Analysis of Incentives Influence on Great Social Groups' Behavior in Stackelberg game

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***Abstract*—We consider the encouragement of the great social groups (agents) to the socially optimal behavior by an example of the volunteering. We search for the optimal actions vector of these social groups, i.e., the equilibrium in the incentives allocation game. On the basis of the game-theoretic model with Stackelberg leadership, under conditions of the awareness asymmetry, the possible equilibrium variants are investigated. In the case of a linear decreasing incentive function and linear cost functions of the agents, Nash equilibrium conditions in Stackelberg game are proved. For various types of the agents' tendency to altruism, the analytical formulas for calculating the equilibria are derived. On the basis of the Russian population statistics, we simulate the behavior of the volunteers groups.**

Keywords—incentive system, Stackelberg game, Nash equilibrium, volunteer

I. Introduction

The encouragement in social systems is used to purposefully change of the social groups' behavior patterns. For this purpose, the incentives are calculated from the optimality conditions of the social criteria, which are established by the governments of these systems. Most often, at the state level, the goal of the incentives is to encourage citizens to perform actions that maximize the collective utility function. Hereinafter, these actions are referred to the socially optimal actions or the volunteering. This encouragement is caused by the need to overcome the trends of individual rationalism [1,2], and it is expressed in the implementation of the social national programs [3,4], including the information systems development programs [5].

For the practical implementation of the incentive system, methods and algorithms were developed [6], and the game-theoretic model of the social groups (hereinafter, agents) behavior was formulated [7] in the form of the non-cooperative game. The model was based on a compensatory linearly decreasing stimulation function, for which the conditions of the individual rationality, Pareto efficiency, and non-manipulation were proved [8–17].

The model [7] describes the dependence of the citizen's individual utility function on the distribution of his disposable time fund, the degree of propensity towards the altruism and the incentive, i.e., the price of the socially optimal action. In turn, the incentive is calculated as a decreasing function of the total number of all volunteers' actions. Based on the optimization of the individual utility functions of all citizens, the model enables us to calculate the vector of socially optimal actions, which satisfies the interests of all citizens, i.e., it is Nash equilibrium. In addition, the model takes into account the interests of the state (meta-agent), which is aimed at the rational increase in the volunteer activities. The meta-agent chooses the coefficients of the incentive function from the following condition: if the incentive is equal to the average wage, then at least half of the available time fund of citizens is allocated for volunteering.

On the basis of this model, the equilibrium conditions were derived, and the formulas for calculating the socially optimal actions vector were obtained. In this case, when choosing actions, the social groups do not take into account each other's behavior. In the game theory, this condition was called Cournot hypothesis [19], and it expresses the symmetry of the players due to the a priori information unawareness of the player about the actions of other players (hereinafter, environment). However, in reality, some social groups may be informed about the activity of other social groups, which leads to a situation of the awareness asymmetry, therefore, in the game, the asymmetry of the equilibrium arises. In the case of the awareness asymmetry, the game of the social groups describes the behavior of agents, who are informed about the optimal choice of the environment; such agents become Stackelberg leaders [20]. In this case, the environment has the followers status, whose behavior is described by Cournot hypothesis.

Further article is structured as follows: the description of the agent incentive system according to [8], the analysis of the principles of choosing the actions in Stackelberg game, the investigation of the stratifying the agents into leaders and followers, the formulation of the equilibrium model, the development of analytical formulas for calculating the equilibrium in Stackelberg game

II. Methods

We consider as the object of stimulation the social system, for example, citizens of a country or employees of a corporation, which are divided into K groups (agents). These agents differ by attribute that affects the effectiveness of stimulation, which is further called the agent type parameter. In other words, all individuals in the group k have a predictable identical reaction to equal incentives. The number of individuals in the group k is indicated $n_k, k \in K$, the symbol K denotes a set of social groups and the number of elements of this set.

