The General Dynamic Market Model and Software Application for Support Modeling Process

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Abstract. Economics has entered the stage of deep transformation of its bases. The traditional method of constructing a scientific theory is first to synthesize and investigate mathematical framework; this traditional approach was taken as a principle of our research. Finally the mathematical theory of the general dynamic market model has recently been constructed, the main elements of which are given in this paper. The next step is to build models of specific real markets based on the general theory. The C# application *Model* was created especially to support the synthesis of concrete models based on the general theory. The most important goal of this paper is to propose cooperation in such research.

Results of the research: the crucial factors, which ensure the market stability, are the market coherence and the market intention to adaptive expectations. If no any firm uses naive expectations in the market then with sufficiently small incoherence there is unique Nash equilibrium, which is stable for all acceptable values of parameters. The increase of naive expectations leads to stability loss, to flip bifurcations and finally to chaos. The increase of number of firms also as a rule leads to stability loss and finally to chaos. At sufficiently small changes in production per step, systems of general dynamic market model turns into systems of neoclassical microeconomics.

Keywords: modeling, computer simulation, C# desktop application, dynamic, economics, general market model, adaptive expectations.

1 Introduction

Increasingly, processes and systems are researched or developed through computer simulations and this trend is likely to continue [1]. Information technology in the economy made it possible to model artificial societies and study economic models through the computer simulation [2]. Economics has entered the stage of deep transformation of its bases. In recent years the researchers are renouncing the assumption of perfect rationality as unconditional basis of economic agents' behavior [3], [4].

The real economic processes make a clear demonstration that neoclassical "rational man" is not their subject. In real economy "optimal imperfect decisions" are taken by simple and non-expensive calculations, well adapted to frequent repetitions, to evolution [5]. All it means that the real economy is dynamical system, and real processes of economy are iterative processes of this system.

Now institutional school of economics analyzes economic systems as a result of evolutionary process of participants' interaction [6]. New paradigm of economics is a mix of the nonlinear dynamical system theory and mathematical programming, including game theory and optimal control theory [7]. And the main tool of new economics is simulation modeling grounded on the basis of 3 computer paradigms (object-oriented, dynamic and multi-agent system) [8].

Modern development of dynamic paradigm in economics is a wide stream of researches. However it is a stream of examples which are not developing in the general theory; their relations with real markets are often problematic [9]. The traditional method of constructing a scientific theory is first to synthesize and investigate mathematical framework – the general market model according to the new dynamic paradigm of economics. And then we can study complex real systems which are grounded on this basis. This traditional approach was taken as a principle of our research.

Indeed, the idea of this research arose in the course of computational experiments with the two-dimensional market model [10], [11], its particular case, which became the foundation of the general theory. The general market model was synthesized in [12]. A computational study of the model was demonstrated there, and on this basis some hypotheses about the general theory, formalizing the calculation results, were formulated. Finally, the mathematical theory of the general dynamic market model has recently been constructed, the main elements of which are given in this paper.

The next step is to build models of specific real markets based on the general theory. In this direction, we have just started: we explored the Australian retailer market, based on the general market model. For this purpose Data Republic technologies were used, which provided comprehensive initial data for research, as well as new methods of machine learning [13]. But hardly such a task can be realized by one person. The most important goal of this paper is to propose cooperation in future research. For this purpose, we offer not only the general market theory, but also a specialized software application. The C# application *Model* was created especially to support the synthesis of concrete models based on the general theory. To date, we are not aware of applications created for such purpose.

Generally speaking, computational modeling derives from two steps: (i) modeling, i.e. finding a model description of a real system, and (ii) solving the resulting model equations using computational methods. In the natural sciences it is often not so difficult to find a suitable model, however the resulting equations tend to be very difficult to solve, and can in most cases not be solved analytically at all [14], [15]. On the other hand, in subjects that are not as well described through a mathematical framework and depend on behavior of objects whose actions are impossible to predict deterministically (such as humans), it is much more difficult to find a good model to describe reality [16], [17], [18]. As a rule, in these disciplines (such as economics) the resulting equations are easier to solve, but they are harder to find and the validity of a model needs to be questioned much more [19]. Therefore, in these applications it turns out to be expedient and convenient to use a computer already at the stage of synthesizing the model. It is such desktop *Model* is proposed in this paper.

The **paper goal** is to introduce the general dynamic market model; to introduce the specialized C# application *Model* and to demonstrate the use of the *Model* in building the general market model; the most important goal is to propose cooperation in future research.

