# Modeling of the TV3-117 Aircraft Engine Technical State as Part of the Helicopter Power Plant in the Form of the Markov Process of Death and Reproduction

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**Abstract.** In this work, the actual scientific and practical problem of modeling the technical state of the TV3-117 aircraft engine as a part of the helicopter power plant in the form of the Markov process of death and reproduction, the results of which are applicable in the form of a stochastic network of the GERT type (Graphical Evaluation and Review Technique) for formalizing the behavioral activities of the helicopter crew in emergency situations. The results obtained allow us to simulate the development of flight situations in the direction of complication and vice versa. The resulting information model for determining the failure of the TV3-117 aircraft engine is a graph of the standard deviations of the state vector of the probabilities of its state, obtained as a result of flight tests, with a graph for its ideal state.

**Keywords:** Aircraft engine, power plant, modeling, technical state, GERT-systems, information criterion, Markov process

## 1 Introduction

The probability of TV3-117 aircraft engine failure at present level of reliability of the power plants is very small. Engine failure during take-off and climb stages is an even less likely event, since, firstly, these stages last for a relatively short period of time, and secondly, an aircraft thorough inspection and inspection of all engines is carried out immediately before takeoff. However, it is impossible to underestimate, though negligible, the possibility of engine failure at the specified stages.

For the formalization of the behavioral activity of an aircraft crew in emergency situations and to simulate the corresponding development of flight situations, models in the form of stochastic network of GERT type (Graphical Evaluation and Review Technique), which allow to model the development of flight situations in the direction of complication and vice versa. GERT is an alternative probabilistic network planning method used in case of organization activities, where follow-up actions may start only after some previous actions have been completed, therefore allowing for cycles and loops. There are several possible consequences of an emergency situation, its elimination, localization and the development of an emergency situation in the direction of deterioration, so the use of GERT-type networks is advisable.

The use of GERT-type networks in the model of decision support systems will allow us to model the prediction of the emergencies development. Here is a transition from one flight situation related to the failure of the TV3-117 aircraft engine as a part of the helicopter propulsion system to another in the form of Markov process of death and reproduction. In order to determine the probabilities of a system state at any given time, it is necessary to use mathematical models of Markov processes with continuous time (continuous Markov processes). If two continuous chains of a Markov network have the same state graphs and differ only in intensity values, then it is possible to find the limit probabilities of the states for each graph separately [1–5].

The homogeneous Markov process of flight situations development with continuous time can be interpreted as the process of change of states under the influence of some flow of events – environmental factors. That is, the probability of a transition can be interpreted as the intensity of the flow of events that translate the system from one state to another – the influence of environmental factors, factors affecting the aircraft crew decision making to be professional or unprofessional.

## 2 Application of Markov process equations for control and diagnostics of technical condition of TV3-117 aircraft engine as a part of helicopter power plant and mathematical description of its failure process

Given the fact that the helicopter has two engines as parts of the power plant, it is believed that the helicopter is a system that has reliable operation ($P \geq 0.9$), which consists of $i$ units of engines. Let the random operating time of one engine have a positive probability distribution and does not depend on the state of other units of engines [6, 7]. In a more general mathematical model of the system, consisting of $i$ units, which takes into account the relationship between the units of engines, we can assume [7] that the exponential distribution has a random time $\tau_i$ of joint work before the failure of one of the available units of engines, that is:

$$P\left\{\tau_i \leq t\right\} = 1 - e^{-\varphi_i t}; \tag{1}$$

where $\varphi_0 = 0$, $\varphi_i > 0$ at $i = 1, 2, \ldots$

In general case $P_{ij}(t)$ represents the probability of the presence at the time $t$ of serviceable $j$ units of engines, provided that at the initial time $t = 0$ there were $i$ units of serviceable engines. The first step in diagnosing and predicting the probability of failure of one of the helicopter engines is to obtain an equation for the transition probabilities of failure of one of the helicopter engines.

