

# Towards an answer set programming methodology for constructing programs following a semi-automatic approach

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**Abstract.** Answer Set Programming (ASP) is a successful rule-based formalism for modeling and solving knowledge-intense combinatorial (optimization) problems. Despite its success in both academic and industry, open challenges like automatic source code optimization, and software engineering remains. This is because a problem encoded into an ASP might not have the desired solving performance compared to an equivalent representation. Motivated by these two challenges, this paper has three main contributions. First, we propose a developing process towards a methodology to implement ASP programs, being faithful to existing methods. Second, we present ASP encodings that serve as the basis from the developing process. Third, we demonstrate the use of ASP to reverse the standard solving process. That is, knowing answer sets in advance, and desired strong equivalent properties, “we” exhaustively reconstruct ASP programs if they exist, paving the road towards a benchmarking procedure of ASP programs.

## 1 Introduction

The automatic generation of solutions for declaratively specified search-problems is one of the most successful areas of artificial intelligence [1], where Answer Set Programming (ASP; [2]) highlights due to its full support on a compact representation of search problems. ASP is a rule-based formalism for modeling and solving knowledge-intense combinatorial (optimization) problems.

ASP’s attractiveness consists of the combination of a declarative modeling language with highly effective solving engines, allowing to specifying a given (search) problem rather than programming the algorithm for solving it. In other words, given a search problem, a programmer specifies the search space domain and problem-specific properties. Combined, let an ASP solver propose solutions called answer sets.

Currently, ASP is robust and mature enough, offering many important language constructs like aggregation, (weak) constraints, different types of negations, and optimization statements to mention a few, as well as high-performance solvers. An example of a *state-of-the-art* and *award-winning* ASP solvers is *clasp* [3] demonstrating its competitiveness and versatility, by winning first places at various solver contests since 2011 (eg. ASP, CASC, MISC, PB, and SAT competitions).<sup>3</sup>

<sup>3</sup> For more details of clasp trophies and tracks, see <http://potassco.sourceforge.net/trophy.html>.

*clasp*, combined with the grounder *gringo* [5], composes *clingo* [6], an ASP system to ground and solve logic programs. For the reader interested in learning more about ASP, including theoretical works, implementations, and applications, see [9,7,4,6].

Despite the success of ASP in both academic and industry, <sup>4</sup> in areas like planning, scheduling, configuration, design, and diagnosis (to mention a few), challenges like automatic source code optimization, and software engineering remain open, where there is a need to integrate software engineering methodologies and tools into ASP [1].

To the best of our knowledge, [10] is the only approach describing a standard software engineering process consisting of the development and the design of ASP programs in an industrial context. Other works in ASP that have some relationship with software engineering, concerns Inductive Logic Programming (ILP) [11,12], Procedural Content Generation (PCG) [13], ASP Debuggers [14] (including Meta-Programming [15]), and an IDE for ASP called ASPIDE [16].

The need for automatic source code optimization, and software engineering tools and methodologies into ASP come hand in hand. Inspired (among others) by circumstances where a problem encoded into an ASP might not have the desired solving performance compared to an equivalent representation.

Motivated by these two challenges, this paper has three main contributions. First, we propose a developing process towards a methodology to implement ASP programs, being (as much as possible) faithful to the method proposed by [10]. Second, we present ASP encodings that fall under the category of *meta-programs* [8] serving as the basis from the developing process. Third, we demonstrate the use of ASP to reverse the standard solving process. That is, knowing answer sets in advance, and desired strong equivalent properties, exhaustively reconstruct ASP programs if they exist, following the approaches from [20,21], paving the road towards a benchmarking procedure of ASP programs, to find an optimal representation. Informally, strong equivalence (*SE*) means that we can safely replace a piece of knowledge representation with another regardless of the context. In other words, we can safely change code without modifying the semantics of the program.

To motivate this paper, let us set the context of the intended process, and let us illustrate a running example using ASP as an overview to reverse the standard solving process. For more fine-grained details including ASP codifications, we refer to Section 3.

**Example** Let us assume to have a system called *ProgramBuilder* which its core iteratively calls *clingo*, and consists (among other features) in three stages. The first stage takes the answer sets and possible strong equivalent properties as input and delivers an intermediate representation. The second stage takes this intermediate representation to construct a starting propositional formula. The third and last stage takes this formula and proposes a new one strongly equivalent to the initial, according to the user needs.

