Relationship Invariants Based Sojourn Time Approximation for the Fork-Join Queueing System

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Abstract

The approximation method of task sojourn time in the fork-join queuing system based on relationship invariants is proposed. The idea is in application of intuitive proportion between the certain queuing systems time characteristics. It is shown that, compared with the known methods, the proposed approximation is more accurate with server utilization higher then 0.7.

1 Introduction

To evaluate the efficiency of query processing in systems using distributed and parallel computing technologies, queuing systems like Split-Merge and Fork-Join are used.

The general idea of the Split-Merge and Fork-Join systems functioning is as follows: tasks received in the system are "split" into n sub-tasks, each of which is sent to the channel with numbers $1, 2, ..., n$, respectively. Processed subtasks fall into the synchronization buffer, where they wait for the servicing completion of their related subtasks. At the end servicing of the last related subtask, synchronization occurs, i.e. aggregation, after which task leaves the system. It is regarded that synchronization occurs instantly.

The difference between the Split-Merge and Fork-Join systems is shown in Figures 1 and 2 for the example of 3 service channels. In the case of Fork-Join, the freed channel may be occupied by the subtask of the next task (Figure 1). When organizing the Split-Merge service (Figure 2), a block occurs at the time of task entering, and the freed channels are idle, waiting for the last of the subtasks of the current task to be serviced.

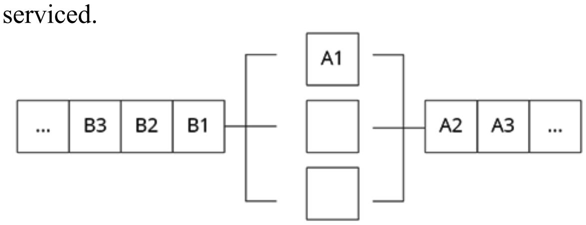

Figure 1: Task service chart in Fork-Join queueing system

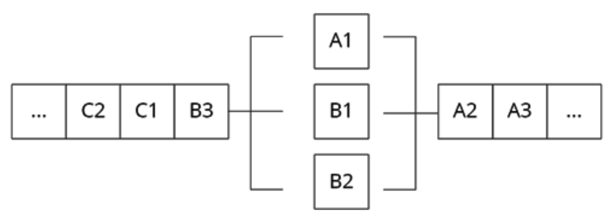

Figure 2: Task service chart in Split-Merge queueing system

A rather large number of works has been devoted to the Fork-Join and Split-Merge processes [Alom2014, Bacc1985, Bacc1989, Fior2015, Flat1979, Harr2003, Khab2019, Nels1988, Olv2014, Ryzh1980, Ryzh2015, Qui2015, Var2002, Varma1994, Wright1992]. For the Split-Merge system in [Harr2003], an exact solution was obtained to determine the maximum service time of independent channels with exponential service time and various intensities, as well as approximations for the case of a general distribution. In [Fior2015], the mentioned distribution was obtained for homogeneous and heterogeneous servers, and its representation in matrix-exponential form made it possible to find both the first and the moments of higher orders. It should be noted that this method is characterized by high

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computational complexity, which is a consequence of the laborious operations of inversion and the Kronecker product of matrices that are included in it. The use of Kronecker algebra is associated with a significant additional memory consumption, as well as many redundant operations with zero operands. In [Ryzh2019], an exact solution was found for an arbitrary distribution of services based on Chebyshev-Laguerre numerical integration. The solution has a relatively low complexity and high accuracy.

For Fork-Join systems, the exact expression for the average sojourn time with an arbitrary service distribution was obtained only for a system with two channels [Bacc1985, Fior2015]. For the case $n > 2$ and exponential service, using various methods, approximations of the average sojourn time were obtained in [Nels1988, Var2002, Varma1994]. We give below a brief summary of the results.

