Descriptive model of temporal features of multivariate time series based on granulation

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***Abstract*—Modern systems are characterized by high rates and volumes of receipt of numerical data. The number of indicators of economical, biological, and technical systems, including Autonomous ones, is increasing, generating large amounts of numerical data of observation in real time. These data have a multidimensional structure and binding to time points, which allows us to consider them in the form of numerical multivariate time series. As part of the descriptive analysis of these data, the article presents new model of representation of local features, considered at different levels of granulation, in respect to temporal features of a multivariate time series in terms of general tendencies. For this purpose, the provisions of the theory of fuzzy sets and fuzzy time series were applied in descriptive model, which provided a linguistic description of tendencies, understandable to the expert. Carried out results in modelling of local feature in terms of tendency in descriptive analysis of COVID-19 spread showed effectiveness and operability of proposed approach.**

Keywords—multivariate time series, fuzzy time series, granulation, general tendency

I. Introduction

Data sets of numerical data in the form of numerical multidimensional time series (MTS), describing the behavior of complex objects, are a source of hidden knowledge necessary when analyzing the feature of processes in many applied systems, including telecommunications, industry, healthcare, meteorology, biology, sociology, public administration, medicine, computer networks and financial applications. By feature we mean a characteristic, distinctive property of object that distinguishes it from other objects or determines its similarity with other objects. The specified semantics of feature of objects allows us to distinguish two ways in the analysis of features of objects: analysis of the features of an individual object and analysis of the features of a set of objects. In each of these areas, one can formulate a typical set of stages of the analysis of features, such as descriptive, diagnostic, predictive, prescriptive, and cognitive analysis. In this case, descriptive (descriptor) analysis is the first stage that determines the effectiveness of subsequent stages of the analysis of objects.

The main task of descriptive analysis of MTS can be considered as the task of extracting, describing and object-oriented interpretation of its features observed in a given time interval, and is to answer the question "What happened?". The MTS data structure is complex. Therefore, when extracting and analyzing the features of such structures, it is advisable to consider MTS in various aspects both as a separate complex object with global (integrative) features, and as a set of one-dimensional time series (TS) forming it. At the same time, the one-dimensional TS can also be described on the basis of its global, local and temporal features. This allows us to consider the features of MTS from the point of view of global and local granules obtained by granulating MTS at the micro and macro levels determined by the consideration aspect.

Usually, the representation of features is considered as a set (or vector) of numerical attributes, each of which numerically summarizes a separate feature of a one-dimensional time series (TS). This representation does not take into account the features of the two-dimensional structure of the MTS, which allows one to extract more complex structures in the form of micro and macro granules and on this basis describe its local and global features of temporal patterns, local and global tendencies, fuzzy and associative rules. In this study, granulation refers to the automatic processing of MTS to extract features aimed at understanding its behavior, according to the approach of R. Yager and J. Kacprzyk [1]. The granular presentation of MTS will allow describing its features within the part of one methodological basis, will reduce the dimension of MTS, develop new methods for their classifying, predicting, clustering, and on this basis, deepen scientific knowledge in a subject-oriented field.

Considering MTS as the object of descriptive analysis, it should be noted that linguistic interpretation of the extracted granules representing the temporal features of MTS is most required for domain experts. Such a linguistic interpretation can be obtained by combining domain-specific knowledge in the field of MTS analysis and fuzzy models integrating numerical and linguistic values. The use of fuzzy models is caused, on the one hand, by the need to represent temporal features MTS that contain inaccuracies and distortions, and, on the other hand, by the ability to obtain interpreted information granules. This is in demand by domain experts, analysts, and intellectual assistants to select and apply adequate models in the subsequent stages of the analysis of complex objects presented by MTS.

The goal of the paper is to develop a descriptive model for mining and representing temporal features of MTS based on fuzzy time series, granulation and tendency.

