Didactic Sequence for the development of variational thinking of university engineering students

Alejandro M. Ecos Espino[1], Joffré Huamán Núñez[2], Zoraida R. Manrique Chàvez[3]
[1] Universidad Nacional de Moquegua, Ilo, Perú.
alejandroecos2013@hotmail.com
[2] Universidad Nacional Micaela Bastidas de Apurímac, Abancay, Perú.
joffrehn13@hotmail.com
[3]Universidad Nacional Intercultural de la Amazonia, Pucallpa, Perú.
zoramanrique@hotmail.com

Abstract. This article is about the effect of a didactic sequence on the development of the variational thinking of university students entering engineering careers. For this purpose, an applied research was carried out with a quasi-experimental design. They were handled with 104 civil engineering students distributed in three groups: one of control (43) and two experiments (40 and 21) of a population of 329 students. For data collection, a questionnaire was designed and validated with 05 items related to the analysis of functions as representation models of variation and change. The methodology of the didactic sequence was based on variational activities carried out in 7 sessions with the experimental groups, while the traditional teaching methodology was used in the control group. The achieved results showed that there was significant influence of the didactic sequence in the development of the variational thinking of the students.

Keywords: didactic sequence, variational thinking, function.

1 Introduction

Differential Calculation represents one of the main achievements of humanity, becoming one of the quantitative tools for the research of scientific problems, which, for the most part, have been framed in phenomena of variation and change. Within this area of knowledge, the development of variational thinking constitutes an important element for the understanding of the change of phenomena in real life. His understanding is an essential axis in the mathematical training of university students, especially those who pursue engineering careers.

In higher education, teaching has as its fundamental purpose the training of professionals with skills to appropriate different mathematical concepts and apply them in different contexts [1]. The development of mathematical thinking allows students to describe, organize, deduct consequences, identify patterns and regularities, build arguments, manage different representation systems, interpret and relate to situations where reality is mathematized [2].

In university classrooms, the learning of mathematics constitutes a relevant problem that manifests itself through the high number of students who disapprove the subjects of this area and by the poor interpretation of the concepts that characterize higher mathematics. This problem tends to increase due to the tendency on the part of the teachers to the traditional teaching and the passive form with which the students assume their learning.

The traditional teaching of calculus has focused its activity on the handling of problem-solving algorithms in many cases far from reality and circumscribed to the mathematical field, not allowing an effective development of mathematical thinking, which creates difficulties to make them really enter the field of the calculation and to make them reach a satisfactory understanding of the concepts and methods of mathematical thinking [3], without being able to develop the capacity in the students to assign a broader meaning to the notions involved in their understanding [4]

Our experience working with university students supports the above. Students give preference to their performance mechanically, restricting themselves to the application of known formulas to determine immediate results; and the algorithmic management of the variables that are part of a mathematical model, without questioning how these variables act and the type of dependence that exists between them. In this context, the concept of function plays an important role. Functions, due to their nature and definition, change in different ways, so understanding their behavior implies understanding how they change and quantifying these changes, which forces them to vary the discourse in the classroom and focus teaching on procedures that prioritize variation. The development of variational thinking implies conceptual changes and ruptures with previous or spontaneous knowledge, which generates the presence of obstacles that are caused by cognitive conflicts in students in the reading and interpretation of the behavior of functions through their graphs or other forms of representation [5].

The above reflects the need to design and implement didactic situations that prioritize the study and analysis of change and evaluate its effect on the development of variational thinking, as an essential axis in the mathematical formation of university students. The research question was: What is the effect of a didactic sequence on the development of variational thinking of engineering university students? The research hypothesis was: The implementation of a didactic sequence based on the study and analysis of change would significantly develop the components of variational thinking, compared to the traditional teaching methodology.