In [Flat1979], an exact formula is given for the average sojourn time for $n = 2$, and the exponential distribution of the service time

$$v_1 = \left(H_2 - \frac{\rho}{8} \right) \cdot v_1^{(M)}, \tag{1}$$

where $H_n = \sum_{i=1}^{n} 1/i$ for $n = 2$, $H_2 = 1.5$,

$\rho = \lambda / \mu$ – system utilization,

$v_1^{(M)} = 1/(\mu - \lambda)$ – average sojourn time in $M/M/1$ queueing system.

In [Nels1988], Nelson and Tantawi proposed an approximation for the average sojourn time in a Fork-Join queueing system with n service channels

$$v_1 \approx \left[\frac{H_n}{H_2} + \frac{4}{11}\left(1 - \frac{H_n}{H_2}\right)\rho \right] \frac{12-\rho}{8} \frac{1}{\mu - \lambda}, n \ge 2. \tag{2}$$

In [Var2002], Varki and Merchant proposed a formula

$$v_1 \approx \frac{1}{\mu}\left[H_n + \frac{\rho}{2(1-\rho)}\left(\sum_{i=1}^{n} \frac{1}{i-\rho} + (1-2\rho)\sum_{i=1}^{n} \frac{1}{i(i-\rho)} \right)\right], n \ge 2. \tag{3}$$

In [Varma1994], Varma proposed a method for approximating the sojourn time based on a combination of high and low system utilization interpolation methods:

$$v_1 \approx \left[H_n + (V_n - H_n)\rho \right] \frac{1}{\mu - \lambda}, \tag{4}$$

where $V_n = \sum_{i=1}^{n} \binom{n}{i} (-1)^{i-1} \sum_{m=1}^{i} \binom{i}{m} \frac{(m-1)!}{i^{m+1}}$.

In this paper, we propose a method for finding the average sojourn time for a Fork-Join queueing system with an arbitrary service distribution based on the approximation of relationship invariants.

2 Method idea

In [Ryzh2015], to find the time characteristics of multi-channel queueing system with priorities, a method was used based on the invariants idea of the relationship between the desired characteristics:

$$\overrightarrow{M_k} / \overrightarrow{G_k} / n \approx \overrightarrow{M_k} / \overrightarrow{G_k} / 1 \cdot \frac{M/G/n}{M/G/1}. \tag{5}$$

All designations are given in Kendall notation. Calculation methods for the systems indicated on the right are considered known. In particular, iterative methods of Takahashi-Takami or matrix-geometric progression are used to calculate systems [Ryzh1980, Taka1976]. For the non-priority systems $M/G/1$ and $M/G/n$ with a non-uniform task flow, the total intensity of the task flow and the weighted average moments of the servicing distribution were used.

As an approximation of the service time, it is proposed to use a second-order hyperexponential distribution H_2 applicable for values of the coefficient of variation, both smaller and larger than 1.0. For example, when replacing the gamma distribution with the parameter $1 < \alpha < 2$ by H_2-approximation, one of the “probabilities” will be negative, and the other will exceed 1.0. As computational experiments show [Ryzh2015], these paradoxical intermediate results do not interfere with the successful calculation of queueing system using known numerical methods [Taka1976].

Here we present the scheme proposed in [Ryzh2015] for the implementation of conversion invariants for an *n*-channel priority system:

a) By multiplying the channel rate by *n*, we obtain the equivalent single-channel system rate.

b) Calculate the waiting time distribution moments $\{w_{i,j}\}$ for all types of requests *i* and the order of the moments *j* = 1, 3.

c) For the same initial data on the weighted average service times and the total intensity of the incoming flow Λ as applied to the $M/H_2/1$ system, calculate the number of requests stationary distribution $\{p_i\}$ and the average requests number in the queue:

$$Q(1) = \sum_{i=1}^{\infty} (i-1) p_i .$$

d) For the $M/H_2/n$ system, it is similar to carry out the calculation of state probabilities at initial service intensities and obtain the average queue length:

$$Q(n) = \sum_{i=n+1}^{\infty} (i-n) p_i .$$

e) Recalculate the waiting time distribution moments in the system with priorities for all *i* and *j* for the multichannel case:

$$W_{i,j} = w_{i,j} \cdot Q(n)/Q(1) .$$

f) Using them, according to relation (5), obtain the sojourn time distribution moments of each task type in a multichannel system.