The agent's type parameter is determined by his altruism, i.e., the propensity to charity, and it is estimated by the coefficient of the charity time elasticity with respect to the disposable time fund $\delta_{ak} \in [0,1]$. The agent is more inclined to altruism, if the coefficient δ_{ak} is closer to one. Actual values of the agent's altruism coefficient are estimated from the following function $a_k = D^{\delta_{ak}}, k \in K$, which describes the dependence of the time interval of socially optimal actions a_k in the absence of any stimulation on the available time fund *D*. On the basis of this function and taking into account the statistics of the volunteer time, the altruism coefficient is calculated by the following formula

$$\delta_{ak}(a) = \frac{\ln a_k}{\ln D}, k \in K, \ \delta_{ak} > 0{,}1 \forall a_k > 1 . \quad (1)$$

The incentive system includes the subsystem for recording the actions a_k and the subsystem for paying incentive. The incentive is equal to the product of the incentive price p_k and the action value, i.e., $p_k(\mathbf{A})a_k$. The incentive price is calculated on the basis of the following incentive function [7]:

$$p_a(\mathbf{A}) = b_1 - b_2 \sum_{k \in K} n_k a_k, k \in K, b_1, b_2 > 0 , \quad (2)$$

where $\mathbf{A} = \{a_k, k \in K\}$ is the vector of the socially optimal actions; b_1, b_2 are constant coefficients that are independent of the vector **A** in the current period. These coefficients are calculated by formulas that depend on the vector $\mathbf{A}_0 = \{a_{0k}, k \in K\}$ of the agents' actions in the previous period[1]:

$$b_1 = p_d \frac{A_0}{A^D - A_0}, b_2 = \frac{p_d}{A^D - A_0}, A_0 = \sum_{k \in K} a_{0k}, A^D = \frac{D}{2} \sum_{k \in K} n_k , \quad (2a)$$

where p_d is the price (tariff rate) of the working time. It should be noted that the coefficients b_1, b_2 are calculated according to formulas (2a), if the incentive fund is not fixed, and the administration (state) is aimed at ensuring a balance between the working and the volunteer time. In the case of the fixed incentive fund (let is equal to F), the coefficients of the incentive function are calculated by the following formulas [7]:

$$b_1 = \frac{F - n\varphi^{\min}}{A_0}\left(1 + \frac{n}{2}\right), b_2 = \frac{F - n\varphi^{\min}}{A_0^2}\frac{n}{2}, n = \sum_{k \in K} n_k , \quad (2b)$$

where $\varphi^{\min}$ is the minimum guaranteed incentive[2].

[1]Formulas (2a) are obtained from the following conditions: 1) with a low level of socially optimal actions A_0, the administration sets a high incentive price, which is equal to the average wage p_d; 2) if the disposable time fund is divided equally between the working time and the charity time (i.e., *D*/2), then the price of the incentive is zero. Under these conditions, the system of equations $b_1 - b_2 A^0 = p_d, b_1 - b_2 A^D = 0$ leads to solution (2a).

[2]Formulas (2b) are obtained from formulas [7] $b_1 = \frac{F - n\varphi^{\min}}{\sum_{k \in K} u_k} \frac{2\bar{u} + \sum_{k \in K} u_k}{2\bar{u}}, b_2 = \frac{F - n\varphi^{\min}}{2\bar{u} \sum_{k \in K} u_k}$, $\bar{u} = \frac{1}{n}\sum_{k \in K} u_k$, as a result of the following transformations. The following notation is used: $a_k = u_k$, $A_0 = \sum_{k \in K} u_k$, $A_0 / n = \bar{u}$. Therefore, $b_1 = \frac{F - n\varphi^{\min}}{A_0} \frac{2A_0/n + A_0}{2A_0/n} = \frac{F - n\varphi^{\min}}{A_0}\left(1 + \frac{n}{2}\right)$, $b_2 = \frac{F - n\varphi^{\min}}{2\frac{A_0}{n}A_0} = \frac{F - n\varphi^{\min}}{A_0^2}\frac{n}{2}$.