II. Related works

Features of MTS are usually represented as numerical characteristics by mapping to a low-dimensional feature space using various transforms, such as locality preserving projections (LPP)[2], which preserves the nearest neighbor relation, singular value decomposition (SVD)[3], and multidimensional wavelet transforms [4]. However, the attributes thus obtained may not have a semantic interpretation and may not express the inherent features of MTS behavior. Chris Aldrich shows the challenges and makes a review of approaches to extracting the MTS features in the problem of defect detection in real dynamic systems based on principal component analysis (PCA) [5]. The author considers classical approaches for extracting features with respect to MTS, considered as a sequence of images. However, the characteristics obtained in these approaches are global, which usually lose local data characteristics.

In order to extract local features from the MTS data, some scientists are expanding the methods of representing the features of time series by a combination of shapelets of various variables [6] to generate associative rules and in the tasks of early classifying of MTS. Note that this approach does not take into account the interpretation of the behavioral characteristics in terms of tendencies of MTS. In addition, studies in the work [7] have shown that there are deviations between the extracted shapelets and the essential features of MTS, therefore, the shapelets cannot fully express the essential characteristics of multidimensional time-series data.

The application of fuzzy transformations and rules for extracting static features of TS was considered in [8–10]. It uses the numerical characteristics of TS, such as average, variation, minimum and range, which are too common for TS. Granulation methods, which are based on the theory of fuzzy sets [11], are used in TS analysis and decision making [12-15]. A scaling and granulation of linear trend patterns using fuzzy models for producing interpretable TS segments in different aspects of perception based time series data mining were discussed by I. Batyrshin and L. Sheremetov in the work [14]. In the problem of representing features of TS by granules in terms of fuzzy values and tendencies was studied and applied in software engineering domains [12, 13].

The book [15] notes that TS granulation is the most adequate method for extracting TS features in the temporal and spatial aspects. Another interesting approach is related to the clustering of granules represented in symbolic form. The granular representation of TS was studied in the prediction problem in the work [16-17]. The application of linguistic summary to granular data is given in [1] as a method of granulation of quantifiers in propositions. An algorithm for finding intervals of monotonous behavior of TS was suggested in [18] and then approach to automatic summarization of information on time series based on intermediate quantifiers (a constituent of fuzzy natural logic) and generalized Aristotle's syllogisms was showed.

The analysis of the current state of the MTS features representation area in the MTS granulation problem for extracting local features allows us to draw the following conclusion. Models for representing features of MTS are under development, while the fuzzy models are a promising approach due their opportunity to give linguistic interpretation of features in different levels of TS granulation.

III. Descriptive model of features of MTS temporal behavior

We consider MTS as an abstract object, representing some observation of a set of changing characteristics of process or of object, of which we do not know anything and assume some noise in their values. Changing characteristics of a process or of object represented by one-dimensional TS and their properties could be considered from different points of view, consequently they may have different interpretation associated with domain semantics. Therefore, before conducting a diagnostic or predictive analysis for them, it is necessary to find out and to describe their temporal properties, which we will call features. In our study, we focus on one point of view for all TS of MTS, which consists in its temporal features, presented by a linguistic description of the TS tendency extracted using fuzzy TS [19]. This approach corresponds to provisions of the work [14]. We apply fuzzy representation of TS to deal with uncertainty in data produced by noise and to create meaningful granules in linguistic form of MTS behavior. That description for each TS represents its global feature and, at the same time, the local MTS feature. In a sense, such a representation of the temporal properties of MTS corresponds to the result of visual analysis of MTS by an expert. We consider the summarization of the set of these local features as a global feature of MTS.

Let $X = (x_{tj}), (j = 1,2, \dots, m; t = 1,2, \dots, n)$ be numeric MTS. Here j is index of one-dimensional TS, m is number of one-dimensional numeric TS in MTS and n is number of observations.

To represent the local feature of MTS, we use the «behavior» characteristic with respect to the general tendency of the j-th TS, for which this feature will be global.

To describe the global feature of «behavior» for one-dimensional TS $x_{tj} \in X$ we use the concept of general tendency [16] introduced for fuzzy TS [19], where fuzzy TS is understood as TS, the levels (values) of which are presented by fuzzy sets forming some linguistic variable $\tilde{Z} = \{\tilde{z}_i | i = 1,2, \dots, r, r < n\}$ [20, 22]. This linguistic variable should be built on the set of admissible values of W of each numerical one-dimensional TS x_{tj}. It is assumed that the indices i of fuzzy labels $\tilde{z}_i$ correspond to partially ordered intervals on W, that are carriers of fuzzy labels $\tilde{z}_i$.

