

The Algorithm of Selecting Candidates for IT Projects Based on the Simplex Method

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Abstract. In the most cases, solution of linear optimization problems is searched for by the simplex method. However, this classic algorithm of solving linear optimization problems may create additional iterations in the procedure of immediate calculation. If we break the standard simplex method algorithm in some of its components, we can accelerate the simplex calculation convergence – reduce the number of simplex tables. For acceleration of the simplex method convergence, it is proposed to deviate from the canonical algorithm. It is required to choose not the neighbor apex as the next problem plan, but the verified apex selected according to evaluation of the biggest and the smallest target function values. The application aspect of the approach proposed is in usage of the obtained research result for providing the possibility to simplify the numeric algorithm based on reducing the number of iterations. This creates conditions for further development and improvement of similar approaches in linear optimization problems. Solution of the model example, that was found by following the classic algorithm and by breaking it, confirms the hypothesis put forward.

Keywords: Linear Optimization, Polyhedron, Target Function, Simplex Method, Basis Vectors, Primary Plan, Reference Plan, Polyhedron Apex

1 Introduction

The intensive development of IT technologies is generated by the high-qualification staff potential available in Ukraine. According to the data of IT Ukraine Association,

the IT industry in Ukraine shows an annual growth of 20%. By results of 2018, the IT industry takes the second place with the volume of services exported. The significance of IT services within the structure of export is growing as well. IT companies strengthen their positions owing to simultaneous implementation of a significant number of projects (up to 300) on the order from customers leading in various branches: automotive, healthcare, TV communication and finance. Employees who realize the projects are the main value for IT companies. Formation of the project team is one of the first-priority objectives in the modern project management. It is the smooth teamwork that represents an important factor of successful project implementation. At the same time, the team formation process is one of the most complicated aspects of project management. A project team is mostly created just for the project implementation period and may consist of specialists in different professions. The team formation procedure is rather difficult and requires using innovative methods with account taken of the fact that the created team has to work like a well-adjusted system[1, 3-7].

2 Research Paper Study and Problem Statement

In multiple cases, mathematic models of active systems management are interpreted in the form of linear optimization problems [2,8,11,14]. Solution of linear optimization problems is based on algorithm of the classic or a common simplex method. It consists in intellectual iteration over polyhedron apexes Ω_I (allowable area of optimization problem). The plan or an apex of polyhedron Ω_I is specified by a system n of basis vectors $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$. The number of possible apexes of polyhedron equals to the number of combinations C_n^m (n – problem measurability, and $m = rang(\Omega_I)$). Real linear optimization problems that interpret models of candidates selection are characterized by big values of m . In view of this, we have to develop an algorithm ensuring ordered iteration over angular points of the polyhedron. Such a method was developed [1, 2] and is called simplex method. It allows obtaining the optimum optimization problem solution from the known primary reference plan X_0 , within a finite number of steps. Each iteration step of a simplex method corresponds to competences of the new candidate that improves the target function value. The algorithmic process continues until finding the optimum value of target function or the absence of optimization problem solution.

The number of simplex method iterations is determined by the primary reference plan X_0 and the number of angular points Ω_I . As there are several “ways” of transition from X_0 to the optimum X_{opt} , we encounter a problem of finding the shortest (in terms of the number of apexes) “way” of iteration. Now there are not any publications with such assessments and their correlation to the classic simplex method algorithm.

3 The Objective and the Tasks of Research

The research objective provides for development of the algorithm for selecting candidates to an IT project implementation team with use of the classic simplex method for reduction of the number of iterations. For achievement of the objective stated, the following tasks were specified:

- Develop the algorithm for selection of IT project implementation personnel with use of simplex method;
- Provide model example calculation confirming reduction of the number of iterations as compared with the classic calculation [9,10,12].

4 General Statement of a Linear Optimization Problem. Reference Plan Drafting by Following the Classic Algorithm and by Breaking it

Project activities in the IT sector require formation of an implementing team. The team is a small group (from 3 to 12 persons) having a brightly expressed target orientation and intensive interaction between the team members while fulfilling a joint task. Efficient and fruitful activities of the team in general depend on the competences of each of the team members.