The effectiveness of the incentive system is evaluated according to the following individual agent's utility function:

$$U_k(a_k) = \left(p_a(\mathbf{A}) - p_d^{1-\delta_{ak}}\right)a_k, k \in K , \quad (3)$$

where $U(\bullet)$ is the continuously differentiable agent's utility function. Function (3) is used on the basis of the following hypothesis of the altruism influence on the agent's behavior: an increase in the propensity to altruism leads to a decrease in the utility of wages.

The problem of searching for Nash equilibrium vector A from the maximization of function (3) under condition (2) in the case of a constant number of the social groups (i.e., $\frac{\partial n_k}{\partial a_k} = 0 \forall k \in K$) enable us to obtain the following system of equilibrium conditions [8]:

$$b_1 - b_2 \sum_{j \in K} n_j a_j - b_2 n_k a_k \left(1 + \sum_{j \in K \setminus k} \rho_{kj}\right) - p_d^{1-\delta_{ak}} = 0, k \in K , \quad (4)$$

subject to

$$\sum_{j \in K \setminus k} \rho_{kj} > -2 , \quad (5)$$

where $\rho_{kj} = \frac{\partial a_j}{\partial a_k}$ is the conjectural variation in the equation of the agent *k*, i.e., the expected change in the action of the agent *j* in response to a single increase in the action of the agent *k*.

The conjectural variation expresses the effect of the agent's awareness asymmetry on the resulting equilibrium (i.e., the actions vector $\mathbf{A}^*$), which is the solution of system (4). The symbol «*» indicates the equilibrium values. In the case of Cournot game, when $\rho_{kj} = 0 \forall j, k \in K$, all agents symmetrically do not change the actions in response to the environment's actions, therefore, the asymmetry of the resulting equilibrium [7] depends on the differentiation of the agents by types. Further, we investigate the case of Stackelberg game (i.e., $\rho_{kj} \neq 0 \forall j, k \in K$), when some agents (leaders) may choose the actions taking into account the principles of choosing actions by other agents (followers). This is another reason for the asymmetry of the resulting equilibrium, and it is the research question of our study.

III. Results and Discussion

We introduce the following notation: $q_k = n_k a_k$ is the aggregate action of the social group *k*; $q_{-k} = n_{-k} a_{-k}$ is the aggregate action of the environment; $q = \sum_{k \in K} q_k$ is the aggregate action of all agents in the system. The environment includes all agents except the agents of the social group *k*. In this case, the system of equations (4) may be transformed as follows:

$$b_1 - b_2(q_k + q_{-k}) - b_2 q_k \left(1 + \sum_{j \in K \setminus k} \rho_{kj}\right) - p_d^{1-\delta_{ak}} = 0\ ,$$

$$\frac{b_1}{b_2} - 2q_k - q_{-k} - q_k \sum_{j \in K \setminus k} \rho_{kj} - \frac{p_d^{1-\delta_{ak}}}{b_2} = 0\ ,$$

$$2q_k + q_k \sum_{j \in K \setminus k} \rho_{kj} + q_{-k} - \frac{b_1 - p_d^{1-\delta_{ak}}}{b_2} = 0\ ,$$

and we write the system in the following resulting form:

$$f_k(q_k, q) = q_k\left(2 + \sum_{j \in K \setminus k} \rho_{kj}\right) + q_{-k} - \alpha_k = 0, k \in K\ , \quad (6)$$

where $\alpha_k = \frac{b_1 - p_d^{1-\delta_{ak}}}{b_2}$. The function $f_k(q_k, q)$ is the reaction function of the agent *k*, because it expresses implicitly the dependence of the optimal action of the agent *k* on the actions of the environment.

We describe the leader appearance process for a social system consisting of two agents:

$$\begin{cases} f_1(q_1, q) = q_1(2 + \rho_{12}) + q_2 - \alpha_1 = 0, \\ f_2(q_2, q) = q_2(2 + \rho_{21}) + q_1 - \alpha_2 = 0. \end{cases} \quad (7)$$

The reaction functions may be expressed explicitly from system (7) as follows:

$$q_1 = \frac{\alpha_1 - q_2}{2 + \rho_{12}}, q_2 = \frac{\alpha_2 - q_1}{2 + \rho_{21}}. \quad (8)$$

If the second agent is not informed about the reaction function of the first agent, then, in accordance with the Cournot hypothesis, in the second equation of system (8), the conjectural variation is zero (i.e., $\rho_{21} = 0$), therefore this equation may be written in the form:

$$q_{2(F)} = \frac{\alpha_2 - q_1}{2}. \quad (8a)$$

In formula (8a), the index «*F*» is introduced for the second agent, because, according to the accepted assumption, he is the follower.