Definition 1. The general tendency (GT) of a one-dimensional TS x_{tj} is a linguistic label $y \in Y$, $Y = \{\text{Stability}, \text{Growth}, \text{Fall}, \text{Systematic Fluctuations}, \text{Chaotic Fluctuations}\}$ expressing its temporal behavior in total.

We assume that general tendencies $'\text{Growth}'$, $'\text{Fall}'$ and $'\text{Chaotic Fluctuation}'$ correspond to non-stationary behavior of TS, while ‘Systematic Fluctuation’ and ‘Stability’ characterize in some sense its stationary property. Representation of TS behavior in the form of GT terms is common to all one-dimensional TS and provides additional knowledge about temporal changes, useful both for experts and for automation further analysis. Therefore, in this study the local features of MTS are considered as the set of the above linguistic terms related to each numerical one-dimensional TS $x_{tj} \in X$.

In real application the set of labels for linguistic describing TS general tendency could be expanded by new ones or reduced as well.

Definition 2. The general linear tendency of TS is a linguistic label $y \in Y, Y = \{\text{Stability}, \text{Growth}, \text{Fall}\}$ expressing its temporal behavior in total. Below we suggest the following designation of GT $\text{Y} = \{y_k | k = 1,2, \dots, kn\}$, where kn is equal to quantity of GT labels.

Definition 3. The global feature of the j-th TS $x_{tj} \in X$, characterizing its behavior on the interval $t = 1,2, \dots, n$ by GT, is kn-dimensional vector $H_j = (h_{jk}), \Big(k = 1,2, \dots, kn,$

where $h_{jk} = \mu_{y_k}(x_{tj}))$, where $\mu_{y_k}(x_{tj})$ denotes a degree of belonging TS x_{tj} to $y_k \in Y$.

Then, to determine TS global feature in terms of GT the membership degree of TS x_{tj} to y_k should be calculated. Since there is challenge to create membership functions for linguistic terms in Y in next Section we propose the technique of micro and macro granulating to obtain TS global feature and calculate the degrees of belonging $h_{jk} = \mu_{y_k}(x_{tj})$.

Definition 4. The descriptive model of local feature of MTS X presented in linguistic terms of GT is the set of TS global features, presented by following expression:

$$Loc(X) = \{L_j, \mu_j | j = 1,2,\dots,m\}, \quad (1)$$

$$L_j = y_c, \mu_j = h_{jc}, c = argmax_{k=1,2,\dots,kn}(h_{jk}). \quad (2)$$

Here $L_j \in Y$ denotes linguistic label having maximal membership degree μ_j among other labels $y_k \in Y$, and c is the number of this label.

The proposed descriptive model of local feature of GT represents its behavior generically and concisely and makes it possible to use it in a diagnostic, predictive and prescriptive analysis of the underlying process or object. At the descriptive stage, the frequency analysis of linguistic labels in $Loc(X)$ may provide the knowledge about global feature of MTS temporal behavior in general.

Using this approach, the MTS global features could be extracted in respect to stationary or non-stationary MTS temporal behavior. Also, such summing propositions could be formed as "In MTS all tendencies referred to Fall", "In MTS less than half tendencies referred to Chaotic Fluctuation" and others to describe temporal changes in MTS using general tendency. The techniques of such summarization were considered in [1,18, 21].

Based on the introduced concepts of local and global features, we define a process of descriptive modeling of MTS temporal behavior in terms of GT by following sequence of expressions:

$$x_{tj} = f1(X), \quad (3)$$

$$\tilde{X}_{jt} = f2(x_{tj}, \tilde{Z}), \quad (4)$$

$$Loc(X) = f3(\tilde{X}_{tj}, Y), \quad (5)$$

$$L = f4(L_j). \quad (6)$$

In this descriptive model (3-6), transformations $f1$ and $f2$ refer to micro granulation of numeric MTS, and the result of the transformation (4) is a fuzzy time series $\tilde{X}_{tj}$, obtained for a one-dimensional j-th TS $x_{tj} \in X$. Micro granulation is considered as the process of creating small granules by decomposing MTS into components. In this case, the relationship of "fragmentation" between the MTS and its micro granules, is established. Macro granulation establishes the "generalization" relation and is represented by transformations $f3$ and $f4$, which form larger granules characterizing of MTS temporal behavior in the form of its local and global features in terms of GT. Based on this descriptive model, knowledge about the local and global features of MTS, characterizing its behavior, is extracted. This knowledge is expressed in a concise linguistic form, understandable to the expert and useful for methods of diagnostic, predictive, prescriptive and cognitive analysis.