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2 Theoretical Background

2.1 The theory of semiotic records

Semiotic representations of an object are necessary when we have to represent an idea or a mathematical object. Its role is fundamental in the development of mental representations, in the fulfillment of different cognitive functions and in the production of knowledge, with which it allows to increase the thinking capacity of the subject about that object and therefore his knowledge of it. In this way, a representation record is a set of signs used to represent an idea or a mathematical object [6]. In order for a semiotic system to be considered a representation register, it must have three fundamental characteristics: to be identifiable, to allow treatment, and finally, to enable conversion. The transformations between systems of representation on a mathematical object can facilitate the emergence of such objects in students, and therefore favor their learning [7].

Mathematical activity is always based on some sequence of successive changes from one representation to another. The use of different representations for the same object increases the subject's ability to think about that object and therefore his knowledge of it. That is why it is important to keep in mind that there are always many possible representations of the same object. Each representation provides a different content according to the system used for its production, but always the object represented remains invariant [8].

In general, training and representation treatment is taken into account in teaching but not conversion. If no activities are proposed that allow students to coordinate at least two representation records, then there is no learning [6]. In this way, if the conceptualization and learning imply a coordination of representation records, the basic learning in mathematics can not only bet on the automation of certain treatments or the understanding of notions, but also must bet on the coordination of the different representation records mobilized for these treatments or for this understanding [9].

In this sense, learning mathematics consists in the development of progressive coordination between several semiotic systems of representation; discriminate and coordinate semiotic representation systems to become able to transform any representation [8]; So the concepts represented in different registers and from different points of view, allows a better organization of knowledge by students, represented a necessary condition for their learning [10].

2.2. Theory of Teaching Situations

With this theory, didactic phenomena in the teaching process are studied and modeled. In this context, the words teach, learn, think, know, acquire different meanings [4].

The mathematics is an organized set of knowledge produced by culture. Teaching thus becomes an activity that reconciles two processes, one of enculturation and another of relatively independent adaptation [11]. The important elements to consider within this theory are the following:

2.2.1. Mathematical knowledge

Knowledge is the main object of teaching, has a particular history, maintains cultural and social relations with the outside of the class, which largely determines the content to be taught and how to do it. The constituted knowledge is presented in different forms, for example questions and answers. [11]

2.2.2. Student

The student's work should, at least in part, reproduce the characteristics of the scientific work itself, as a guarantee of effective construction of relevant knowledge. A good reproduction for the student of a scientific activity would require him to act, formulate, test, build models, languages, concepts, theories, to change them for others, to recognize those that adapt to his culture, to resort to those that are useful, etc. [11]. The student learns by adapting to a medium that is a factor of contradictions, difficulties and imbalances.

A didactic situation is a set of relations explicitly or implicitly established between a student or group of students, in a certain medium, eventually comprising instruments or objects and, an educational system (the teacher) in order to enable students a knowledge constituted or in the process of constitution. [11]

- Situations of action.
- Situations of formulation.
- Situations of validation.
- Sitautions of institutionalization.

2.2.3. The teacher

The teacher must cause the student to acquire knowledge whose meaning and functioning are satisfactory. For this, an adequate choice of the problems proposed is essential, in order to raise the desired adaptations. In this sense, the work of the teacher must be, to some extent, inverse to that of the researcher, providing students with the means that allow them to recontextualize and repersonalize their knowledge. [11]

The student learns math by doing. The teacher must propose mathematical situations that students can experience and build mathematical knowledge as a solution to them. For a teacher, teaching refers to the creation of conditions that will produce the appropriation of knowledge by students [12].