We propose a similar approach for finding the average sojourn time in the Fork-Join queueing system with the number of channels. We rewrite the proportion (5) in the following form

$$FJ_n \approx FJ_2 \cdot \frac{M/G/n}{M/G/2} . \qquad (6)$$

Here FJ_n and FJ_2 – Fork-Join queuing systems with *n* and 2 channels, respectively. To find the average sojourn time in FJ_2, we use the formula (1). The calculation of the remaining queueing systems from the right-hand side of (6) will be carried out by known methods [Ryzh1980, Taka1976]. To ensure the equality of system utilization values, when calculating $M/G/n$ queuing systems, the initial moments of service time for were multiplied by $n/2$.

3 Calculation results

The following are the results of calculations of the average sojourn time in the queueing system (Figures 3-8) in comparison with the methods proposed in [Nels1988,Var2002,Varma1994] (formulas (2), (3), (4)) and simulation data. The comparison was carried out depending on the system utilization and the number of service channels at three values of the service time coefficient of variation {0.5, 1.0, 1.5}.

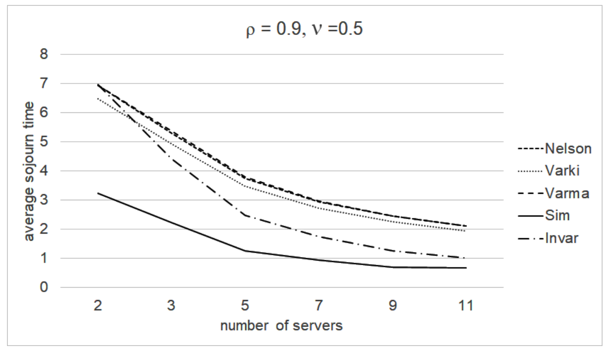

Figure 3: Average sojourn time depending on the number of service channels, υ = 0.5, ρ=0.9

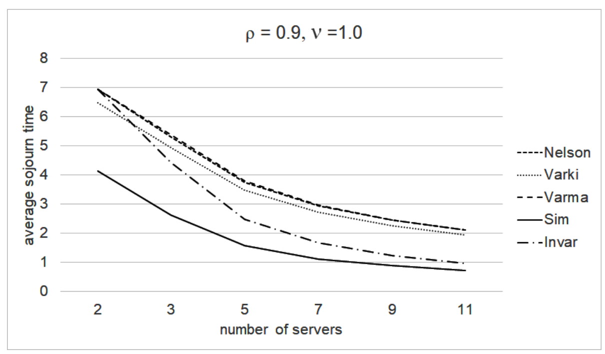

Figure 4: Average sojourn time depending on the number of service channels, υ = 1.0, ρ=0.9

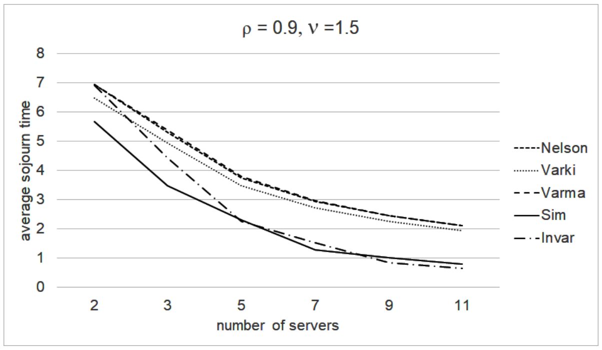

Figure 5: Average sojourn time depending on the number of service channels, $\upsilon = 1.5$, $\rho=0.9$