A. Micro granulation in MTS

Let us consider micro granulation of numeric MTS as the process of transforming its set of one-dimensional numerical TS into fuzzy TS according to expression (4). We denote some one-dimensional numeric TS included in the MTS as follows:

$$\{x_t | x_t \in W, W \subseteq \mathbb{R}, t = 1,2,\dots,n\}. \quad (7)$$

Suppose that a linguistic variable $\tilde{Z}$ [20, 22] is created on the set W (domain of TS values) with r linguistic terms:

$$\tilde{Z} = \{\tilde{z}_i | i = 1,2,\dots,r, r < n\}. \quad (8)$$

Note the number of generated fuzzy terms r of linguistic variable $\tilde{Z}$ for each TS could be set by an expert or determine automatically.

We assume the set W is covered by partially ordered intervals and each linguistic term $\tilde{z}_i \in \tilde{Z}$ is constrained by its corresponding interval.

To convert a numerical TS x_t into a fuzzy TS $\tilde{X}_t$, we use the NFLX-transforming TS («conversion from numeric to fuzzy linguistic» values) according to expression (4) as was described in [22]:

$$NLFX: \{x_t | t = 1,2,\dots,n\} \mapsto \{\tilde{X}_t | t = 1,2,\dots,n\}, \quad (9)$$

The fuzzy TS $\tilde{X}_t$ is formed as follows:

$$\mu_{\tilde{x}_t}(x_t) = max_{i=1,2,\dots,r}(\mu_{\tilde{z}_i}(x_t), \ s \in \{1,2,\dots,r\}, \quad (10)$$

$$\tilde{x}_t = \tilde{z}_s, \ s = argmax_{i=1,2,\dots,r}(\mu_{\tilde{z}_i}(x_t)), \quad (11)$$

$$\tilde{X}_t = \{\tilde{x}_t, \mu_{\tilde{x}_t}(x_t) | t = 1,2,\dots,n\}, \quad (12)$$

where $\tilde{x}_t$ is a linguistic term equal to a linguistic term $\tilde{z}_s$ with a maximum degree of membership for TS at time *t*, s is the number of this linguistic term, and $\mu_{\tilde{x}_t}(x_t)$ is the degree of belonging x_t to this linguistic term at time *t*.

In that way the fuzzy values of numeric TS are formed using a linguistic variable $\tilde{Z}$, the fuzzy terms of the latter are ordered by increasing their indices i (according to assumptions about the linguistic variable).

Then the values of two neighboring fuzzy values $\tilde{x}_t$ and $\tilde{x}_{t-1}$ in fuzzy TS may be represented by linguistic labels as follows for: $t = 2,3,\dots,n$:

$$\tilde{x}_{t-1} = \tilde{z}_{s(t-1)}, \quad (13)$$

$$\tilde{x}_t = \tilde{z}_{v(t)}, \quad (14)$$

where $s(t-1)$ and $v(t)$ denote the indices of fuzzy labels of the linguistic variable $\tilde{Z}$, associated with time instants $(t-1)$ and t respectively.

Since fuzzy terms in the linguistic variable $\tilde{Z}$ are ordered by indices (according to the assumptions about the linguistic variable), we use these indices to determine the intensity of change for two neighboring fuzzy values of fuzzy TS in direction of their increasing and decreasing. We suppose that between two neighboring fuzzy values there can also be no changes. Taking in account the expressions (13) and (14), the intensity of change of two neighboring fuzzy values in fuzzy TS for observation t is presented as:

$$\alpha_t = v(t) - s(t-1),\ t = 2,3,\dots,n. \quad (15)$$

Thus, at the stage of micro granulation of MTS, for each value of TS, we obtain the degree of its belonging $\mu_{\tilde{x}_t}(x_t)$, the corresponding linguistic label $\tilde{x}_t$, and the intensity of changes in neighboring values α_t.

