendert van der Torre, pp. 287–408.
- [MWR17] Fabrizio Macagno, Douglas Walton, and Chris Reed. “Argumentation Schemes. History, Classifications, and Computational Applications”. In: *IFCoLog Journal of Logics and Their Applications* 4.8 (2017). URL: <http://dougwalton.ca/papersinpdf/17IFCoLogSCHEMES.pdf>.
- [Pfe01] Frank Pfenning. “Logical Frameworks”. In: *Handbook of Automated Reasoning*. Ed. by Alan Robinson and Andrei Voronkov. Vol. I and II. Elsevier Science and MIT Press, 2001.
- [Pra06] Henry Prakken. “Formal systems for persuasion dialogue”. In: *The Knowledge Engineering Review* 21.2 (June 2006), pp. 163–188. DOI: 10.1017/s0269888906000865.
- [Pra17] Henry Prakken. “Historical Overview of Formal Argumentation”. In: *IFCoLog Journal of Logics and Their Applications* 4.8 (2017), pp. 2183–2262. URL: <http://www.cs.uu.nl/groups/IS/archive/henry/history.pdf>.
- [Pro] *Project Debater - How does it work?* Accessed: 2019-24-06. URL: <https://www.research.ibm.com/artificial-intelligence/project-debater/how-it-works/>.
- [Rab08] Florian Rabe. “Representing Logics and Logic Translations”. PhD thesis. Jacobs University Bremen, 2008. URL: <http://kwarc.info/frabe/Research/phdthesis.pdf>.
- [RATIO] *RATIO: Robust Argumentation Machines*. Home Page of the DFG Special Research Action (SPP) 1999. URL: <http://spp-ratio.de> (visited on 06/17/2018).
- [Ree+17] Chris Reed et al. “The Argument Web: an Online Ecosystem of Tools, Systems and Services for Argumentation”. In: *Philosophy & Technology* 30.2 (May 2017), pp. 137–160. DOI: 10.1007/s13347-017-0260-8.
- [RK13] Florian Rabe and Michael Kohlhase. “A Scalable Module System”. In: *Information & Computation* 0.230 (2013), pp. 1–54. URL: <http://kwarc.info/frabe/Research/mmt.pdf>.
- [TM07] Nava Tintarev and Judith Masthoff. “A Survey of Explanations in Recommender Systems”. English. In: *Data Engineering Workshop*.

- Ed. by G. Uchyigit. IEEE Computer Society, Dec. 2007, pp. 801–810. DOI: 10.1109/ICDEW.2007.4401070.
- [Tou03] Stephen E Toulmin. *The uses of argument*. Cambridge university press, 2003.
- [Wak17] Toshiko Wakaki. “Assumption-Based Argumentation Equipped with Preferences and its Application to Decision Making, Practical Reasoning, and Epistemic Reasoning”. In: *Computational Intelligence* 33.4 (2017), pp. 706–736. DOI: 10.1111/coin.12111. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/coin.12111>.