For determining a performer of certain processes, we need to carry out the analysis of (and actually to iterate over) the competences of each of the candidates. The synergic effect of a joint teamwork is qualitatively higher than the effect of single persons' activities, i.e. a joint work of specialists can in total give you much more than the results of their individual work.

For large- and medium-scale projects, teams may count tens, hundreds and thousands of participants responsible for particular activities.

The team is the main element of project structure as this is the team who ensures implementation of the project idea. The team leader knows the abilities and skills of the members and uses them for work on the project in accordance with the need. For assurance of efficient high-synergy work of the team, it is necessary first to plan its composition to determine the desired professional characteristics of its members. Most frequently, project managers fail to do it intentionally or replenish the team, as new tasks appear that cannot be solved by efforts of its existing members. In some cases, the project manager composes the team but does not deem it necessary to introduce its members to each other; as a result, the complete composition of the team is only known to the project manager. Such a behaviour shows that there is at least a failure to understand the significance of joint efforts for achievement of the maximum synergy. The main integrating factor of team creation and team activities is the strategic objective of the project implementation [15-19]. According to this objective, the project manager defines the required number of specialists – team members, their qualification, carries out selection and hiring of employees.

The classic method of team members selection was based on involving professional experts. Advisors were involved for selection of candidates by means of interview. Later, this function was fulfilled through creation of a standard of competences.

For improving the efficiency of this process, it was proposed to use an algorithm based on usage of a simplex method.

Without loss of generality, we may assume to have a standard form of a linear optimization problem record

$$W_I = \sum_{j=1}^n c_j x_j \rightarrow \max,$$

$$\Omega_I : \sum_{j=1}^n a_{ij} x_j \leq b_i, \quad i=1, \dots, m,$$

$$x_j \geq 0, \quad j=1, \dots, n,$$

where $b_i \geq 0, \quad i=1, 2, \dots, m$.

Adding a balance nonnegative variable to each inequality $x_i \geq 0, \quad i=n+1, n+2, \dots, n+m$ and recording the problem in a vector form allow obtaining the canonical recording form of an optimization problem

$$W_I = (\mathbf{c}, \mathbf{x}) \rightarrow \max,$$

$$\Omega_I : (\mathbf{a}_j, \mathbf{x}) = \mathbf{b},$$

$$\mathbf{x} \geq 0,$$

or in expanded form:

$$x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + \dots + x_m \mathbf{a}_m + x_{m+1} \mathbf{a}_{m+1} + \dots + x_n \mathbf{a}_n = \mathbf{b},$$

where

$$\mathbf{x} = [x_1, x_2, \dots, x_n]^T, \quad X \in \mathbf{R}^n, \quad \mathbf{c} = [c_1, c_2, \dots, c_n],$$

$$\mathbf{a}_1 = [a_{11}, a_{21}, \dots, a_{m1}]^T, \mathbf{a}_2 = [a_{12}, a_{22}, \dots, a_{m2}]^T, \dots, \mathbf{a}_n = [a_{1n}, a_{2n}, \dots, a_{mn}]^T,$$

$$\mathbf{a}_{n+1} = \mathbf{e}_{n+1} = [1, 0, \dots, 0]^T, \mathbf{a}_{n+2} = \mathbf{e}_{n+2} = [0, 1, \dots, 0]^T, \dots, \mathbf{a}_{n+m} = \mathbf{e}_{n+m} = [0, 0, \dots, 1]^T,$$

$$\mathbf{b} = [b_1, b_2, \dots, b_m]^T.$$

Vectors $\mathbf{a}_{n+1}, \mathbf{a}_{n+2}, \dots, \mathbf{a}_{n+m}$ are unit vectors. These vectors are linearly independent vectors and constitute the basis. The right sides vector resolution of the optimization problem set of constraints has the following form:

$$\mathbf{b} = b_1 \mathbf{e}_{n+1} + b_2 \mathbf{e}_{n+2} + \dots + b_m \mathbf{e}_{n+m}.$$

As all $b_i \geq 0$, we obtain the allowable primary reference plan \mathbf{X}_0 . The following basis resolution corresponds to the primary plan:

$$\mathbf{X}_0 = b_1 \mathbf{e}_{n+1} + b_2 \mathbf{e}_{n+2} + \dots + b_m \mathbf{e}_{n+m} = [0, 0, \dots, 0, b_1, b_2, \dots, b_m]^T.$$

