Information in Hierarchical systems

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Abstract. The paper discusses the decentralization of the organizational system control in the presence of external uncertainty factors. The quantity of information that he or she can timely receive is assumed to be limited.

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1 Introduction

Experience shows that, in practice, the control of rather complex organizational systems is based on the hierarchical principle. Hence, it can be concluded that decentralized control is more efficient. However, it is rather difficult to explain the reason for the effectiveness of such decentralization. Such explanation was proposed in the early 70s of the last century by Yu.B. Germeyer and N.N. Moiseev [1]. It consists in the following.

Virtually always, the result of control depends not only on the controls selected by the operating party, but also on external factors that are beyond the decision maker's control. Control will be effective if the decision maker correctly takes into account all information about the external environment. But this is often impossible due to the large quantity of such information. If the decision maker delegates some of his or her decision-making powers to some agents, by joint efforts, they will be able to process large amounts of information in a timely manner, thereby making the control more efficient. However, there is another problem. Having the right to choose controls, agents will have their own interests and choose control pursuing their own goals. This may reduce the overall system control efficiency. However, if the interests of all agents are well coordinated, it is still possible to benefit from the decentralization.

Currently, the ability to handle large amounts of information is rapidly increasing; however, there is no obvious tendency to centralize the control of complex systems. This is probably due to the simultaneous processes of complicating the ties both between the individual elements within a controlled system and the system's ties with the external environment. Therefore, the quantity of information necessary for effective control is also increasing. In addition, we need to somehow separate the essential information from the non-essential one; moreover, the separation method also depends on the quantity of information that can be processed in a timely manner.

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At the verbal level, the above idea set forth by Yu.B. Germeyer and N.N. Moiseev has been discussed rather widely. However, for a long time, it was not possible to build formal mathematical models that allow for describing this effect. There are objective reasons for this.

Obligatory such a model should include:

– the description of several agents (including the operating party);

– the description of each agent's capabilities and goals;

– the existence of external uncertainty;

– the description of each agent's attitude towards uncertainty arising both from insufficient information about the external environment and from unknown choices of partners' controls;

– a sufficiently detailed description of the agents' awareness, allowing, among other things, to estimate the quantity of information processed.

Thus, the desired model shall be very complex.

It is worthwhile to dwell on the last item in this list. The very concept of "information" is very complex and ambiguous. In some models of control decentralization, we can be limited only by the description of the quantity of information; however, it is not easy to do. There are several approaches to determining the quantity of information [2]. This very fact suggests that, in some cases, each of them is not suitable for modeling reality; therefore, the choice of the desired method is a non-trivial task. In addition, each of the existing models of the quantity of information is quite complex; hence, when incorporated it into the general decentralization model, we get a very difficult object to study.

Probably, a model combining game-theoretic constructions with a quantitative description of information exchanges was proposed for the first time in [3]. It uses the approach to determining the quantity of information that A.N. Kolmogorov referred to as the combinatorial one. The same approach is used below.

This paper accepts the following structural assumptions ensuring the presence of a "radial structure" in the control system under consideration:

– it is believed that the set of controls can be presented as a Cartesian product of several simpler sets;

– it is assumed that, in the decentralized version, control is carried out without feedback, i.e. the top-level element has no information about controls chosen by other elements;

– with decentralized control, it is believed that the lower-level elements have complete and, therefore, identical information about external uncertainty factor.

The rejection of any of these assumptions leads to the fact that the lower-level elements can and shall interact with each other. As a result, a number of interesting and important problem statements have been created; but this is a subject for a separate research.

2 Control System

Let us consider the following controlled system model. The decision maker may choose any control w at his or her discretion from the set W. In addition to this choice, the control result is influenced by some uncertainty factor α from the set A, the value of which is behold the control of the operating party (the decision maker). The control efficiency is estimated by value $g(w,\alpha)$ of the function $g: W \times A \to \mathbb{R}$. Let us assume that the goal of control is to maximize this value.

Let us make another assumption reflecting the concept of "technological structuredness" of the controlled system under consideration. Let us assume that the set W can be presented as a Cartesian product $W = U \times V^1 \times V^2 \times ... \times V^n$. Then, every element $w \in W$ can be recorded as $w = (u, v^1, v^2, ..., v^n)$, where $u \in U$, $v^i \in V^i$, $i = 1, 2, ..., n$. We will use this form of recording where it is convenient, without special reservations. In addition, we assume that, on the set A, the probabilistic measure $\wp$ is given, which is known to the operating party.