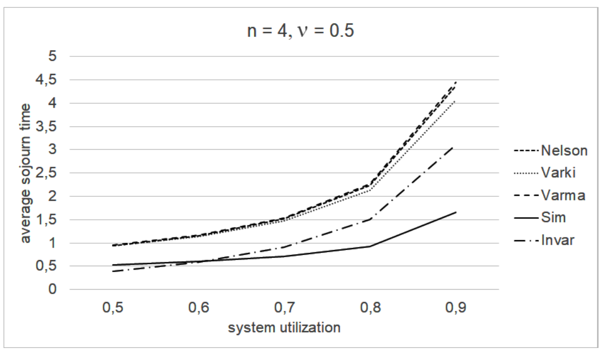

Figure 6: Average sojourn time depending on the system utilization, $\upsilon = 0.5$, n=4

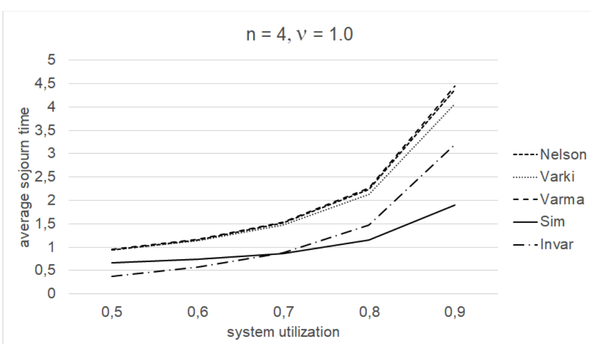

Figure 7: Average sojourn time depending on the system utilization, $\upsilon = 1.0$, n=4

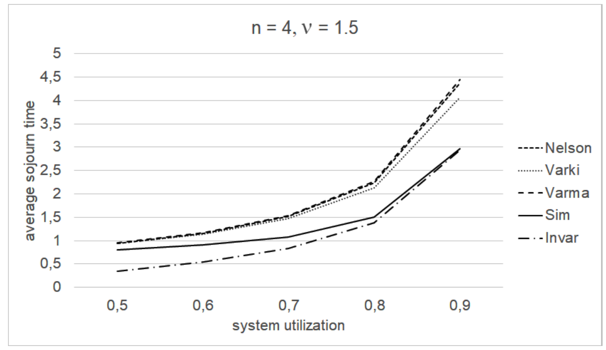

Figure 8: Average sojourn time depending on the system utilization, $\upsilon = 1.5$, n=4

As can be seen from the figures, the accuracy of approximation by the method of invariants of the ratio is higher than calculated by the formulas (2-4). Moreover, at low ρ values, the method of invariant relations gives underestimated estimates of the average sojourn time, especially when increasing the coefficient of variation of the service time. Since the proposed method (as well as other approximation methods compared here) allows us to estimate only the average sojourn time of task in the system, if higher moments are necessary, the Split-Merge calculation method mentioned above should be used, since the estimated values of the sojourn time estimates in this case are the upper bound stay in Fork-Join queuing system.

4 Summary

The relation invariants method showed well average sojourn time approximation accuracy for Fork-Join queueing system. Calculation results and simulation data comparison showed that the proposed approximation accuracy increases as system utilization increasing. At high the service time coefficient of variation values, the method gives underestimated sojourn time of the task in the system. If it is necessary to find the upper bounds of sojourn time higher moments, it is recommended to use the method proposed in [Khab2019,Ryzh2019]

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References

[Alom2014] Alomari F., Menasce D.A. Efficient Response Time Approximation for Multiclass Fork and Join Queues in Open and Close Queueing Networks. IEEE Transaction on Parallel and Distributed Systems, vol. 25. pp.1437-1446, 2014.

[Bacc1989] Baccelli F., Makowski A.M., Shwartz A. The Fork-Join Queue and Related Systems with Synchronization Constraints: Stochastic Ordering and Computable Bounds. Advances in Applied Probability, vol. 21. No. 3. Pp. 629-660, 1989.

[Bacc1985] Baccelli F. Two parallel queues created by arrivals with two demands. The M/G/2 symmetrical case. Technical report INRIA-Rocquencourt, no. 426, 1985.