The system benefits from the declarative approach of ASP, having a series of underlying programs comprises in a single one, called *SPF* that faithfully represents the entire system workflow. To describe this workflow, let us consider a very simple example with the intention to transmit our approach very clearly. Interested in finding a propositional

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<sup>4</sup> An incomplete but vast list of ASP applications:  
<https://www.dropbox.com/s/pe261e4qi6bcyyh/aspAppTable.pdf>

formula of two variables  $p$  and  $q$ , such that it has  $\{p\}$  and  $\{q\}$  as unique answer sets, and discarding the empty set and  $\{p, q\}$ .

**First Stage** Departing from known answer sets as input, the system calls *clingo* and let it guess for a formula that satisfies the previous conditions. However, with these conditions, *clingo* finds over 300 different intermediate representations (potential formulas) that satisfies the given input.

As mentioned before, the user can benefit from *SE* properties to delimit more the search. This means the user can straightforwardly specify in *SPF* the desired properties to satisfy. For example, the user can ask for a representation that satisfies *commutativity*, *associativity*, and *identity*. Calling again *SPF* coupled with the user-given properties, *clingo* encounters four intermediate representations.

Let us mention that these intermediate representations consist of a 3x3 matrix based on the 3-valued logic of  $G_3$ . Now suppose we have two users, the first one, decides to refine more the search by adding another property, for example *idempotency*. Now *clingo* yields a single matrix, allowing the user to move to the second stage. The second user instead, asks *ProgramBuilder* to take two matrices  $M_1$  and  $M_2$  from the four remaining, postponing the decision, and moving also to the second stage.

**Second Stage** If the user has more than one matrix, he or she could enter into a dialog process until one solution is selected. However, the user could also keep the matrices and continue the workflow.

Suppose the user has two matrices  $M_1$  and  $M_2$ , he or she wants to decide for one of them. To be more specific, let  $M_1$  and  $M_2$  be the matrices from tables 1a, and 1b respectively. We can see that both truth tables 1a, and 1b differ in a single value when both inputs are 1.<sup>5</sup> As mentioned before, these intermediate representations serve

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(a) Truth table from $M_1$	(b) Truth table from $M_2$																																

Table 1:  $M_1$  and  $M_2$  differing where both inputs equals to 1 in the logic of  $G_3$ .

to construct initial propositional formulas. *ProgramBuilder* takes each matrix and constructs its corresponding formula. Let us state that each formula is a disjunction of clauses, where each clause corresponds to the interpretations where the result equals to 2, and let the function  $F$  be responsible for the construction of the following formulas:

$$\begin{aligned}
 F_1 &= F(M_1) = (p \wedge \neg q) \vee (q \wedge \neg p) \vee (p \wedge \neg\neg q) \vee (q \wedge \neg\neg p) \vee (p \wedge q) \\
 F_2 &= F(M_2) = (p \wedge \neg q) \vee (q \wedge \neg p) \vee (p \wedge \neg\neg q) \vee (q \wedge \neg\neg p) \vee (p \wedge q) \vee (\neg\neg p \wedge \neg\neg q)
 \end{aligned}$$

*ProgramBuilder* can warn the user, that if you add another program  $Q$ , consisting of the rules  $(p \leftarrow \neg q) \wedge (q \leftarrow \neg p)$  to both formulas  $F_1$  and  $F_2$ , then,  $AS(F_1 \wedge Q) =$

<sup>5</sup> We present the tables for the reader and the sake of clarity. However, they could be irrelevant for the user. For the interested reader, the semantics of  $G_3$  can be found in [34].

$\{\{p\}, \{q\}\}$ , while  $AS(F_2 \wedge Q) = \{\}$ . In other words, for the first case, we have the single answer set  $\{p, q\}$ , and for the second case, there are no answer sets (unsatisfiable). This means, that  $F_1$  and  $F_2$  are not strongly equivalent. It is relevant to mention that it is up to the user to pick one of them or to continue to the third stage.