If at the same time the first agent is informed about the reaction function (8a) of the second agent, then he calculate the conjectural variation ρ_{12} as follows:

$$\rho_{12} = \frac{\partial a_2}{\partial a_1} = \frac{\partial\left(q_2 / n_2\right)}{\partial\left(q_1 / n_1\right)} = \frac{n_1}{n_2}\frac{\partial q_2}{\partial q_1} = -\frac{n_1}{2n_2}, \quad (9)$$

where the following relation is taken into account: $a_k = \frac{q_k}{n_k}$.
A substitution of formula (9) into the first equation of system (8) leads to the explicit reaction function of the second agent:

$$q_{1(L)} = \frac{\alpha_1 - q_2}{2 - \frac{n_1}{2n_2}}. \quad (10)$$

In formula (10), the index «*L*» is introduced for the first agent, because according to the accepted assumption, he is the leader.

Taking into account the introduced notation and the transformations, we write the reaction system (8) in the case of the first agent's leadership as follows:

$$q_L = \frac{\alpha_L - q_F}{2 - \frac{n_L}{2n_F}}, q_F = \frac{\alpha_F - q_L}{2}. \quad (11)$$

Thus, the process of the agents' stratification into leaders and followers proceeds in accordance with the sequence, which is demonstrated in Fig. 1.

Because, in the considered social system, the equilibrium action vector $\left(q_L^*, q_F^*\right)$ is defined as the intersection point of reactions (11) on the plane q_L, q_F, the ratio of equilibrium actions $\frac{q_L^*}{q_F^*}$ depends on the ratios of the slopes and free terms of reaction (11). An analysis of reactions (11) is illustrated in Fig. 2.

We introduce the relative indicators of the system state: η is the ratio of the leaders group number to the followers group number, β is the ratio of the constants in equations (6), μ is the ratio of the agents' type parameters. These indicators are calculated by using the following formulas:

$$\eta = \frac{n_L}{n_F}, \beta = \frac{\alpha_L}{\alpha_F}, \mu = \frac{\delta_{aL}}{\delta_{aF}}. \quad (12)$$

Given these notations, solving of system (11) allows us to write the following expressions of the Stackelberg equilibrium vector coordinates:

$$q_L^* = \frac{2\alpha_L - \alpha_F}{3 - \eta}, q_F^* = \frac{\left(2 - \frac{\eta}{2}\right)\alpha_F - \alpha_L}{3 - \eta}. \quad (13)$$

The Stackelberg equilibrium vector is indicated in Fig. 2 by a point E_S in contrast to the Cournot equilibrium vector, which is indicated by a point E_K.

The following assertion, the proof of which is placed in the appendix, defines the conditions for the equilibrium existence in the system.