The main idea of using the simplex method-based algorithm is sequential iteration over the competences of a new candidate to become member of an IT project team. One vector is excluded from and another included to the basis by the Gauss-Jordan

method.[13] Subject to compliance with these criteria, we have to build a chain. The beginning of the chain is located at the starting apex \mathbf{X}_0 of polyhedron Ω_l and corresponds to the first simplex table of calculation. Moving to the next candidate \mathbf{X}_1 by following the classic algorithm corresponds to transition to the neighbor apex. Actually, each table is a numeric description of apexes Ω_l . The process is to be continued till finding the optimum apex \mathbf{X}_{opt} or confirming its absence.

At the arbitrary step of calculation by following the common simplex method algorithm, we have the possibility to move not to the neighbor apex, but to the arbitrary apex located around the optimum apex. Such an apex can be selected based on multiple evaluation methods, e.g. the half-interval method. For this selection, the alternative chain of simplex calculation may have a much smaller number of iterations.

Let us consider a model example of a two-dimensional linear optimization problem solution to confirm this case, first by following the standard procedure and then by breaking the rule of basis vectors combination selection.

Model example

The total competence of candidate W_i consists of several separate competences, with factors determined by the experts

$$W_l = 2x_1 + 3x_2 \rightarrow \max,$$

$$\Omega_l : \begin{cases} 3x_1 - 4x_2 \leq 6, \\ x_1 + 2x_2 \leq 12, \\ -x_1 + x_2 \leq 3, \\ x_2 \leq 4, \\ x_j \geq 0, j = 1, \dots, 4. \end{cases}$$

Basis nonnegative unknowns are to be added to left sides of each inequality x_3, x_4, x_5, x_6 . As a result, we obtain a canonical form of a linear optimization problem:

$$W_l = 2x_1 + 3x_2 \rightarrow \max,$$

$$\Omega_l : \begin{cases} 3x_1 - 4x_2 + x_3 = 6, \\ x_1 + 2x_2 + x_4 = 12, \\ -x_1 + x_2 + x_5 = 3, \\ x_2 + x_6 = 4, \\ x_j \geq 0, j = 1, \dots, 6. \end{cases}$$

We have the primary characteristic of candidate $\mathbf{X}_0 = [0, 0, 6, 12, 3, 4] \in \Omega_l$. Let us draw the reference simplex table (Tab. 1).

Table 1. Simplex table vertex X_0

			a_1	a_2	a_3	a_4	a_5	a_6	$\{b_j/a_{ij}\}$	
<i>Basis</i>	<i>C</i>	<i>B</i>	2	3	0	0	0	0		X_i
a_3	0	6	3	-4	1	0	0	0		X_0
a_4	0	12	1	2	0	1	0	0	6	
a_5	0	3	-1	1	0	0	1	0	3	
a_6	0	4	0	1	0	0	0	1	4	
Δ_j	$W_1(X_0) = 0$		-2	-3	0	0	0	0		

Index row Δ_j has two negative evaluations meaning that plan $\mathbf{X}_0 = [0, 0, 6, 12, 3, 4] \in \Omega_1$ is not optimum and can be improved. The pivot column can be found by the rule of selecting the smallest negative value of evaluations. This is column \mathbf{a}_2 as $\min\{-2, -3\} = -3 \rightarrow \mathbf{a}_2$.

The pivot row is to be set by the rule of selecting the smallest simplex ratio for positive components of the pivot column. We have

$$\left\{ \frac{b_i}{a_{i2}} \mid a_{i2} > 0, i = 4, 5, 6 \right\} = \min\left\{ \frac{12}{2}, \frac{3}{1}, \frac{4}{1} \right\} = 3 \rightarrow \mathbf{a}_5$$

Our solving element is $a_{52} = 1$. For it, we make a Gauss-Jordan transformation and by following the algorithm we select (Tab. 2):