B. Macro granulation in MTS

Macro granulation is considered as process of combining micro granules into larger ones, obtained by expressions (1) an (2). Using the proposed MTS descriptive model, macro granulation is considered according to expression (3) to produce local features of MTS temporal behavior in terms of GT. Since determine the membership functions μ_{y_k} for linguistic terms of variable Y is challenge we propose the approach to calculate the degree of belonging TS $x_{tj} \in X$ to each $y_k \in Y$.

In this Section the technique for assessing GT L_j as global characteristic of each numeric TS $x_{tj} \in X$ is presented using fuzzy TS (see expression (12)), the indices of its two neighboring fuzzy values and the set of linguistic labels

$Y = \{$Stability, Growth, Fall, Systematic Fluctuation, Chaotic Fluctuation$\}$.

The task is to determine GT $L_j \in Y$ (see definition 3) for TS which is presented by fuzzy TS using linguistic variable $\tilde{Z}$ and to describe the local feature of MTS in respect to definition 4. Consequently, the membership degrees $\mu_{y_k}(x_{tj})$ of TS x_{tj} to $y_k \in Y, k = 1,2,..,5$ should be calculated.

For this purpose, we suggest rule-based technique of assessing local features of MTS in terms of GT which includes following steps:

Step 1. Pre-processing.

Step 1.1. Micro granulation of MTS according to expressions (7-15) and consideration j-th TS $x_t \in X$.

Step 1.2. Based on the values $\alpha_t, t = 2,3,\dots,n$, calculated according to expression (15), for a TS x_t determining its total intensities of changes for growth and for fall:

$$For\ t = 2,3,\dots,n$$

$$If\ \alpha_t > 0, then\ \alpha_{growth} = \alpha_{growth} + abs(\alpha_t),$$
$$If\ \alpha_t < 0, then\ \alpha_{fall} = \alpha_{fall} + abs(\alpha_t).$$

Step 1.3. Initialization of membership degrees for all linguistic labels in Y:

$$For\ k = 1,2,\dots,5\ \ \mu_{y_k}(x_t) = 0.$$

Step 2. Assessing membership degrees and linguistic labels of global feature of TS temporal behavior in GT terms.

Step 2.1. If $(\alpha_{growth} = 0$ and $\alpha_{fall} = 0)$, then $y_1 = $ '$Stability$', $\mu_{y_1}(x_t) = 1$,

Step 2.2. If $\alpha_{growth} > 2 * \alpha_{fall}$, then $y_2 = $ '$Growth$', $\mu_{y_1}(x_t) = 0$, $\mu_{y_2}(x_t) = \frac{\alpha_{growth}}{(r-1)*(n-1)}, \mu_{y_3}(x_t) = \frac{\alpha_{fall}}{(r-1)*(n-1)}$,

Step 2.3. If $\alpha_{fall} > 2 * \alpha_{growth}$, then $y_3 = $ '$Fall$', $\mu_{y_1}(x_t) = 0, \mu_{y_2}(x_t) = \frac{\alpha_{growth}}{(r-1)*(n-1)}, \mu_{y_3}(x_t) = \frac{\alpha_{fall}}{(r-1)*(n-1)}$,

Step 2.4. If $(0{,}85 * \alpha_{fall} < \alpha_{growth} < 1{,}15 * \alpha_{fall})$ or $(0{,}85 * \alpha_{growth} < \alpha_{fall} < 1{,}15 * \alpha_{growth})$, then $y_4 = $ '$Systematic\ Fluctuation$', $\mu_{y_1}(x_t) = 0,\ \mu_{y_2}(x_t) = \frac{\alpha_{growth}}{(r-1)*(n-1)}, \mu_{y_3}(x_t) = \frac{\alpha_{fall}}{(r-1)*(n-1)}$, $\mu_{y_4}(x_t) = 1$, else $y_5 = $ '$Chaotic$ Fluctuation', $\mu_{y_1}(x_t) = 0, \mu_{y_2}(x_t) = \frac{\alpha_{growth}}{(r-1)*(n-1)}, \mu_{y_3}(x_t) = \frac{\alpha_{fall}}{(r-1)*(n-1)}$, $\mu_{y_2}(x_t) = \frac{\alpha_{growth}}{(r-1)*(n-1)},\ \mu_{y_3}(x_t) = \frac{\alpha_{fall}}{(r-1)*(n-1)},\ \mu_{y_4}(x_t) = 0$, $\mu_{y_5}(x_t) = 1$.