**Third Stage** The system takes each formula and proposes a new strongly equivalent alternative. *ProgramBuilder* is equipped with an algebra of logical transformations (respecting *SE*), that can translate  $F_1$  into a given normal-form. For this case,  $F_1$  is translated into to the logical disjunction  $p \vee q$ . With the example above, we propose a first step methodology with the possibility to implement it into an interactive software that construct ASP programs through defined properties. As mentioned above, this software could include transformation modules to visualize the constructed programs in multiple forms. We present this toy example on purpose to make it easy to follow. Nevertheless, this example inspires the conception of a more general framework.

The remainder of the paper is structured as follows: Section 2 informally introduces ASP, followed by the formal definition of strong equivalence. Then, we focus on the introduction of non-standard concepts needed in this paper, like the approach to construct formulas from an interpretation in the 3-valued logic of  $G_3$ . We also mention the straightforward relationship with the logic of of Here-and-There (*HT*). We close this section with the best practices for designing and developing ASP programs. Section 3 describes our methodology by complementing the running example, illustrating with more complex examples as well as ASP programs. Finally, we discuss the conclusions of the paper, and direct future work in Section 4.

## 2 Background

In this section, we present theoretical and practical aspects that would be of interests in our proposed approach such as formal definition of strong equivalence, the formalities to construct propositional formulas using Gödel's 3-valued logic ( $G_3$ ) [24], and its relationship with the logic of Here-and-There (*HT*) [23], among others. Lastly, we recapitulate the design and development process of ASP programs from [10].

### 2.1 Strong Equivalence

The term *Strong Equivalence* [17], concerning ASP programs, means that, having two programs (formulas)  $F_1$  and  $F_2$ ,  $F_1$  is strongly equivalent to  $F_2$  if  $F_1$  is equivalent to  $F_2$  in the Gödel's 3-valued logic ( $G_3$ ), which is equivalent to the logic of *Here-and-There* (*HT*). Also, via the *reduct* [27],  $F_1$  is strongly equivalent to  $F_2$  if for each set  $X$  of atoms both *reducts*  $F_1^X$  and  $F_2^X$  are equivalent in classical logic [18,19]. It is relevant to remark the importance of *Strong Equivalence* into a software engineering perspective, which not only  $F_1$  and  $F_2$  comprise the same answer sets (meaning  $F_1 \equiv F_2$ ) but, we can extend both formulas with another one  $R$  such that  $F_1 \cup R$  and  $F_2 \cup R$  yield the same answer sets (represented by  $F_1 \equiv_{SE} F_2$ ).

## 2.2 Constructing formulas from an interpretation in $G_3$ or $HT$

For software engineering purposes, it is possible to construct propositional formulas (hence, ASP programs) from an interpretation in  $HT$  [20,21]. Yet, it is also possible to use  $G_3$  logic, which it is equivalent to  $HT$ , and the relationship is straightforward.<sup>6</sup>

For the  $G_3$  values 0, 1, and 2, 0 equals  $\perp$  or false *There*, 1 equals false *Here* but true *There*, and 2 equals  $\top$  or true *Here*. Therefore, considering that both logics  $G_3$  and  $HT$  are equivalent, we keep  $G_3$  for the remainder of the paper. That is, given a  $G_3$ -interpretation  $I$  (as shown in Tables 1a and 1b), we apply the following specification or clause  $C$  from [21].<sup>7</sup> To create a clause, we apply the formula below whenever an interpretation equals to 2. A more detailed example is shown in Table 2 from Section 3.

$$\left( \bigwedge_{I(v)=2} v \right) \wedge \left( \bigwedge_{I(w)=0} \neg w \right) \wedge \left( \bigwedge_{I(x)=1} \neg \neg x \right) \wedge \left( \bigwedge_{I(y), I(z)=1, y \neq z} (y \rightarrow z) \right) \quad (1)$$

Then, to construct the propositional formula, we need to apply disjunctions over the resulting clauses, as shown in  $F_1$  from the running example. Lastly, the formula can be simplified according to (but not necessarily all) [27,30,31] Taking back the running example, the formula  $F_1$  is reduced to the disjunction  $p \vee q \vee (p \wedge q)$ , that is strongly equivalent to the constructed formula  $p \vee q$ .<sup>8</sup>

On the other hand, we can apply the same procedure to find a counter-example for two programs  $P_1$  and  $P_2$  such that  $P_1 \equiv P_2$  (yield the same answer sets), but instead of applying disjunctions over the resultant clauses, we conjunct them [26]. This counter-example serves to prove if both programs are strong equivalent, meaning that  $P_1 \equiv_{SE} P_2$  as shown in the previous section.