Table 2. Simplex table vertex X_1

			a_1	a_2	a_3	a_4	a_5	a_6	$\{b_j/a_{ij}\}$	
<i>Basis</i>	<i>C</i>	<i>B</i>	2	3	0	0	0	0		X_i
a_3	0	18	-1	0	1	0	4	0		X_1
a_4	0	6	3	0	0	1	-2	0	2	
a_2	3	3	-1	1	0	0	1	0		
a_6	0	1	1	0	0	0	-1	1	1	
Δ_j	$W_1(X_1) = 9$		-5	0	0	0	3	0		

From the second table (Tab. 2):

$$\mathbf{X}_1 = [0, 3, 18, 6, 0, 1], W_1(\mathbf{X}_1) = 9.$$

The index row Δ_j has a negative evaluation. Plan $\mathbf{X}_1 = [0, 3, 18, 6, 0, 1]$ is not optimum and it can be improved. The pivot column is \mathbf{a}_2 , as only this column contains negative evaluation $\Delta_1 = -5$. We select a pivot row from the condition of the smallest simplex ratio for positive components of the pivot column. We have

$$\left\{ \frac{b_i}{a_{i1}} \mid a_{i1} > 0, i = 4, 6 \right\} = \min\left\{ \frac{6}{3}, \frac{1}{1} \right\} = \{2, 1\} = 1 \rightarrow \mathbf{a}_6$$

In the new basis, instead of \mathbf{a}_6 we involve \mathbf{a}_1 . After respective calculation, we have the third simplex table (Tab. 3).

Table 3. Simplex table vertex X_2

			a_1	a_2	a_3	a_4	a_5	a_6	$\{b_j/a_{ij}\}$	
<i>Basis</i>	<i>C</i>	<i>B</i>	2	3	0	0	0	0		X_i
a_3	0	19	0	0	1	0	3	1	19/3	X_2
a_4	0	3	0	0	0	1	1	-3	3	
a_2	3	4	0	1	0	0	0	1		
a_1	2	1	1	0	0	0	-1	1		
Δ_j	$W_1(X_2) = 14$		0	0	0	0	-2	5		

From the third table (Tab. 3):

$$\mathbf{X}_2 = [1, 4, 19, 3, 0, 0], W_1(\mathbf{X}_2) = 14.$$

The index row Δ_j has a negative evaluation. Plan $\mathbf{X}_2 = [1, 4, 19, 3, 0, 0]$ is not optimum and can be improved. The pivot column is \mathbf{a}_5 , as only this column contains negative evaluation $\Delta_5 = -5$. We select the pivot row by the condition of the smallest simplex ratio for positive components of the pivot column. We have

$$\left\{ \frac{b_i}{a_{i5}} \mid a_{i5} > 0, i = 3, 4 \right\} = \min \left\{ \frac{19}{3}, \frac{3}{1} \right\} = 3 \rightarrow \mathbf{a}_4.$$

In the new basis, instead of \mathbf{a}_4 we involve \mathbf{a}_5 . After respective calculation, we have the fourth simplex table (Tab. 4).

Table 4. Simplex table vertex X_3

			a_1	a_2	a_3	a_4	a_5	a_6	$\{b_j/a_{ij}\}$	
<i>Basis</i>	<i>C</i>	<i>B</i>	2	3	0	0	0	0		X_i
a_3	0	10	0	0	1	-3	0	10	1	X_3
a_5	0	3	0	0	0	1	1	-3		
a_2	3	4	0	1	0	0	0	1	4	
a_1	2	4	1	0	0	1	0	-2		
Δ_j	$W_1(X_3) = 20$		0	0	0	2	0	-1		

From the fourth table (Tab. 4):

$$\mathbf{X}_3 = [4, 4, 10, 0, 3, 0], W_1(\mathbf{X}_3) = 20.$$

The index row Δ_j has a negative evaluation. Plan $\mathbf{X}_3 = [4, 4, 10, 0, 3, 0]$ is not optimum and can be improved. Our pivot column will be \mathbf{a}_6 , as only this column contains negative evaluation $\Delta_6 = -1$. We select the pivot row by the condition of the smallest simplex ratio for positive components of the pivot column. We have

$$\left\{ \frac{b_i}{a_{i6}} \mid a_{i6} > 0, i = 2, 3 \right\} = \min \left\{ \frac{4}{1}, \frac{10}{10} \right\} = 1 \rightarrow \mathbf{a}_3.$$

In the new basis, instead of \mathbf{a}_3 , we involve \mathbf{a}_6 . After respective calculation, we have the fifth simplex table (Tab. 5).