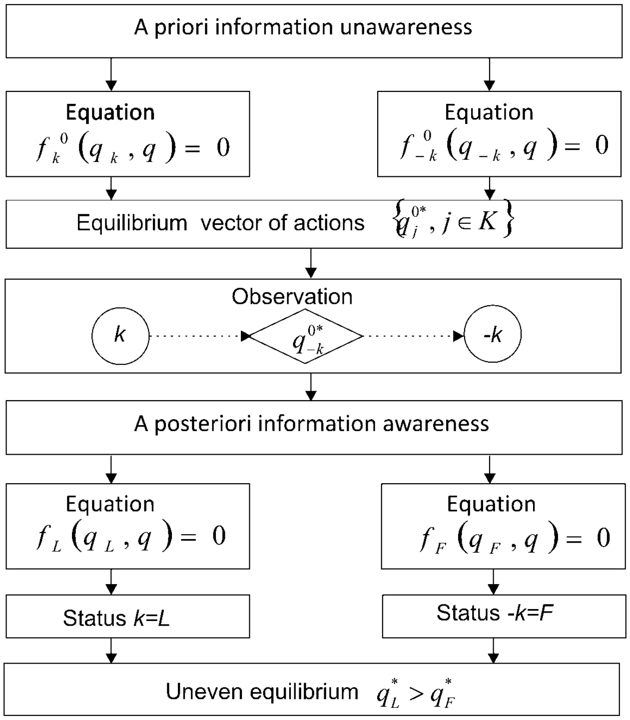

Fig. 1. Diagram of agents' stratification process.

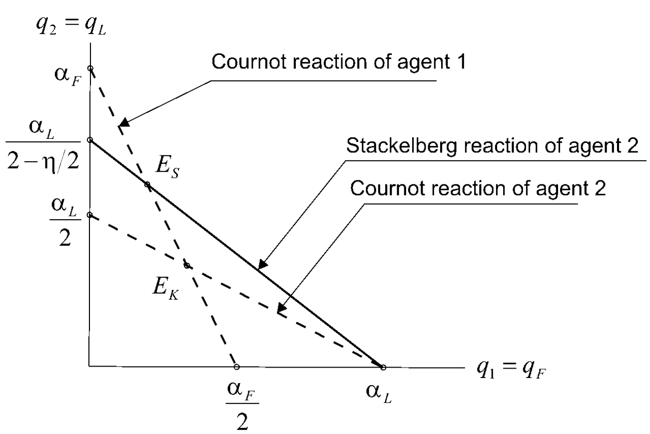

Fig. 2. Graphical analysis of equilibria in social system.

Assertion 1. The social system is in the equilibrium, i.e., the equilibrium actions are non-negative $q_L^* \ge 0 \wedge q_F^* \ge 0$, if the following conditions are satisfied:

$$\frac{1}{2} \le \beta \le 2 - \frac{\eta}{2} \forall \eta < 3, \text{ and } \frac{1}{2} \ge \beta \ge 2 - \frac{\eta}{2} \forall \eta > 3, \tag{14}$$

and if the approximating function of the following form $\beta = \mu^{\gamma}, 0 < \gamma << 1$ is exists, then conditions (14) have the form:

$$e^{\frac{\ln 0,5}{\gamma}} \le \mu \le e^{\frac{\ln(2-0,5\eta)}{\gamma}} \forall \eta < 3, \text{ and } e^{\frac{\ln 0,5}{\gamma}} \ge \mu \ge e^{\frac{\ln(2-0,5\eta)}{\gamma}} \forall \eta > 3. \tag{14a}$$

The various variants of the approximating function $\beta = \mu^{\gamma}$ are investigated in Fig. 3. With $p_d \ge 100$ and $\frac{b_1}{p_d} \ge 1{,}1$, the index of power has the following limitation $\gamma \le 0{,}3$.

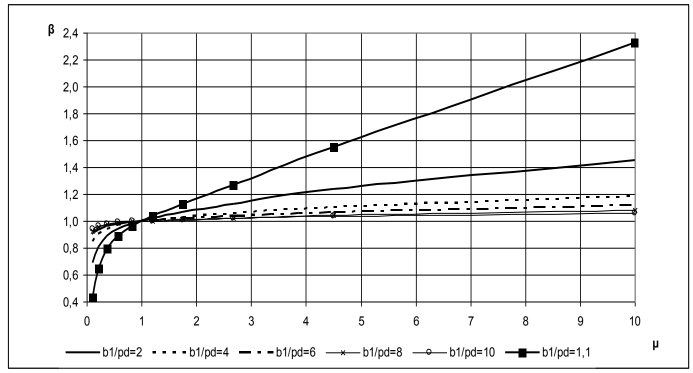

Fig. 3. Analysis of the approximating function $\beta = \mu^{\gamma}$.

On the basis of Assertion 1, we may derive the following practical conclusion.