## 2.3 Software Engineering

The work from [10] proposes a six steps methodology for the development of ASP programs, following the project management (PM) standard ISO 21500:2012, also coordinated with the principles behind the life cycles development from the Project Management Body of Knowledge (PMBOK) [22]. We recapitulate the six areas and let us point out the intersection with the aforementioned stages from our methodology.

1. **Identify the needs** Find opportunities where ASP is stronger than conventional methods. Define and document the application requirements properly (first stage).
2. **Design a valid specification of the problem** Implement an ASP specification of the core problem with small instances for testing. Take advantages of ASP which allows interactive problem refinement and tuning (second stage with support from the first stage).

<sup>6</sup> We only mention the needed concepts from the logics of  $G_3$  and  $HT$ . For more information, we may refer the reader to [28,29].

<sup>7</sup> The original formula is in the context of  $HT$ . To be consistent, we adapted for the  $G_3$ .

<sup>8</sup> For more details about the simplification, we refer to [21].

3. **Performance engineering** Explore alternatives of ASP program implementations (as shown in the third stage with the support of the other two stages), and evaluate their performance considering “real-world” size instances.<sup>9</sup> For a prototyping process, like our methodology, we focus more on readability rather than performance.
4. **Integrate into the existing environment** Choose the best ASP program alternative from the feasibility study and implement a clean ASP program which processes the transformed data. Evaluate if is possible to use incremental solving, and consider the manipulation of answer sets. Design interfaces and implement a complete and efficient transformation from legacy input data to ASP and back. It is now possible to integrate ASP solvers like *clingo*, into legacy systems in a more natural way due to a complete API in languages like Python or C++.
5. **Testing and debugging** Ensure high-quality via automated tests, and debugging of ASP programs if applicable. For instance, ASP Debuggers like [14,15].
6. **Maintenance** Focus on a well-defined structure of the program, and benefit from ASP’s modularity for further adaptations.

Also, [10] stated that in this development process, they consider knowledge base design and performance engineering as the most important and most different steps from conventional software engineering. Our method falls in these two steps, particularly, covering the first three, letting glimpse opportunities to develop the last three steps.

Furthermore, in [10], they use an Object-Oriented approach (OO) into ASP called OOASP, which allows analyzing OO software models and their instances employing ASP. The OOASP approach has been successfully implemented in Siemens, as an extension to any OO modeling environment. It has been evaluated together with Siemens internal tools. This modeling approach is currently out of the scope of this paper, but it will be considered for future work development.

### 3 Methodology and Approaches

This section describes and exemplifies our methodology, delving more into the underlying ASP encodings. That is, we retake our running example, and decompose the three stages with more complex examples. Let us motivate again with the definition of the problem.

**Problem definition.** Given (an incomplete set of) answer sets, and possible strong equivalent properties as input, search and construct a single or several propositional formulas, hence, an ASP program satisfying these conditions. To do so, we follow the standard *guess-and-check* paradigm of ASP where solution candidates are tested for feasibility with the possibility of yielding none, one, or multiple answer sets. Typically, these answer sets serve as the solutions of an encoded program, but for our purposes, they are interpretations in  $G_3$  which allow us to construct formulas.

Before delving into the three stages, let us explain that we have two types of answer sets (due to our *meta-programming* approach), the answer sets given as input, and the answer sets as intermediate representations. From now on, we easily differentiate

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<sup>9</sup> Particularly, for the third area, we only focus on the exploration of ASP program alternatives and their implementations. Their performance evaluation concerning “real-world” size instances, is left for future work.

them as  $answer\_set(s)_{input}$  and  $answer\_set(s)_{output}$  respectively, and we use them interchangeably.