We make the following standard assumptions. Let us assume that, on the sets $u \in U$, $v^i \in V^i$, $i = 1, 2, ..., n$ and A, topologies are given in which these sets are compact. Let us consider the function g as continuous in the Cartesian product topology $U \times V^1 \times V^2 \times ... \times V^n \times A$. Let us consider the measure $\wp$ as a Borel one.

We assume that the operating party has the opportunity to obtain information about the realized uncertainty factor value, but the quantity of information that such party is able to obtain and timely process is limited. Namely, let us assume that the operating party can use l bits of information, and there are no other restrictions on the use of such information.

The above can be formalized as follows. Let introduce a notation. Hereinafter, $\Phi(X,Y)$ will denote the family of all functions maping the set X into the set Y.

The assumption made means that all information about the uncertainty factor, that is available to the operating party, can be encoded with words $s = (s_1, s_2, ..., s_l)$ from zeros and ones of the l length. Let us denote the set $\{0,1\}^l$ (the Cartesian degree of the set $\{0,1\}$) with the letter S. Since the operating party has no restrictions on access to information about the uncertainty factor, the choice of "encoding method" $P: A \to S$ would be its prerogative. In addition, depending on the information obtained $s \in S$, the operating party has the right to choose any control $w \in W$. That means, in fact, it can choose the function $w_*: S \to W$. If the operating party fixes the encoding method $P \in \Phi(A,S)$ and the control choice rule $w_* \in \Phi(S,W)$, and the value of the indefinite factor $\alpha \in A$ is realized, the operating party will receive the message $P(\alpha)$, choose the control $w_*\left(P(\alpha)\right)$, and its gain will be $g\left(w_*\left(P(\alpha)\right), \alpha\right)$.

We will consider two control schemes of the described system. In the first of them, the control w is selected centrally. In the second one, the decision maker delegates the right to choose controls v^i to n agents: the agent number i gets the right to choose the control $v^i \in V^i$ $(i = 1, 2, ..., n)$. The operating party (the Center) reserves the choice of control $u \in U$. We believe that, at the time of decision making, agents know exactly the realized value of the uncertainty factor α.

The i agent's right to influence the situation inevitably entails the appearance of his or her own goals. The process of forming such goals is complex and insufficiently studied. In this model, such goals are considered as exogenous. We assume that the purpose of the agent i is described by the desire to maximize the value of the function $h^i\left(u, v^i, \alpha\right)$. It is significant that this function depends on his or her own control, the Center's control, and the uncertainty factor; but it does not depend on choices of other agents. We will assume that the functions h^i are known to the Center. The functions h^i are considered to be continuous.

Let us consider two systems of models differing from each other by the attitude of the operating party towards uncertainty.

3 Interval Uncertainty

First, let us consider the case where the operating party is careful in relation to uncertainty.

In the case of centralized control, its maximum guaranteed result will be

$$R_0\left(l\right) = \sup_{(w_*, P) \in \Phi(S,W) \times \Phi(A,S)} \inf_{\alpha \in A} g(w_*(P(\alpha)), \alpha) .$$

For comparison, in the absence of restrictions on the quantity of information obtained, the maximum guaranteed result is

$$R_0\left(\infty\right) = \sup_{(w_\#, P) \in \Phi(A,W)} \inf_{\alpha \in A} g(w_\#(\alpha), \alpha).$$

Theorem 1. The following equalities are true:

$$R_0\left(l\right) = \max_{(w_0, w_1 ..., w_{m-1}) \in W^m} \min_{\alpha \in A} \max_{s = 0,1,...,m-1} g(w_s, \alpha),$$

$$R_0\left(\infty\right) = \min_{\alpha \in A} \max_{w \in W} g(w, \alpha).$$

In the case of decentralized control, under the assumptions made, the Center can assume that agent i will choose its control from the set

$$BR^i\left(u_*, P, \alpha\right) = \left\{v^i \in V^i : h^i\left(u_*\left(P\left(\alpha\right)\right), v^i, \alpha\right) = \max_{v^i \in V^i} h^i\left(u_*\left(P\left(\alpha\right)\right), v^i, \alpha\right)\right\}$$

therefore, the maximum guaranteed result of the Center will be

$$R_1(l) = \sup_{(u_*, P) \in \Phi(S,U) \times \Phi(A,S)} \min_{\alpha \in A} \min_{v^1 \in BR^1(u_*, P, \alpha)} \dots \min_{v^n \in BR^n(u_*, P, \alpha)} g(u_*\left(P\left(\alpha\right)\right), v^1, ..., v^n, \alpha).$$