The paper is organized as follows: in part 2 we introduce the general dynamic market model; in part 3 we demonstrate desktop application *Model*; part 4 concludes.

We had to omit proofs of propositions in order to fit the paper format.

2 General Market Model

2.1 Main definitions

First of all let's introduce the basic concepts of the model. We consider a market of homogeneous product, where exogenous parameter $n(t)$ indicates how many firms operate at time t. Each firm produces output $x_i(t)$, where $i = 1,...,n(t)$. Thus the industry output of the market is $Q(t) = \sum_{i=1}^{n} x_i(t)$ at time t. Product price $P(t)$ is given by isoelastic demand function $P(t) = b(t)/Q(t)$ ($b(t) > 0$).

Formally the firm is defined by its objective function. Firm maximizes both its own profit $\pi_X = (P - v)\cdot x - fc$, where $v = v(t)$ is the firm's competitive marginal cost per unit in the market, $fc = fc_i(t)$ is fixed cost, and consumer surplus $CS = CS_i(t)$ is a difference between maximum price which consumer can pay and real price. Here $CS = \Theta \cdot \left(\int_{\varepsilon}^{Q} P(q)dq - P \cdot Q \right)$, where parameter ε is the minimal technologically possible product quantity, $\Theta = \Theta_i(t)$ specifies the segment of the market, which the firm believes its own and optimizes; usually $\Theta_i(t) = \frac{\gamma_i(t)}{n(t)}$, where $\gamma_i(t)$ is the given model parameter. Then

$$CS = \Theta(b \cdot \ln(\frac{Q}{\delta}) - \frac{b}{Q} \cdot Q) = b\Theta \cdot (\ln(\frac{Q}{\delta}) - 1) = b\Theta \cdot \ln\frac{Q}{\hat{\delta}} ,$$

where $\hat{\delta} = \delta \cdot e$ (specific choice of δ does not affect the model dynamics and so further we suppose $\hat{\delta} = 1$). Then general profit function $\Pi = \Pi_i(t)$ ($i = 1,...,n(t)$) of firm is:

$$\Pi = \alpha\pi + \beta \cdot CS = \alpha \cdot ((P - v)x - fc) + \beta \cdot b\Theta \cdot \ln\frac{Q}{\delta},$$

where $\alpha = \alpha_i(t)$ is share of short-run own profit $\pi = \pi_i(t)$ in the objective function, $\beta = \beta_i(t) = 1 - \alpha$ is share of consumer surplus CS, $fc = fc_i(t)$ is a fixed cost. As a matter of fact Π is the weighted average of short-run profit π and expected stable long-run profit.

Adaptive planning

The methods used by firms for planning are extremely diverse and hardly a general uniform description of them is possible in principle. Here, in the General dynamic market model, we use only one obvious and universal consideration. If firms i and j are identical at moment t and in particular they have the same planning at moment t, then it is natural for firm when planning to suppose that their production quantities will be equal at next moment $t+1$ too.

In most general case for mixed naïve and adaptive expectations

$$x_{ij}^e(t+1)=\delta_{ij}(t)x_i(t+1)+\chi_{ij}(t)x_j(t) \quad (0\le\delta_{ij}(t)\le r_{ij}(t)\ ,\ \chi_{ij}(t)=(1-p_{ij})+(r_{ij}(t)-\delta_{ij}(t))\ge 0\ ). \quad \textbf{(1)}$$

Here $\delta_{ii}(t)\equiv 1$, $\chi_{ii}(t)\equiv 0$ for all i and t. Thus according to (1) prospective industry output of a market expected by a firm i during next time period $t+1$ is equal to

$$Q_i^e(t+1)=\sum_{j=1}^{n(t)}x_{ij}^e(t+1)=\sum_{j=1}^{n(t)}\delta_{ij}(t)\cdot x_i(t+1)+\sum_{j=1}^{n(t)}\chi_{ij}(t)x_j(t) \quad \textbf{(2)}$$

Then under planning of their quantity $x_i(t+1)$ in next period $t+1$ each firm i maximizes objective function Π_i^e $i=1,...,n(t)$. Thus we obtain the equations of dynamics of the general market model [12]

$$c_i x_i(t+1)=\sqrt{\frac{b}{v}w_i(t)+d_i^2}+d_i-w_i(t) \qquad (i=1,...,n(t)), \quad \textbf{(3)}$$

where $c_i=c_i(t)=\sum_{j=1}^{n(t)}\delta_{ij}(t),\ w_i(t)=\sum_{j=1}^{n(t)}\chi_{ij}(t)x_j(t),\ d_i=d_i(t)=\frac{1}{2}\frac{b\Theta_i}{v}\frac{\beta_i}{\alpha_i}c_i(t).$

In this paper we consider all actions, expectations and strategies of firms in short-run time period Δ, therefore the equations parameters n, c_i, d_i, δ_{ij}, χ_{ij} etc. are assumed further as constants which are independent of time.