Our implementation of the core problem into ASP follows the common practices of ASP, by separately provide an instance and an encoding. As stated above in the definition of the problem, the instance corresponds to  $answer\_sets_{input}$  and strong equivalent properties. On the other hand, the encoding consists of means to prove the existence or the lack of a propositional formula. Before addressing the three stages, let us illustrate our workflow with another example. Originally, a motivation behind this work was the question about the existence of an interpretation, that satisfies the four essential properties from the exclusive disjunction (XOR).

Let us request a propositional formula, keeping the same variables  $p$  and  $q$ , and the same unique  $answer\_sets_{input} \{p\}$  and  $\{q\}$ , while discarding again the empty set and  $\{p, q\}$ . We represent each input answer set with the atom `answer_set` having a string value as its argument, as shown in Listing 1.1.

```

1 :- not answer_set("p").
2 :- not answer_set("q").
3 :- answer_set("").
4 :- answer_set("p q").

```

Listing 1.1: Answer sets as part of the instance (`answer_sets.lp`).

Since the requirements are clear beforehand, and as part of the first stage, we can represent the four essential properties from the classical logic XOR as part of our input, where two of them are *commutativity* and *associativity*. The other two properties are *self inverse*, meaning that any input XORed with itself is false, and *identity*, where an input XORed by false, yields the double negation of the entry.<sup>10</sup> To see these properties in the context of an instance, we refer to Listing 1.2.

```

1 %% Commutativity : X xor Y = Y xor X
2 :- op(X,Y,R1), op(Y,X,R2), R1!=R2.

4 %% Associativity : (X xor Y) xor Z = X xor (Y xor Z)
5 left(X,Y,Z,R) :- op(X,Y,W1), op(W1,Z,R). %% Left
6 right(X,Y,Z,R) :- op(Y,Z,W1), op(X,W1,R). %% Right
7 :- left(X,Y,Z,R1), right(X,Y,Z,R2), R1 != R2.

9 %% Self Inverse : X xor X = 0
10 :- op(X,X,R), R!=0.

12 %% Identity : X xor 0 = not not X
13 :- op(X,0,Y), neg(X,X1), neg(X1,Z), value(Y), Y != Z.

```

Listing 1.2: Essential *SE* properties of the classical XOR operator (`xor-strong.lp`).

From the code above, we represent each property as an integrity constraint, where the atom `op(X, Y, R)` corresponds to the desired operator of two arguments (variables)  $X$  and  $Y$  and its result  $R$ . Let us allow to get ahead, and mention that this atom is part

<sup>10</sup> By letting us expressing that a variable  $p$  XORed with a false constant as  $p \oplus \perp$ , equals  $\neg\neg p$ . This is represented in ASP as a constraint of the form: `:- not p.`

of the *answer\_sets\_output* or intermediate representation. Before we move to the second stage, we describe the encodings that yield the intermediate representations.

The encoding consists of four parts, the guessing of the intermediate representation, the definition of the logical operators, the theory completion, and  $G_3$  persistency properties. Let us start with the intermediate representation guessing, which is no other than a choice rule with both boundaries set to one, asking for an operator  $\text{op}(X, Y, R)$ , from any to values  $X$  and  $Y$ , resulting in  $R$  (Listing 1.3).

```
1 1 { op(X,Y,Z) : value(Z) } 1 :- value(X), value(Y).
```

Listing 1.3: Guess formula via an interpretation in  $G_3$  (*guess\_formula.lp*).

The second part is the definition of the logical operators in  $G_3$ , shown in Listing 1.4.

```
1 value(0..2). %% G3 values

3 and(X,X,X) :- value(X).
4 and(X,Y,X) :- value(X), value(Y), X<Y.
5 and(X,Y,Y) :- value(X), value(Y), Y<X.

7 or(X,X,X) :- value(X).
8 or(X,Y,X) :- value(X), value(Y), Y<X.
9 or(X,Y,Y) :- value(X), value(Y), X<Y.

11 neg(0,2). neg(2,0). neg(1,0).

13 implication(X,Y,2) :- value(X), value(Y), X <= Y.
14 implication(X,Y,Y) :- value(X), value(Y), X > Y.
```

Listing 1.4:  $G_3$  values and logical operators (*logical\_operators.lp*).