Corollary 1: for the equilibrium existence in the social system, the number of the leaders should not exceed the number of the followers by more than 4 times.

We introduce the indicator of the equilibrium actions unevenness ξ^*, which is determined by the following formula:

$$\xi^* = q_L^* / q_F^*. \tag{15}$$

The following assertion, the proof of which is placed in the appendix, estimates the influence of the state parameters ratio on the equilibrium actions unevenness.

Assertion 2. In the social system, an increase in the ratio of the leaders number to the followers number η increases (decreases) the equilibrium actions unevenness for a given value of β according to the following rule:

$$\frac{\partial \xi^*}{\partial \eta} \begin{cases} > 0 \forall \beta > 0{,}5, \\ < 0 \forall \beta < 0{,}5; \end{cases} \tag{16a}$$

an increase in the ratio β increases (decreases) the equilibrium actions unevenness for a given value of η according to the following rule:

$$\frac{\partial \xi^*}{\partial \beta} \begin{cases} > 0 \forall \eta < 3, \\ < 0 \forall \eta > 3; \end{cases}. \tag{16b}$$

an increase in the factor η more (less) affects the change in the equilibrium actions unevenness than an increase in the factor β, under the following conditions:

$$\left|\frac{\partial \xi^*}{\partial \eta}\right| > \left|\frac{\partial \xi^*}{\partial \beta}\right| \; if \; \frac{|2\beta - 1|}{2} > |3 - \eta| \; and \; < \left|\frac{\partial \xi^*}{\partial \beta}\right| \; if \; \frac{|2\beta - 1|}{2} > |3 - \eta|. \tag{16c}$$

On the basis of Assertion 2, we formulate the following practical conclusions.

Corollary 2. In the social system

1) under the conditions $p_d \geq 100$ and $\frac{b_1}{p_d} \geq 1{,}1$, an increase in the leaders group number n_L in comparison with the followers group number n_F leads to a shift of the equilibrium actions unevenness towards the leaders, if the propensity to altruism of the followers exceeds this indicator of the leaders by more than 10 times (i.e., $\mu > 0{,}1$);

2) an increase in the leaders' propensity to altruism δ_L in comparison with this indicator of the followers δ_F leads to a shift of the equilibrium actions unevenness towards the leaders, if the number of the followers is more than 3 times the number of the leaders.

We simulate equilibrium (14) and sensitivity indicators (16) by an example of the social groups of Russian volunteers, the number of which in 2016 was 1.435 million, or about 1% of the population[3]. The volunteers were divided into 9 groups according to the propensity to altruism [7]. In our case, we divide the volunteers into 2 groups. The type parameters are calculated (Table 1) with the following constant values: D=112 hours per week, $p_d = 240$ rub. per hour. Into the leaders group (the second group), we combine the groups 2–9 from the article [7], because the numbers of these groups individually are small in comparison with the first group. The coefficients of the incentive function calculated by formulas (2a) are $b_1 = 284$, $b_2 = 0{,}0035$. The ratio of the leaders number to the followers number, calculated by formula (12), is η=0.44.

In this system, if the number of social groups is equal (i.e., when η=1), the Cournot equilibrium is shifted toward the second agent, which has the higher propensity to altruism (Fig. 4). The Stackelberg equilibrium at η=1 leads to greater unevenness toward the second agent (i.e., the leader), and at η=0.44 this equilibrium, on the contrary, shifts toward the first agent, the group of which has the predominant number. In all these cases, the aggregate equilibrium actions significantly exceeds the actual indicator A_0 (Table I), which is a consequence of the stimulation effect.

Fig. 5 illustrates features (16) of the Stackelberg equilibria. According to conditions (16a), with an increase in the ratio of the leaders number to the followers number η, in the case of $\beta < 0{,}5$, the equilibrium actions unevenness grows, and in the case of $\beta > 0{,}5$, the parameter ξ^* decreases. According to conditions (16b), in the case of η <3, an increase in the parameter β causes an increase in the equilibrium actions unevenness ξ^*, and in the case of η>3, this leads to a decrease in the parameter ξ^*. The values of the parameter ξ^* in the negative half-plane correspond to the case of non-existence of the equilibrium according to conditions (14) in the case of η<3 for $\beta < 0{,}5$ or for $\beta > 2 - \frac{\eta}{2}$, and in the case of η>3 for $\beta > 0{,}5$.