The mathematical model under development is the Markov process of failure of one of the helicopter engines $\xi_t$, $t \in [0, \infty)$, on many states $N = 0, 1, 2, \ldots$, in which transition probabilities $P_{ij}(t) = P\{\xi_t = j \mid \xi_0 = i\}$, $i, j \in N$ are presented at $t \to 0+$ as [5]:

$$P_{i,i-1}(t) = \varphi_i t + o(t); \quad (2)$$

$$P_{ii}(t) = 1 - \varphi_i t + o(t). \quad (3)$$

Jumps in the process of simple engine operating states change $\xi_t$ are presented on fig. 1. Assume that at $t = 0$ the process is in its initial state $i$. At time $\tau_i$ $P\{\tau_i \le t\} = 1 - e^{-\varphi_i t}$; the process transitions to the state $i - 1$ etc.

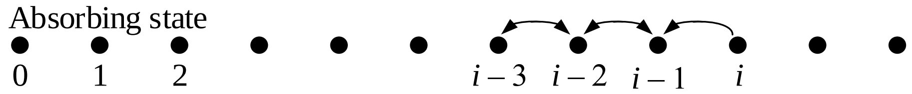


**Fig. 1.** Graphs changes in the state of the TV3-117 aircraft engine as part of the helicopter power plant

The first (reverse) system of Kolmogorov differential equations for transition probabilities in the case of the transition of the aircraft engine TV3-117 as part of a helicopter power plant from one state to another takes the form [7]:

$$\frac{dP_{oj}(t)}{dt} = -\varphi_0 P_{oj}(t); \quad (4)$$

$$\frac{dP_{ij}(t)}{dt} = \varphi_i P_{i-1,j}(t) - \varphi_i P_{ij}(t); \quad (5)$$

where $i = 1, 2, \ldots$ with initial conditions $P_{ii}(0) = 1$, $P_{ij}(0) = 0$ at $i \ne j$.

The second (direct) system of Kolmogorov differential equations for transition probabilities in the event of a change in the state of operation of the TV3-117 aircraft engine as a part of the helicopter power plant is:

$$\frac{dP_{i0}(t)}{dt} = -P_{i0}(t)\varphi_0 + P_{i1}(t)\varphi_1; \quad (6)$$

$$\frac{dP_{ij}(t)}{dt} = -P_{ij}(t)\varphi_j + P_{ij+1}(t)\varphi_{j+1}; \quad (7)$$

where $j = 1, 2, \ldots$ with initial conditions $P_{ii}(0) = 1$, $P_{ij}(0) = 0$ at $i \ne j$.

According to [8], after performing a series of mathematical transformations, we arrive at a closed expression for the double generating function

$$F(t;z;s) = e^{\frac{z}{\lambda}\left(1+(s-1)e^{-\lambda t}\right)}. \quad (8)$$

From here and from the definition $F(t;\, z,\, s)$ we get:

$$F_i(t;s) = \left(1 - e^{-\lambda t} + se^{-\lambda t}\right)^i; \quad (9)$$

at $i \in N$.

Relation (9) means that the random operating times of each of the available $i$ engine units are independent of each other; this independence property holds only for a linear process type.

For applications in the mathematical theory of reliability [6, 7], it is of interest to find a similar (9) closed integral representation for the generating function $F_i(t;\, s)$, as solutions to the Kolmogorov equations (4)–(7) for the failure process of one of the engines (by summation of the Fourier series), under particular assumptions about the function $\varphi_i = \varphi(i)$ [8].

In the case of a quadratic type process [8], we have a system of equations:

$$\frac{dF}{dt} = \lambda z^2 \left( \frac{dF}{dz} - \frac{d^2 F}{dz^2} \right); \tag{12}$$

$$\frac{dF}{dt} = \lambda \left( s - s^2 \right) \frac{d^2 F}{ds^2}; \tag{13}$$

with initial condition $F(0; z; s) = e^{zs}$.

Thus, the obtained equations for describing the failure of one of the engines show that the value of $\varphi_i$ depends on the intensity of occurrence of the event $\lambda$, despite the type of process (quadratic, polynomial, power-law or Poisson), which makes it possible to use standard methods for constructing an algorithm for determining continuously Markov process when simulating the operation of the TV3-117 aircraft engine as a part of the helicopter power plant.