2.2 Main propositions of the theory

Let's define the key notion of *market coherence* in the general market model. Let $x_{ik}^e(t+1)$ is quantity of output of firm k expected by a firm i, $Q_i^e(t+1)=\sum_{k=1}^{n}x_{ik}^e(t+1)$ is prospective industry output of a market expected by a firm i during next time period $t+1$ from (2). Then for firms i and j we put $\varepsilon_{ij}=\max_{t\in\Delta}\frac{|Q_i^e(t+1)-Q_j^e(t+1)|}{Q(t)}$, where $Q(t)$ is industry output of the market in period t, Δ is the time period considered. Value ε_{ij} characterizes *incoherence* in expectations of firms i and j. Therefore value $\varepsilon=\max_{i,j}\varepsilon_{ij}$ we will call the level of incoherence of market expectations or simply the *market incoherence*. Finally, the value $1-\varepsilon$ we will call the level of coherence of market expectations or simply the *market coherence*.

Let $P=\min_{t\in\Delta} P(t)$, $\Theta=\max_i \Theta_i$, $B=4\frac{v}{P}n$, $D=2\Theta n$, $\beta=\max_i \frac{\beta_i}{1-\beta_i}$, $c_i=\sum_{k=1}^{n}\delta_{ik}$, $C_{ij}=\frac{c_i}{c_j}$, $\lambda_{ij}(t)=\frac{x_i(t)}{x_j(t)}$, where $i,j=1,\ldots,n$.

Proposition 1. (Main lemma) Let $2Q(t)>Q_i^e(t)$ for all $i=1,\ldots,n$, $t\in\Delta$ and j be the number of a firm with a production volume $x_j(t)$ above the average: $x_j(t)\ge\frac{\sum_{k=1}^{n}x_k(t)}{n}=\frac{Q(t)}{n}$ ($t\in\Delta$). Then for sufficiently small β, for all $i=1,\ldots,n$ and all $t\in\Delta$

$$\left|\lambda_{ij}(t)-C_{ij}(1\pm D\beta)\right|\le B\varepsilon\cdot$$

Let's define the second key notion in the general market model. According to (1), the higher intention of a firm to plan with adaptive expectations the closer δ_{ij} to r_{ij}. So in the notations (1) the value

$$\Lambda=\frac{\sum_{i,j=1(i\neq j)}^{n}\delta_{ij}x_j}{\sum_{i,j=1(i\neq j)}r_{ij}x_j}\qquad(0\le\Lambda\le1)$$

we will call the intention to plan with adaptive expectations in the market or simply the *market intention to adaptive expectations*.

Proposition 2. At $\Lambda=1$ with all sufficiently small ε and β there is the unique Nash equilibrium for the dynamic system (3) and this equilibrium point is stable for all admissible values of the parameters.

Proposition 3. At $\Lambda=0$ with all sufficiently small ε and β there is the unique Nash equilibrium and this equilibrium point is unstable and hyperbolic for all admissible values of the parameters of the almost any dynamic system (3).

But what is the behavior of system (3) for $0<\Lambda<1$?

Proposition 4. For dynamic system (3) with all sufficiently small ε and β with decrease in Λ from 1 to 0 flip bifurcations (cycle doubling bifurcations) occur throughout the Sharkovskii's order and finally the state of dynamic chaos arises.

This proposition, unlike the previous ones, even formally uses the results of computational studies of dynamics of the two-dimensional market models. For such application computations through *Model* were carried out.

Proposition 5. For dynamic system (3) there is such a segment $[a;b]$ ($0<a<b<1$), that for all $\Lambda\in[a;b]$, with all sufficiently small ε and β, with increasing n flip bifurcations (cycle doubling bifurcations) occur throughout the Sharkovskii's order and finally the state of dynamic chaos arises.

Proposition 6. For sufficiently small $x_i(t+1) - x_i(t)$ and respectively small β_i ($i = 1,..., n$, $t \in \Delta$) the dynamics of system (3) has the unique stable Nash equilibrium point with an equilibrium price in the market.