In the model without restrictions on the quantity of information obtained, similar result is equal to

$$R_1(\infty) = \sup_{u_\# \in \Phi(A,U)} \min_{\alpha \in A} \min_{v^1 \in BR^1(u_\#,\alpha)} \ldots \min_{v^n \in BR^n(u_\#,\alpha)} g(u_\#(\alpha), v^1, \ldots, v^n, \alpha),$$

where

$$BR^i(u_\#, \alpha) = \left\{ v^i \in V^i : h^i\left(u_\#(\alpha), v^i, \alpha\right) = \max_{v^i \in V^i} h^i\left(u_\#(\alpha), v^i, \alpha\right) \right\}.$$

Theorem 2. We have the following equalities:

$$R_1 = \sup_{(u_0,u_1,\ldots,u_{m-1}) \in U^m} \min_{\alpha \in A} \max_{s=0,1,\ldots,m-1} \min_{v^1 \in E^1(u_s,\alpha)} \min_{v^2 \in E^2(u_s,\alpha)} \ldots \min_{v^n \in E^n(u_s,\alpha)} g(u_s, v^1, v^2, \ldots, v^n, \alpha),$$

$$R_1(\infty) = \min_{\alpha \in A} \max_{u \in U} \min_{v^1 \in E^1(u,\alpha)} \min_{v^2 \in E^2(u,\alpha)} \ldots \min_{v^n \in E^n(u,\alpha)} g(u, v^1, v^2, \ldots, v^n),$$

where

$$E^i(u_s, \alpha) = \left\{ v^i \in V^i : h^i(u_s, v^i, \alpha) = \max_{\omega^i \in V^i} h^i(u_s, \omega^i, \alpha) \right\}.$$

For comparison of the two control methods, let us note that the values $R_0(l)$ and $R_1(l)$ do not decrease with increasing l. Of course, for any l, the following inequalities are true: $R_0(l) \le R_0(\infty)$ and $R_1(l) \le R_1(\infty)$. Besides, $R_0(\infty) \ge R_1(\infty)$, and in general, the inequality is strict. True is the

Lemma 1. The following equality holds $\lim_{l \to \infty} R_0(l) = R_0(\infty)$.

This simple and seemingly purely technical result is the key to proving the following substantive statement.

Theorem 3. All controlled systems can be divided into two classes. For games of the first class, regardless of l, the following inequality is true: $R_0(l) \ge R_1(l)$. For games of the second class, there is such natural L that for all $l \ge L$, there is true the inequality $R_1(l) > R_0(l)$. Both classes are not empty.

4 Stochastic Uncertainty

Now, let us assume that the operating party is risk neutral, that is, it focuses on the mathematical expectation of its gain.

With centralized control method, the maximum expected result of the operating party is, by definition, equal to

$$R_2(l) = \sup_{(w_*,P) \in \Phi(S,W) \times \Phi(A,S)} \int_A g\left(w_*\left(P(\alpha)\right), \alpha\right) \wp(d\alpha).$$

A similar result in the model without restrictions on the quantity of information obtained is given by the condition

$$R_2(\infty) = \max_{w_\# \in \Phi(A,W)} \int_A g(w_\#(\alpha), \alpha) \wp(d\alpha).$$

Theorem 4. The following equalities are true:

$$R_2(l) = \max_{(w_0, w_1, \ldots, w_{m-1}) \in W^m} \int_A \max_{s=0,1,\ldots,m-1} g(w_s, \alpha) \wp(d\alpha),$$

$$R_2(\infty) = \int_A \max_{w \in W} g(w, \alpha) \wp(d\alpha).$$

With decentralized control method, the maximum expected result of the Center is determined by the condition

$$R_3(l) = \sup_{(u_*, P) \in \Phi(S,U) \times \Phi(A,S)} \int_A \min_{v^1 \in BR^1(u_*, P, \alpha)} \ldots \min_{v^n \in BR^n(u_*, P, \alpha)} g\left(u_*\left(P(\alpha)\right), v^1, \ldots, v^n, \alpha\right) \wp(d\alpha).$$