Step 3. Determining global feature of TS in terms of GT.

Step 3.1. Calculating the index of linguistic label for TS with maxim membership degree:

$$c = \underset{k=1,2,\dots,5}{argmax}\{\mu_{y_k}(x_t)\}, \mu_j = \mu_{y_c}(x_t).$$

Step 3.2. Determining the linguistic term of GT of *j*-th TS x_t:

$$L_j = y_c.$$

Step 4. Repeat Steps 1-3 for m TS of MTS and determine its local feature:

$$Loc(X) = \{L_j, \mu_j | j = 1,2,\dots,m\}.$$

IV. Descriptive modeling of COVID-19 using granulation and general tendencies

To illustrate the practical application of the proposed model of local feature of MTS in terms of GT, let us consider an example of descriptive analysis of MTS formed by COVID-19 [23] indicators observed in the local territorial region to understand how a pandemic spreads there. Given that the nature and behavior of COVID-19 is poorly understood, and many countries have different policies regarding the intensity and management of quarantine activities, many researchers and ordinary people are interested in the question of when and by what signs it can be judged that the activity of COVID-19 is reduced.

Most researchers suggest evaluating tendencies in COVID-19 prevalence rates [24]. Considering the tendencies in TS of the indicators of this pandemic over some temporal interval, it is possible to make decision and informed recommendations on the weakening of quarantine measures. In our study, the MTS characterizing COVID-19 spreading is defined by a set of TS that represent daily changes in the total number of detected cases of infection (Sv), the total number of patients recovered (Sr) and the total number of patients who died (Sd). As an example of descriptive analysis, we focus on analyzing, extracting and interpretation the tendencies of such MTS, which describe the prevalence of COVID-19 in the city of Moscow of Russian Federation from March 26, 2020 to May 3, 2020 [25].

Using micro and macro granulation of MTS, we extract the global features of its indicators and describe the local feature of COVID-19 activity in terms of the GT with meaningful interpretation. These features, expressed linguistically, will be focused on summarizing the dynamics of COVID-19 spread and the dynamics characterizing to some extent the formation of collective immunity. For this, we use, based on the main indicators presented at [25], the new ones grouped into two types: (1) characteristics of the spread of COVID-19 and (2) characteristics of patient recovery.

In the experimental study of descriptive analysis of COVID-19 activity in Moscow the following variables and indicators were used:

t – this is number of daily observation, $t = 1,2,\ldots,39$.

$Sv(t)$, $Sr(t)$, $Sr(t)$ describe the total number of cases per day of infection, recovery cases and death, respectively.

$Sa(t)$ is TS of the total number of active cases, $Sa(t) = Sv(t) - Sr(t) - Sd(t)$.

$Sn(t)$ designates the daily total number of new infections: $Sn(t) = Sv(t) - Sv(t-1)$.

$n(t)$, $r(t)$, $a(t)$ present the number per day fixed of new cases of infection, recovery and active, respectively. The increase in TS of daily infections, deaths, and active infections indicates a negative trend, while the fluctuation trend can be interpreted as a sign of a transition to a positive trend. The downward trend in $n(t)$ and $a(t)$ will show a positive trend. It is understood that the increase in the number of recovered patients is a good trend.

$Kv(t) = Sa(t)/Sv(t)$ determines TS of proportion of total active cases in relation to all cases of infection. A decrease in this fraction indicates that the distribution activity of COVID-19 is reduced. This indicates a positive trend.

$Kr(t) = Sr(t)/Sa(t)$ – this is TS of proportion of total cases of recovery in relation to active cases. The growth of this share shows that the number of ill and received immunity increases, which is positive in terms of the formation of collective immunity.