2.2.4. The middle

In the process of knowledge production, the student's interactions with a certain problematic situation and the teacher's interaction with the student are basic. From this raises the need for a medium. First, the medium must be a factor of contradictions, difficulties, imbalances and, therefore, adaptation for the student. In order for this to happen and the interactions with the environment are, in this way, producers of new knowledge, it is necessary that the student can use the knowledge available to try to control this medium, and, in turn, receive feedback from it. to show you the inadequacy of your means of control. Second, the medium must allow the autonomous functioning of the student. For this, it must enable the functioning of knowledge as a student's free production and must allow the student to experiment with the previously acquired knowledge, mobilizing it as operational knowledge in the situation.[11]

2.3. Variational thinking

Variational thinking can be described roughly as a dynamic way of thinking, which attempts to mentally produce systems that relate their internal form variables in such a way that they vary together in similar patterns to the quantities of the same or different magnitudes in the trimmed threads of reality. [13]

This type of thinking includes the mathematics of change on the one hand and thought processes on the other; it implies the integration of the numerical domains, from the natural to the complex, concepts of variable, function, derivative and integral, as well as their symbolic representations, their properties and the domain of the elementary modeling of the phenomena of change. [4]

The study of the concepts, procedures and methods that involve variation are integrated into different systems of graphic representation, tabular, verbal expressions, diagrams, symbolic expressions, particular and general examples, to allow, through them, the understanding of Mathematical concepts In this way, the situations that depend on the systematic study of the variation become significant, since it is obliged not only to express attitudes of observation and registration, but also to processes of treatment, coordination and conversion.

A characterization of the elements of thought and variational language, which is detailed below [14]:

- Variational Situation (VS): This is the set of problems whose treatments demand the implementation of variational strategies and that require the establishment of points of analysis between different states of change. These types of situations can occur both in a purely mathematical setting, and in a context related to other scientific or everyday fields.
- Variational Arguments (VA): These are arguments that resort to the analysis of change and its quantification, and that are used by people when they make use of "maneuvers, ideas, techniques, or explanations that somehow reflect and express quantitative recognition and qualitative of the change in the system or object being studied"
- Variational Codes (VC): They consist of the oral or written expression of change and variation, and which are articulated to generate the VA. These codes may consist of phrases, drawings, tables or gestures, which account for the variational analysis that is performed.
- Specific Variational Structure (SVS): These are specialized tools, processes and procedures in the mathematical or scientific field [9], that function as a support point to address and explain the study of change and variation in VS.
- Variational Strategy (VSt): It consists of a particular way of reasoning and acting before an VS, and that allows the generation of the VAs that explain the situation. Also, for the use of VSts, the SVS is used to study the variation. These strategies are:
  - Comparison
  - Seriation

  - Prediction
  - Estimate
- Variational Tasks (VT): They consist of activities, actions and executions within an SV, which share similarities in terms of their objectives and the contexts in which they are developed. They are characterized by the use of one or more VSt within the same analysis context, which can be numerical, graphic or analytical, which allows organizing the study of the variation in the VS in more specific actions and objectives within these contexts. Some of the VTs identified are the following:
  - Tabulation as numerical variation (TVN)
  - Analysis of data in numerical tables (ADT)
  - Construction of graphs with the variation as reference point (CGV)
  - Graphical analysis with the variation as a reference point (GAV)

3 Method

The present research is an applied one, the experimental method was used, since the independent variable (didactic sequences) will be manipulated to produce an effect on the dependent variable (variational thinking). The design is quasi-experimental, groups of intact subjects previously established will be used, they will go through a pretest, then the treatment condition (presence or absence) will be administered to each group and finally they will go through the posttest [15].

The population was formed by the 329 entrants to the engineering careers of the 03 universities of the province of Abancay (01 from public management and 02 from private management), where the research was conducted. The sample is non-probabilistic and intentional type, consists of 104 entrants to the Civil Engineering career, arranged in two experimental groups: 40 from the public university and 21 from a private university, enrolled in the Mathematics I course. control was made up of 43 students from the other private university, enrolled in the Basic Mathematics course.