Table 5. Simplex table vertex X_{opt}

			a_1	a_2	a_3	a_4	a_5	a_6	$\{b_j/a_{ij}\}$	
<i>Basis</i>	<i>C</i>	<i>B</i>	2	3	0	0	0	0		X_i
a_6	0	1	0	0	1/10	- 3/10	0	1		
a_5	0	6	0	0	3/10	1/10	1	0		
a_2	3	3	0	1	- 1/10	3/10	0	0		X_{opt}
a_1	2	6	1	0	1/5	2/5	0	0		
Δ_j	$W_1(X_{\text{opt}}) = 21$		0	0	1/10	17/10	0	0		

All evaluations are nonnegative $\Delta_j \geq 0$. This means that we have found the optimum solution.

$$\mathbf{X}_{\text{opt}} = [6, 3], W_1(\mathbf{X}_{\text{opt}}) = 21.$$

Therefore, the calculation within the classis simplex calculus contains the following chain of sequential iteration over apexes Ω_j : $\mathbf{X}_0 \rightarrow \mathbf{X}_1 \rightarrow \mathbf{X}_2 \rightarrow \mathbf{X}_3 \rightarrow \mathbf{X}_{\text{opt}}$.

Let us confirm that breaking the canonical simplex method algorithm can essentially reduce the length of the calculation chain – the number of simplex tables. We are not selecting the smallest evaluation like in the common simplex method, but the biggest one. Respective calculation is given in (Tab. 6).

Table 6. Alternative simplex algorithm

Basis	C	B	a_1	a_2	a_3	a_4	a_5	a_6	$\{b_j/a_{ij}\}$	X_i
			2	3	0	0	0	0		
a_3	0	6	3	-4	1	0	0	0		
a_4	0	12	1	2	0	1	0	0	6	
a_5	0	3	-1	1	0	0	1	0	3	X_0
a_6	0	4	0	1	0	0	0	1	4	
Δ_j	$W_1(X_0) = 0$		-2	-3	0	0	0	0		
a_3	0	-30	0	-10	1	-3	0	0		
a_1	2	12	1	2	0	1	0	0		
a_5	0	15	0	3	0	1	1	0		X_4
a_6	0	4	0	1	0	0	0	1		
Δ_j	$W_1(X_4) = 24$		0	1	0	2	0	0		
a_2	3	3	0	1	-1/10	3/10	0	0		
a_1	2	6	1	0	1/5	2/5	0	0		
a_5	0	6	0	0	3/10	1/10	1	0		X_{opt}
a_6	0	1	0	0	1/10	-3/10	0	1		
Δ_j	$W_1(X_{opt}) = 21$		0	0	1/10	17/10	0	0		

The length of calculation chain has been almost twice reduced as $X_0 \rightarrow X_4 \rightarrow X_{opt}$.

The problem considered is two-dimensional. Therefore, we can perform a graphical solution providing us with geometric interpretation of the problem calculation chains.

We set up an equation of limit lines $\omega_1: 3x_1 - 4x_2 = 6$, $\omega_2: x_1 + 2x_2 = 12$, $\omega_3: -x_1 + x_2 = 3$, $\omega_4: x_2 = 4$, $\omega_5: x_1 = 0$, $\omega_6: x_2 = 0$, and set semi-planes determined by respective inequalities of the set of constraints. As a result, we can draw polyhedron Ω_1 (Fig. 1).