In the model without restrictions on the quantity of information obtained, this result is equal to

$$R_3(\infty) = \sup_{u_\# \in \Phi(A,U)} \int_A \min_{v^1 \in Br^1(u_\#, \alpha)} \ldots \min_{v^n \in Br^n(u_\#, \alpha)} g\left(u_\#(\alpha), v^1, \ldots, v^n, \alpha\right) \wp(d\alpha).$$

Theorem 5. We have the following equalities:

$$R_3(l) = \max_{(u_0, u_1, \ldots, u_{m-1}) \in U^m} \int_A \max_{s=01,\ldots,m-1} \min_{v^1 \in E^1(u_s, \alpha)} \ldots \min_{v^n \in E^n(u_s, \alpha)} g(u_s, v^1, \ldots, v^n, \alpha) \wp(d\alpha),$$

$$R_3(\infty) = \int_A \max_{u \in U} \min_{v^1 \in E^1(u, \alpha)} \ldots \min_{v^n \in E^n(u, \alpha)} g\left(u, v^1, \ldots, v^n, \alpha\right) \wp(d\alpha).$$

It is easy to establish that, for any l, the following inequalities are true: $R_2(l) \le R_2(\infty)$, $R_2(l) \le R_2(l+1)$, $R_3(l) \le R_3(\infty)$, $R_3(l) \le R_3(l+1)$ and $R_2(\infty) \ge R_3(\infty)$.

Let assume that $g(u, v^1, v^2, \ldots, v^n, \alpha) = \sum_{i=1}^{n} h^i(u, v^i, \alpha)$. Then, one can demonstrate that $R_1(l) \ge R_0(l)$, and in non-trivial cases, the inequality is strict. Similarly, if $g(u, v^1, v^2, \ldots, v^n, \alpha) = -\sum_{i=1}^{n} h^i(u, v^i, \alpha)$, then $R_1(l) \le R_0(l)$ and, in a typical case, the inequality is strict.

The first hypothesis from the previous paragraph can be interpreted as a “very good” coordination of interests of agents and the system as a whole. The second assumption is interpreted as a “very bad” coordination of interests of the Center and agents.

The analogue of Lemma 1 is as follows.

Lemma 2. We have the equality $\lim_{l\to\infty} R_2(l) = R_2(\infty)$.

True is the

Theorem 6. With a fixed quantity of available information, both the inequality $R_2(l) > R_3(l)$ and the opposite inequality $R_3(l) > R_2(l)$ can be true. However, with any $\varepsilon > 0$, at large enough l, we have the inequality $R_2(l) > R_3(l) - \varepsilon$ and, in a typical case, with large enough l, the inequality $R_2(l) > R_3(l)$ is also true.

5 Conclusion

Qualitative conclusions, that can be made based on the study of both systems models, can be formulated as follows.

In non-trivial cases, the picture is as follows. If the interests of agents are "poorly coordinated" with those of the Center, a centralized control is always more profitable. On the contrary, if the interests of the Centers and agents are "well coordinated", then, with large values of l, centralization of control is more profitable, and with small values of l, a decentralized control is preferable.

In general, these conclusions are consistent with substantive ideas. In our opinion, this suggests that models built reflect correctly the essence of the objects modeled.

Let us note, by the way, that the results of theorems 1,2,4, and 5 allow, at least in principle, to separate the "essential" information from the "non-essential" one. Surely, we are still far from getting any quantitative results, but the qualitative picture becomes clearer.

It is important to note that all results have been obtained with minimal "geometric" constraints on the structure of the studied models.

The second of the considered model systems allows for using not the maximum, but the "average" number of bits in the relevant message to estimate the quantity of information, that is, for using the Shannon entropy. But the corresponding model has not yet been studied.

The above gives reason to hope that the suggested approach can serve as a basis for building models that allow to study issues of the optimal decentralization of the control basis for building models of complex organizational systems on a quantitative or qualitative level.

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