This encoding consisting of the operators *and*, *or*, *negation*, and *implication*, serves twofold. It works for the generation of intermediate representations or to compute the answer sets if given an initial formula.<sup>11</sup> To find intermediate representations (*answer\_sets\_output*), we need to characterize them in terms of what we call a theory completion, as shown in Listing 1.5.<sup>12</sup>

```
1 completion(0,X,Y,R) :- neg(X,X1), neg(Y ,Y1), and(X1,Y1,R).
2 completion(1,X,Y,R) :- neg(X,X1), neg(X1,X2), neg(Y,Y1),
   and(X2,Y1,R).
3 completion(2,X,Y,R) :- neg(Y,Y1), neg(Y1,Y2), neg(X,X1),
   and(X1,Y2,R).
4 completion(3,X,Y,R) :- neg(Y,Y1), neg(Y1,Y2), neg(X,X1),
   neg(X1,X2), and(X2,Y2,R).

6 belongs(1,p). belongs(2,q). belongs(3,p). belongs(3,q).

8 code(0,""). code(1,"p"). code(2,"q"). code(3,"p q").
```

<sup>11</sup> We discuss this point at the end of the section.

<sup>12</sup> For the sake of clarity, we fix this encoding concerning the exemplary signature  $\{p, q\}$ . However, it is possible to generate this encoding for a given signature.



```

10 completion_asp(A_ID,X,Y,R) :- op(X,Y,Z), completion(A_ID,X
    ,Y,C), and(Z,C,R).

12 consistent(A_ID):-completion_asp(A_ID,X,Y,R),value(R),R>0.
13 incomplete(A_ID):-belongs(A_ID,p), completion_asp(A_ID,X,Y
    ,Z), implication(Z,X,R), R<2.
14 incomplete(A_ID):-belongs(A_ID,q), completion_asp(A_ID,X,Y
    ,Z), implication(Z,Y,R), R<2.

16 answer_set(S) :- consistent(A_ID), not incomplete(A_ID),
    code(A_ID,S).

```

Listing 1.5: Theory completion for answer sets (`theory_completion.lp`).

Describing an overview of the main function of this code, the first four rules from Listing 1.5, captures the completions needed for all possible answer sets (related to *answer\_set\_input*) concerning our inputs  $p$  and  $q$ . Then, the facts in line 6, display the correspondence between the constants  $p$  and  $q$  with all the possible answer sets, followed by (facts) mappings into string representations in line 8. Line 10 forms the completion concerning the operator. Then, the completion must be consistent (line 12), and we define what incompleteness is (lines 13 and 14). Lastly, line 16, derives the corresponding answer sets in string representation via their correlated code. These answer sets must satisfy consistency and completeness, as well as the *answer\_sets\_input* (Listing 1.1).

Finally, we need to guarantee  $G_3$  persistence properties [25,21]. They are displayed in Listing 1.6. Here, line 1 states that it is not possible that in case there exist an interpretation 1,0,2, then exist another interpretation with inputs 2 and 0, that evaluates to any other value different than 2. Line 2 describes the commutated property, and line 3 states that, is not possible that an interpretation resulting in 1, comes from inputs different than 1.

```

1 :- op(1,0,2), op(2,0,X), X != 2.
2 :- op(0,1,2), op(0,2,X), X != 2.
3 :- op(X,Y,1), X != 1, Y != 1.

```

Listing 1.6:  $G_3$  persistence (`g3_persistence.lp`).

Solving both, the instance, and the encoding produces the intermediate representations to move to the second stage. However, for this particular example, there are no *answer\_sets\_output*. This means it is not possible to represent an XOR operator as a function of two arguments in ASP, that aside, satisfy all four properties. Despite the negative solution where there is no formula to construct, it is positive in the sense that this methodology can save time, money, and resources. Also, this fits perfectly into iterative software engineering methodologies, taking the user back to the initial or design stage, wondering about the requirements.