The Variational Thought variable was measured with the Variational Thought Questionnaire, which consists of 05 items and was designed taking into account the different representation records of the mathematical function object (Algebraic, Graphical, Tabular, Verbal. In each item, they have been raised a series of activities and actions within a problematic situation of a variational nature. Depending on the type of record, various tasks have been proposed such as tabulation with numerical variation, data analysis in numerical tables, construction of graphs based on variation and graphical analysis based on variation. All these tasks require students to construct variational arguments, and consequently, the implementation of variational strategies that will enhance their variational thinking. The development of variational thinking was measured based on the development of Variational strategies: Comparison, Seriación, Prediction and Estimation. This instrument went through the validity of content, by experts in the field, giving a final opinion of applicable; Likewise, the reliability yielded a Cronchach Alpha of 0.702, considered for performance tests as acceptable.

4 Results

From the students' responses to the questionnaire, the direct scores for each component (Comparison, Seriation, Prediction and Estimation) of the dependent variable Variational Thinking were calculated. The application of the Shapiro Wilks normality test showed that the distributions of the scores in all the components of the study variable are normal, therefore, the Student T-Test was used for hypothesis testing.

Table 1. Analysis in the components of Variational Thinking between the experimental groups and the control group in the Pre-test

Variable	Grupo	n	M	SD	t	p
Comparison	Experimental 1	40	3.3	1.1	-0.43	0.58
	Experimental 2	21	3.2	1.2	-0.46	0.61
	Control	43	3.3	0.9	-0.45	0.59
Seriation	Experimental 1	40	2.7	1.2	-2.21	0.32
	Experimental 2	21	2.5	1.3	-2.51	0.28
	Control	43	2.6	1.1	-1.98	0.29
Prediction	Experimental 1	40	2.6	1.3	1.93	0.38
	Experimental 2	21	2.3	1.2	1.78	0.35
	Control	43	2.5	1.2	1.83	0.37
Estimate	Experimental 1	40	1.9	1.2	-0.55	0.40
	Experimental 2	21	1.7	1.2	-0.59	0.41
	Control	43	1.8	1.1	-0.51	0.38

Note. n: number. M: Mean. SD: Standard Deviation. t: Student T for independent samples

In the Pre-test, no significant differences were found between the average scores obtained by experimental group 1 (n = 40), experimental group 2 (n = 21) and the control group (n = 43) in the Thought components Variational Student T values for independent samples calculated for the components of Variational Thinking between the experimental groups (Comparison t = -0.43, Seriation t = -2.21, Prediction t = 1.93, Estimate t = -0.55), between experimental group 1 and the control group (Comparison t = -0.46, Seriation t = -2.51, Prediction t = 1.78, Estimate t = -0.59), between experimental group 2 and the control group (Comparison t = -0.45, Seriation t = - 1.98, Prediction t = 1.83, Estimate t = -0.51) were not significant ($p> 0.05$), therefore it is established that there are no statistically significant differences working with a confidence level of 0.05, between the means (M) of the experimental and control group. The results obtained in the Pre-test guarantee the homogeneity of the scores in the study variables between the groups and this basic condition allows us to assume that subsequent analyzes will be based on the fact that the different experimental conditions start from an equivalent score in the Components of Emotional Variational Thinking [16]. Thus, the changes that are generated later can be attributed to the manipulation exerted on the independent variable (Didactic Situation) and not to a different starting situation, in both groups, in relation to their variational thinking.

Tabla 2. Analysis in the components of Variational Thinking between the experimental groups and the control group in the Post-test

Variable	Grupo	n	M	SD	t	p
Comparison	Experimental 1	40	4.2	0.9		
	Experimental 2	21	3.9	1.1	2.31	0.02*
	Control	43	3.3	0.9	2.33	0.03*
Seriation	Experimental 1	40	3.8	0.8		
	Experimental 2	21	3.5	0.9	2.28	0.01*
	Control	43	2.8	1.0	2.28	0.01*
Prediction	Experimental 1	40	3.5	1.0		
	Experimental 2	21	3.2	1.1	2.31	0.02*
	Control	43	2.6	1.0	2.37	0.04*
Estimate	Experimental 1	40	2.5	0.9		
	Experimental 2	21	2.2	1.0	2.28	0.01*
	Control	43	1.9	0.9	2.28	0.01*