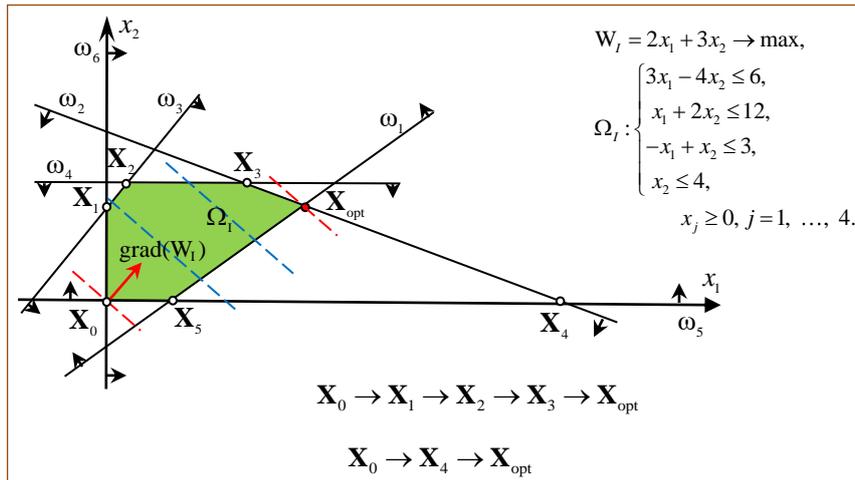


Fig. 1. Interpretation of compliance with the classic simplex method algorithm and of breaking it.

At the coordinate origin point, we draw the gradient vector $\text{grad}(W_1) = [2, 3]$. Perpendicularly to it, we draw the level line. Moving the line in parallel to itself in the gradient direction, we set the maximum point \mathbf{X}_{opt} - the apex of the level lines outreach (Fig. 1). The coordinates of the extreme apex are found as coordinates of the crossing point of respective limit lines:

$$\mathbf{X}_{\text{opt}} : \omega_1 \times \omega_2 \Leftrightarrow \begin{cases} 3x_1 - 4x_2 = 6, \\ x_1 + 2x_2 = 12, \end{cases} \Leftrightarrow \begin{cases} x_1 = 6, \\ x_2 = 3. \end{cases}$$

Therefore, the target function reaches its maximum value at the apex $\mathbf{X}_{\text{opt}} = [6, 3]$ and it is equal to $W_1(\mathbf{X}_{\text{opt}}) = 21$.

The geometric interpretation of the classic simplex calculation consists in the fact that the first simplex table (Tab. 1) corresponds to apex \mathbf{X}_0 . The calculation up to the second table (Tab. 2) corresponds to transition to the neighbor apex \mathbf{X}_1 , in the direction of the biggest target function growth. The third, the fourth and the fifth simplex tables (Tab. 3, Tab. 4, Tab. 5) correspond to transition $\mathbf{X}_1 \rightarrow \mathbf{X}_2 \rightarrow \mathbf{X}_3 \rightarrow \mathbf{X}_{\text{opt}}$ (Fig. 2). Therefore, for solving the problem by following the classic algorithm, we need to set up five simplex tables. For reducing the number of iterations, we break this algorithm and select not the smallest but the biggest negative evaluation $\Delta_1 = -2$ in the initial simplex table. The further calculation is given in Table No. 6. As we can see, the number of simplex tables has been reduced from five to three.

5 Research Results Summary

The example considered shows reduction in the number of numeric calculations of an optimization problem based on the method of breaking the standard algorithm. It visually demonstrates reasonability of using optimization problems for determining variations of deviating from canonical algorithms of linear optimization problems solution. From the practical point of view, the proposed approach allows simplifying the calculation complexity of problems on selecting candidates to an IT project implementation team with account taken of their competences.

Based on comparative solutions of a model problem, it has been proved that the number of iterations can be essentially reduced: the classic calculation has five, and in case of breaking the algorithm there are three iterations only.

The research result obtained allows arriving at the conclusion that in a common case, there is a need to search for reasonability of breaking the standard simplex calculation algorithm.

The application value of the proposed approach consists in using the obtained scientific result for assurance of creating an efficient team for IT projects implementation.

6 Conclusions

It has been determined that using the proposed algorithm in project management is reasonable if applied with breaking the classic method and contributes to acceleration of convergence in the process of obtaining the optimization solution. It has been proved on the example of solving a typical model problem that the proposed approach allows us to essentially reduce the number of iterations. A significant reduction in the computational actions in solving linear optimization problems allows to increase the dimension of the tasks. Such practical expediency stimulates the study of the possibility of constructing more efficient algorithms. The application aspect of the approach proposed is in usage of the obtained research result for providing the possibility to simplify the numeric algorithm based on reducing the number of iterations. This creates conditions for further development and improvement of similar approaches in linear optimization problems.

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