Proposition 6 means fulfilling the premises of classical economic theory at sufficiently small changes in production per step. Speaking informally, at sufficiently small changes in production per step, systems of general dynamic market model turns into systems of neoclassical microeconomics.

3. Desktop Application for Support Modeling Process

C# application *Model* created to support the modeling process. Its main goal is to maximize research support, to provide the best service for a regular cycle: hypothesis → experiment → hypothesis. The basic requirements implemented in the specialized software application are i) obtain a model in the subject language without codes; ii) immediately obtain all the necessary research tools already configured for this model; and most importantly, iii) it is easy to modify the model depending on the results of computer experiments. The main requirement is that new ideas should immediately be put into experiments for verification. In natural experiments it is impossible to immediately implement a new idea, immediately creating a new device. But in *Model* with computer simulations we can do this using the program window with the appropriate tools: new experiment results create new ideas that we test immediately by creating appropriate new windows.

The following figure shows the main program window, which automatically appears when you open the *Model*.

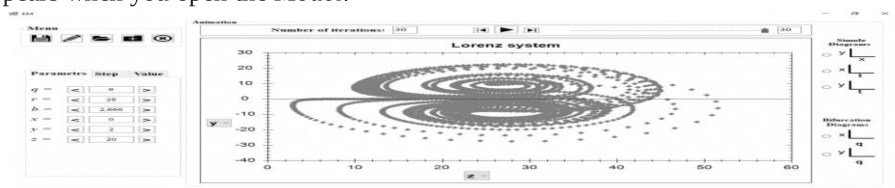

Fig. 1. *Model*'s main window

But the main tool to support the synthesis of concrete models using the *Model* is a simple modification of the current model. After pressing the second menu button (the "Edit" button) we get the window of fig. 8.

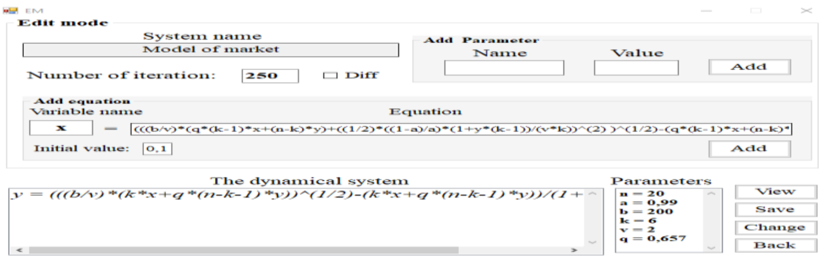

Fig. 2. Window for setting or modifying a dynamic system

The editing window on the screen is located above the current model window, which allows both windows to be used simultaneously. Left-click on the model equation in *The dynamical system* field to go to the *Equation field*, where you can change it. After clicking the *Add button* the modified equation is written back to *The dynamical system* field. Similarly, such a procedure can be performed with parameters in the *Add Parameter* field.

3 Conclusion

Economics has entered the stage of deep transformation of its bases. The traditional method of constructing a scientific theory is first to synthesize and investigate mathematical framework – the general market model according to the new dynamic paradigm of economics. And then we can study complex real systems which are grounded on this basis. This traditional approach was taken as a principle of our research.

The mathematical theory of the general dynamic market model has recently been constructed, the main elements of which are given in this paper. We show that the definition of our model really consists only of simple and obvious constructions; that there is nothing in it that would not inevitably enter into the model of any market of homogeneous products.

Dynamics of the general market model with sufficiently small *incoherence* in the market is stratified on dynamics of two-dimensional markets. The main lemma of theory allows us to generalize properties of simple two-dimensional market models by means of formal reduction to the general market model of this paper.

The crucial factors which ensure the market stability are the *market coherence* and the *market intention to adaptive expectations*. If no any firm uses naive expectations in the market then with sufficiently small *incoherence* there is unique Nash equilibrium which is stable for all acceptable values of parameters. The increase of naive expectations leads to stability loss, to flip bifurcations and finally to chaos.

The increase of number of firms as a rule also leads to stability loss, to bifurcations and finally to chaos in the market. Thus behavior of general view markets is sharply different from their usual behavior in neoclassical microeconomic theory. However at sufficiently small changes in production per step, systems of general dynamic market model turns into systems of neoclassical microeconomics. This means that general dynamic theory does not contradict logically the neoclassical static theory, despite their striking unlikeness.

The next step is to build models of specific real markets based on the general theory. The C# application *Model* was created especially to support the synthesis of concrete models based on the general theory. To date, we are not aware of applications created for this purpose. The most important goal of this paper is to propose cooperation in such researches.

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