[Fior2015] Fiorini P., Lipsky L. Exact Analysis of Some Split-Merge Queues. Performance Evaluation Review, vol. 43. No. 2. pp. 51-53, 2015.

[Flat1979] Flatto, L., Hahn S. Two parallel queues created by arrivals with two demands. SIAM Journal on Applied Mathematics, vol. 44. Pp.1041-1053, 1979.

[Harr2003] Harrison P.G., Zertal S. Queueing models with maxima of service times. Computer Performance Evaluation. Modelling Techiniques and Tools, pp. 152-168, 2003.

[Khab2019] Khabarov R.S., Lokhvitckii V.A. Model' otsenivaniya operativnosti mnogopotochnoi obrabotki zadach v raspredelennoi vychislitel'noi srede s uchetom protsessov Split-Join. Vestnik Rossiyskogo novogo universiteta. Seriya ”Slozhnye sistemy: modeli, analiz i upravlenie” [Efficiency evaluating model of tasks multy-threading processing in a distributed computing environment with Split-Join processes]. Vestnik of Russian New University. Series ”Complex systems: models, analisys, management”, no. 1. Pp. 26-34, 2019 (In Russ.).

[Nels1988] Nelson, R., Tantawi A.N. Approximate analysis of fork/join synchronization in parallel queues. IEEE Transactions on Computers, vol. 37. Pp. 739-743, 1988.

[Olv2014] Olvera-Cravioto M., Ruiz-Lacedelli O. Parallel queues with synchronization. In arXiv preprint https://arXiv:1501.00186, 2014.

[Ryzh1980] Ryzhikov Y.I., Khomonenko A.D. Iterativnyj metod rascheta mnogokanalnyh system s proizvolnym raspredeleniem vremeni obsluzhivaniya [An iterative method for calculating multichannel systems with an arbitrary distribution of service time]. Problemy upravleniya i teoriya informacii [Management problems and information theory], no. 3, Pp. 32-38, 1980 (In Russ.).

[Ryzh2015] Ryzhikov Y.I., Khomonenko A.D. Raschet mnogokanal'nyh sistem obsluzhivaniya s absolyutnym i otnositel'nym prioritetami na osnove invariantov otnosheniya [Calculation of multi-channel queueing systems with preemptive and non-preemptive priorities based on relationship invariants]. Intellektual'nie technologii na transporte [Intellectual Technologies on Transport], no. 3, Pp. 11-17, 2015 (In Russ.).

[Ryzh2019] Ryzhikov Y.I., Lokhvitckii V.A., Khabarov R.S. Metod rascheta dlitel'nosti obrabotki zadach v sisteme massovogo obsluzhivaniya s uchetom processov Split-Join [Method for calculating the duration of processing tasks in a queuing system taking into account Split-Join processes]. Izvestiya vysshikh uchebnykh zavedeniy. Priborostroenie [Journal of Instrument Engineering], vol. 62. No. 5. Pp. 419-423, 2019 (In Russ.).

[Taka1976] Takahashi Y., Takami Y. A numerical method for the steady-state probabilities of a GI/G/c queuing system in a general class. J. of the Operat. res. soc. of Japan, vol. 19(2). Pp. 147-157, 1976.

Qui2015] Qiu Z., Perez J.G., Harrison P.G. Beyond the mean in fork-join queues: Efficient approximation for response-time tails. Performance evaluations, vol. 91. pp. 99-106, 2015.

[Var2002] Varki E., Merchant A., Chen H. The M/M/1 Fork-Join Queue with Variable Subtasks. http://www.cs.unh.edu/varki/publication/2000-nov-open.pdf, Accessed 19 Nov 2019.

[Varma1994] Varma S., Makowski A.M. Interpolation Approximations for Symmetric Fork-Join Queues. Performance evaluations, vol. 20. Pp. 245-265, 1994.

[Wright1992] Wright P.E. Two parallel processors with coupled inputs. Advances in Applied Probability, vol. 24. Pp. 986-1007, 1992.