On the other hand, and following our XOR motivation, we also question ourselves if we can find a formula that semantically behaves as a parity constraint as the ones used in *xorro* [32]. That is, we are searching for a constraint formula that discards candidate answer sets from an independent generation process.<sup>13</sup> We affirmatively answer this

<sup>13</sup> The generation process for every variable  $x$  is represented as  $x \vee \neg x$ .

question, and this comes after an exhaustive search. In other words, we ask *clingo* for all possible intermediate representations, and *clingo* found a single *answer\_set\_output*. This means that there is only one possibility to represent an XOR as a constraint in ASP satisfying the aforementioned properties. By these means, we can confirm that the founded formula semantically behaves as the parity constraints used in *xorro*.

$$\begin{aligned} & \text{op}(0, 0, 0), \text{op}(0, 1, 2), \text{op}(0, 2, 2) \\ & \text{op}(1, 0, 2), \text{op}(1, 1, 0), \text{op}(1, 2, 0) \\ & \text{op}(2, 0, 2), \text{op}(2, 1, 0), \text{op}(2, 2, 0) \end{aligned} \tag{2}$$

With this intermediate representation (2) as a matrix of the form of  $M_1$  or  $M_2$  (from the running example), we can move to the second stage.

For this example, we do not have more than one representation, so we do not have anything else to compare. Hence, the constructed formula, namely  $F_{xor}$  gives the following clauses (2), taking the specification shown in 1. This results in the initial propositional formula:

Table 2: Resulting clauses after the interpretation in  $G_3$ .

Interpretation	Clause
$\{0, 1, 2\}$	$\neg p \wedge \neg \neg q$
$\{0, 2, 2\}$	$\neg p \wedge q$
$\{1, 0, 2\}$	$\neg \neg p \wedge \neg q$
$\{2, 0, 2\}$	$p \wedge \neg q$

$$F_{xor} = (\neg p \wedge \neg \neg q) \vee (\neg p \wedge q) \vee (\neg \neg p \wedge \neg q) \vee (p \wedge \neg q) \tag{3}$$

Lastly, the third stage proposes a transformation for  $F_{xor}$ . For instance, a resulting formula could be  $(\neg \neg p \vee \neg \neg q) \wedge (\neg p \vee \neg q)$ . Nevertheless, it is possible to reverse the presented method by given a propositional formula and let *clingo* search for the answer sets. For example, let our instance be the same  $F_{xor}$  formula using the logical operators from Listing 1.4, as:

```
op(X, Y, Z2) :- or(X, Y, R1), neg(X, X1), neg(Y, Y1), or(X1, Y1, R2),
and(R1, R2, Z), neg(Z, Z1), neg(Z1, Z2).
```

This formula replaces the code from Listings 1.1, and 1.3, and reuse the aforementioned encoding, logical operators (Listings 1.4), theory completion (Listings 1.5), and  $G_3$  persistency properties (Listings 1.6). Therefore, generating candidate answer sets over  $p$  and  $q$ ,  $F_{xor}$  discards the empty set and  $\{p, q\}$ .

Finally, it is worth mentioning that currently, we have an initial and very basic implementation using Python and *clingo*. For more details, go to <https://github.com/flavioeverardo/Propositional-Formula-Builder-PFB>.

## 4 Discussion

Motivated by the need for automatic source code optimization, and the inclusion of software engineering into ASP, we presented a preliminary developing process towards a methodology to implement ASP programs, following existing methods. We captured this developing process into an initial prototype consisting of ASP encodings, that reverses the standard solving workflow towards an exhaustive search for propositional formulas, all within ASP. The resultant formula(s) must satisfy strong equivalent properties as well as known answer sets.

For future work, there is too much to do. First of all, we plan to continue the development of a fully-integrated software concerning the proposed methodology, including the tools for reconstructing more complex formulas. This initial prototype uses ASP in its whole, as it is conceived thanks to high-level interfaces, sophisticated algorithms for grounding and solving, including search heuristics and learning techniques based on nogoods, among others. Hence, this initiative constructs propositional formulas. However, it is far from fully equipped software. One possible extension could be the construction of not only propositional formulas, but *non-ground* ASP programs. That is, including variables. Then, we could benefit from tools like *anthem* [33] to verifying Strong Equivalence of ASP programs in the input language of *gringo*.

In terms of software engineering, both our method and the prototype could benefit from several other techniques from the ASP community, fitting perfectly with the uncovered steps (4,5, and 6) from [10] as well as into the OOASP approach. Some of these techniques, includes Inductive Logic Programming, debugging, Graphical User Interfaces encouraged by ASPIDE, to name a few.

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