[3]Labor and Employment in Russia 2017: Stat. Sat. / Rosstat M., 2017. http://www.gks.ru/free_doc/doc_2017/trud_2017.pdf

TABLE I. CHARACTERISTICS OF SOCIAL GROUPS OF VOLUNTEERS IN 2016

Parameter	Total	Groups	
		1	*2*
Population n_k, thousand	1435	997	438
Average duration of volunteer activities per week a_k, hours	8,64	2.35	23.0
Aggregate duration of volunteer activities per week A_0, thousand hours	12398	2343	10055
Propensity to altruism δ_{ak}		0.18	0.66
Type parameter α		55168	80360

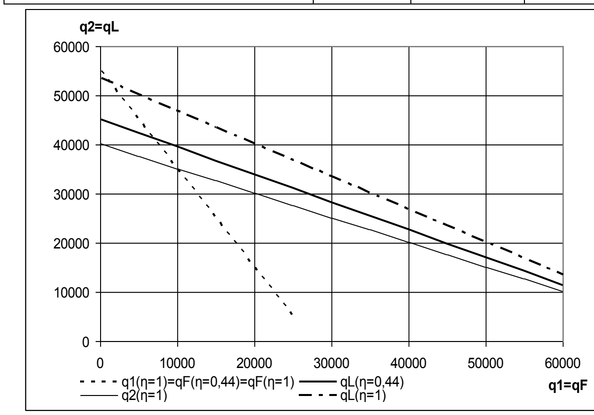

Fig. 4. Analysis of Cournot equilibrium and Stackelberg equilibrium.

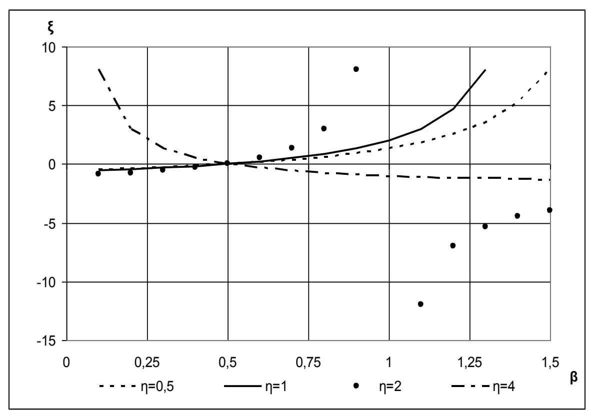

Fig. 5. Analysis of Stackelberg equilibrium sensitivity to state parameters.

IV. CONCLUSION

We investigate the behavior of the volunteers social groups. The study of the game-theoretic model in the framework of the Stackelberg game leads to the following conclusions. First, the equilibrium in the social system exists if the number of the leaders group does not exceed the number of the followers group by more than 4 times. Second, in the real conditions, an increase in the number of the leaders group in comparison with the number of the followers group leads to an increase in the equilibrium actions unevenness towards the leaders, if the followers' propensity to altruism exceeds this indicator of the leaders by more than 10 times. Third, an increase in the leaders' propensity to altruism in comparison with this indicator of the followers leads to an increase in the equilibrium actions unevenness towards the leaders if the number of the followers is more than 3 times the number of the leaders.