$Ks(t) = Sa(t+p)/Sa(t)$ is the coefficient of the delayed effect of total active cases $Sa(t)$ per day with number t on the total active cases that occur by the end of the incubation period $Sa(t+p)$ (according to WHO [23], the duration of incubation period p can be up to 14 days). A decrease in this coefficient indicates that the activity of infection from active cases is reduced. This indicates a positive trend.

$Kn(t) = Sn(t+p)/Sa(t)$ is the coefficient of the delayed effect of the total active cases of $Sa(t)$ per day with number t on the total new cases of $Sn(t+p)$ that occur at the end of the incubation period. A decrease values in this coefficient indicates that the activity of infection from active cases is reduced. This indicates a positive trend.

$Ka(t) = a(t+p)/a(t)$ determines the coefficient of the delayed effect of daily recorded active cases $a(t)$ per day with number t on the occurrence of daily recorded active cases $a(t+p)$ that occur at the end of the incubation period. A decrease in this coefficient indicates that the activity of infection from active cases is reduced. This indicates a positive trend.

To our mind introduced above indicators are necessary in order to be able, on the one hand, to extract additional information about the positive or negative dynamics of COVID-19 activity, and on the other hand, in order to be able to construct linguistic variables and fuzzy sets on intervals of values, which are necessary for the automatic determination of tendencies. Based on the introduced indicators, for descriptive analysis of the dynamics of COVID-19 activity in Moscow, the following MTS was formed:

$$X = \{ r(t), Kr(t), n(t), a(t), Kv(t), Ks(t), Kn(t), Ka(t)\}.$$

To extract its micro granules in the form of fuzzy TS values, the NFLX-transform was used. For this purpose, a preliminary linguistic variable $\tilde{Z}$ with ten fuzzy sets was determined for each of the eight time series included in the MTS. When modeling fuzzy terms, triangular membership functions were used, which were built on partially ordered intervals of the same length. The universal set of each linguistic variable was determined on the basis of an extended range between the maximum and minimum values of each derived indicator, as described in the work [26]. At the stage of macro granulation, to each component of the MTS, the linguistic characteristic of its GT was determined, which made it possible to determine the local feature of the analyzed MTS, presented in Table 1.

The data from table 1 show that 75% of the trends in the dynamics of COVID-19 activity in Moscow are positive according to the descriptive model of local features of MTS. It can be noted that according to the indicators characterizing the recovery of patients in this study, all trends are positive.

TABLE I. RESULTS OF A DESCRIPTIVE ANALYSIS OF MTS BY GT, CHARACTERIZING THE DYNAMIC OF COVID-19 ACTIVITY IN MOSCOW

MTS	Linguistic label of GT	Interpretation of GT
$r(t)$	Growth	Positive
$Kr(t)$	Growth	Positive
$n(t)$	Growth	Negative
$a(t)$	Growth	Negative
$Kv(t)$	Fall	Positive
$Ks(t)$	Fall	Positive
$Kn(t)$	Fall	Positive
$Ka(t)$	Fall	Positive

According to the last column of Table 1, we can conclude that in Moscow by May 3, 2020, only 67% of the distribution indicators of COVID-19 had a positive trend. Negative dynamics trends were observed in the rates of new and active cases of COVID-19 infection recorded daily. It can be assumed that this is due to several reasons, among which should be noted an increase in the number of tests conducted in Moscow. To clarify this, it is necessary to conduct an additional analysis, which may be the subject of a new study.

V. CONCLUSION

The authors propose an approach to descriptive modelling of local feature of MTS that characterizes its behavior in terms of general tendency. The positions and the descriptor model of the MTS, as well as expressions, allowing to generate a linguistic description of its local feature, are considered. The proposed approach is characterized by the use of granulation tools MTS, fuzzy TS and concept of general tendency, which allows you to extract interpretable knowledge that is useful for further analysis of behavior of processes and objects. Application of the proposed model of local feature in terms of GT in descriptive analysis of MTS of COVID-19 spread in Moscow showed its effectiveness and operability while automatically monitoring the situation. Moreover, the obtained knowledge is useful in making decision corresponding to decline quarantine activity.

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