Note. n: number. M: mean. SD: Standard Deviation. t: Student T for independent samples
* $p < 0.05$

The Post-test analysis in the experimental groups reflects the changes that have been generated by the application of the independent variable (Didactic Sequence). Student T values for independent samples calculated for the components of Variational Thinking between experimental group 1 and the control group (Comparison t = 2.31, Seriation t = 2.28, Prediction t = 2.31, Estimate t = 2.28), between the group Experimental 2 and the control group (Comparison t = 2.33, Seriation t = 2.28, Prediction t = 2.37, Estimate t = 2.28) were significant ($p <0.05$), therefore it is established that there are statistically significant differences working with a level of 0.05 confidence, between the means (M) of the experimental groups and the control group

Table 3. Analysis in the components of Variational Thinking between the experimental groups and the control group in the Pre-Test and Post-test

			Pre-test	Post-Test	Diferencias	
Variable	Grupo	n	M	M	t	p
Comparasion	Experimental 1	40	3.3	4.2	2.28	0.03*
	Experimental 2	21	3.2	3.9	2.29	0.04*
	Control	43	3.3	3.3	-0.33	0.21
Seriation	Experimental 1	40	2.7	3.8	2.30	0.04*
	Experimental 2	21	2.5	3.5	2.30	0.04*
	Control	43	2.6	2.8	-0.25	0.11
Prediction	Experimental 1	40	2.6	3.5	2.28	0.03*
	Experimental 2	21	2.3	3.2	2.26	0.02*
	Control	43	2.5	2.6	-0.27	0.18
Estimate	Experimental 1	40	1.9	2.5	2.26	0.02*
	Experimental 2	21	1.7	2.2	2.26	0.02*
	Control	43	1.8	1.9	-0.33	0.21

Note. n: number. M: Mean. SD: Standard Deviation. t: Student T for paried samples
* $p < 0.05$

When comparing the differences between the means of the groups in the Comparison component, statistically significant differences were found in experimental group 1 ($p = 0.03$) and in experimental group 2 ($p = 0.04$), which reflects improvement as a result of Didactic Sequence application. Regarding the Seriation component, statistically significant differences were also found in experimental group 1 ($p = 0.04$) and in experimental group 2 ($p = 0.04$), with an increase in this aspect. Similarly, in the Prediction component significant differences are observed in both experimental groups ($p <0.05$), finding an increase. In the case of the Estimation component, statistically significant differences are found in both experimental groups ($p <0.05$), finding improvement. Regarding the control group, no significant differences were found in any of the components of the Variational Thought, which reflects the usefulness of the Didactic Sequence for the development of the components of the Variational Thought, and consequently in the improvement of the performance of the math students

5 Conclusions

The activities designed in the didactic sequence allowed the students to improve their skills in handling the variational strategies of comparison, seriation, prediction and estimation, and consequently their academic performance. The management of the different representations allowed to favor the development of cognitive processes such as visualization through which they managed to compare states, establish relationships between these states and deduce new states through the analysis of various variation scenarios [17].

The proposed activities allowed the students to better manage the ideas of growth and decrease of the variables, through the establishment of differences between states and the identification of stationary or extreme values. Comparison and prediction strategies were favored when they worked in numerical and graphic register, finding greater difficulties in the algebraic register [18].

The implementation of the didactic situation allowed the students to improve the management of their strategies and variational arguments in the different representation records proposed for the functions, however the use of arguments of variational type was limited by speech under which they have faced to this concept previously [19].

The implementation of the didactic sequence allowed students to manage functions as models of change situations, as well as their analysis from the perspective of variation through the management of strategies and variational arguments that enhance the development of variational thinking [20].

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