Proof of Assertion 1. Equilibrium (13) exists in the first orthant if $q_L^* = \frac{2\alpha_L - \alpha_F}{3-\eta} \geq 0 \wedge q_F^* = \frac{\left(2 - \frac{\eta}{2}\right)\alpha_F - \alpha_L}{3-\eta} \geq 0$; then, taking into account (12), we may write the following system of inequalities:

$$\frac{1}{2} \leq \beta \leq 2 - \frac{\eta}{2} \forall \eta < 3, \frac{1}{2} \geq \beta \geq 2 - \frac{\eta}{2} \forall \eta > 3 . \quad \text{(A1)}$$

Taking into account the notation (6), the ratio $\beta = \frac{\alpha_L}{\alpha_F}$ is associated with the ratio of the agents' type parameters as follows:

$$\beta = \frac{b_1 - p_d^{1-\delta_{aL}}}{b_1 - p_d^{1-\delta_{aF}}} . \quad \text{(A2)}$$

The ratio (A2) is the dependence on the ratio $\mu = \frac{\delta_{aL}}{\delta_{aF}}$. Taking into account that $b_1 > p_d$, according to (2a), and because $\mu \in [0{,}1;10]$ taking into account (1), dependence (A2), as shown by the numerical experiment in Fig. 3, may be approximated by the following function:

$$\beta = \mu^{\gamma} \forall \mu \in [0{,}1;10], 0 < \gamma << 1 . \quad \text{(A3)}$$

In the case of approximation (A3), inequalities (A1) may be written in the form (14a).

Proof of Corollary 1. It follows from formulas (13) that for η=3 the equilibrium is not defined. It follows from formulas (14a) that $2 - \frac{\eta}{2} > 0$, because the logarithm function is defined only with a positive sub-logarithmic expression. Therefore, there is the limitation $\eta < 4$.

Proof of Assertion 2. A substitution of formulas (14) into formula (15) and transformation taking into account formulas (12) allows us to obtain the following expression:

$$\xi^* = \frac{q_L^*}{q_F^*} = \frac{2\alpha_L - \alpha_F}{\left(2 - \frac{\eta}{2}\right)\alpha_F - \alpha_L} = \frac{2\frac{\alpha_L}{\alpha_F} - 1}{2 - \frac{\eta}{2} - \frac{\alpha_L}{\alpha_F}} = \frac{2\beta - 1}{2 - \frac{\eta}{2} - \beta} . \quad \text{(A4)}$$

A differentiation (A4) with respect to η leads to the expression $\frac{\partial \xi^*}{\partial \eta} = \frac{2\beta - 1}{2\left(2 - \frac{\eta}{2} - \beta\right)^2}$, from which inequality (16a) follows. A differentiation (A4) with respect to β leads to the expression $\frac{\partial \xi^*}{\partial \beta} = \frac{3-\eta}{\left(2 - \frac{\eta}{2} - \beta\right)^2}$, from which inequality (16b) follows. A comparison of the modulus of these expressions demonstrates that $\left|\frac{\partial \xi^*}{\partial \eta}\right| > \left|\frac{\partial \xi^*}{\partial \beta}\right|$ if $\frac{|2\beta - 1|}{2} > |3 - \eta|$, therefore, we write conditions (16c).

Proof of Corollary 2. As an analysis of the approximating function $\beta = \mu^{\gamma}$ demonstrates (Fig. 3), for $p_d \geq 100$ and $\frac{b_1}{p_d} \geq 1{,}1$ there is a restriction $\gamma \leq 0{,}3$. A comparison of inequalities (14) and (14a) leads to the conclusion that $\beta \geq 0{,}5 \Leftrightarrow \mu \geq e^{\frac{\ln 0{,}5}{\gamma}} \forall \eta < 3$ and $\beta \leq 0{,}5 \Leftrightarrow \mu \leq e^{\frac{\ln 0{,}5}{\gamma}} \forall \eta > 3$. Because $e^{\frac{\ln 0{,}5}{0{,}3}} = 0{,}1$, it follows from formula (16a) that $\frac{\partial \xi^*}{\partial \eta}\begin{cases} > 0 \forall \mu > 0{,}1, \\ < 0 \forall \mu < 0{,}1, \end{cases}$ i.e. the first part of the corollary is correct. The second part is derived from formula (16b), in which we replace $\frac{\partial \xi^*}{\partial \beta}$ to $\frac{\partial \xi^*}{\partial \mu}$, because if $\frac{\partial \xi^*}{\partial \beta} > 0$, then $\frac{\partial \xi^*}{\partial \mu} > 0$.

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