## 3 Algorithm for determining a continuous Markov process when simulating the operation of TV3-117 aircraft engine

We simulate the operation of a helicopter engine, which can be in the following states: $S_0$ – engine operation in idle mode; $S_1$ – engine operation at rated operation mode; $S_2$ – engine is serviceable, reconfiguration $\lambda_{02} < \lambda_{21}$; $S_3$ – the engine is faulty, repair is underway $\lambda_{13} < \lambda_{30}$.

Let us set the values of the parameters $\lambda$, using the experimental data obtained during the flight tests of the helicopter: $\lambda_{01}$ – engine work flow at nominal operation mode (without reconfiguration); $\lambda_{10}$ – engine maintenance flow; $\lambda_{13}$ – engine failure flow; $\lambda_{30}$ – stream of updates.

The main task of modeling is to obtain a continuous Markov process when simulating the operation of a helicopter engine during its operation in real operating modes, that is, determining the time of transitions from one engine operating mode to another in all possible variants, the parameters of which are set $S$ and $\lambda$ [9].

The simulation algorithm will have the form shown in fig. 2, where

$$\tau_{01} = -\frac{1}{\lambda_{01}} \ln R; \quad \tau_{13} = -\frac{1}{\lambda_{13}} \ln R; \quad \tau_{20} = -\frac{1}{\lambda_{20}} \ln R;$$

$$\tau_{02} = -\frac{1}{\lambda_{02}} \ln R; \quad \tau_{10} = -\frac{1}{\lambda_{10}} \ln R; \quad \tau_{21} = -\frac{1}{\lambda_{21}} \ln R.$$

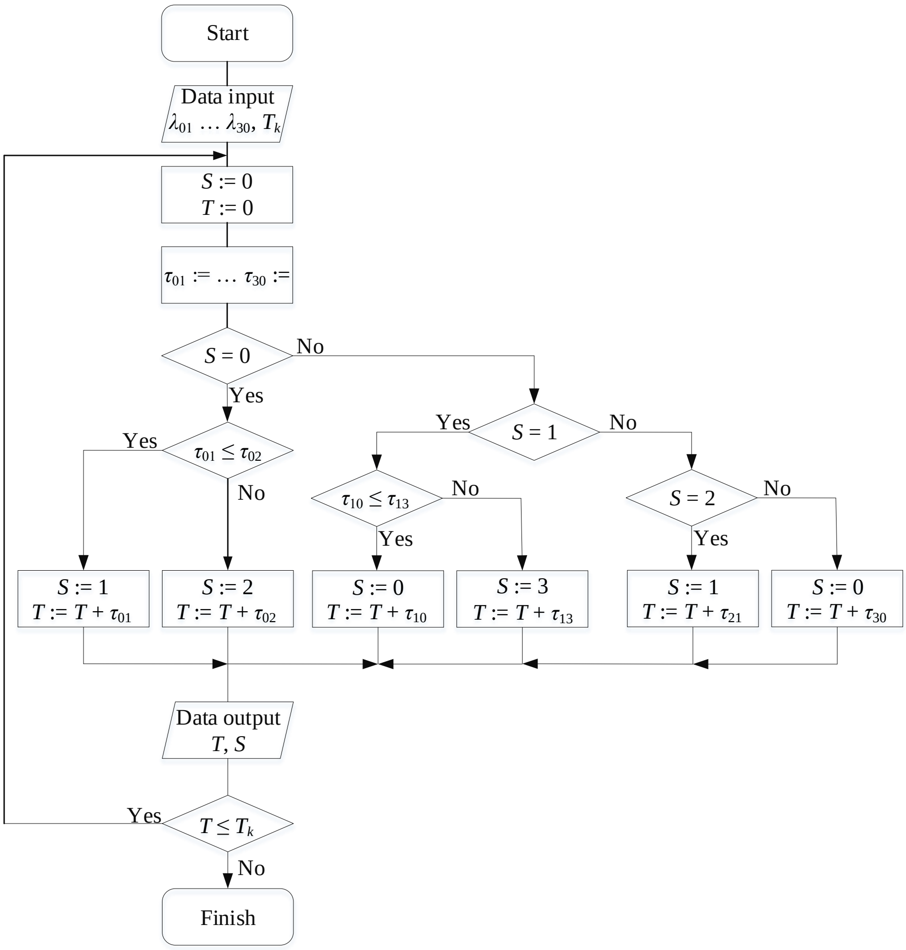


**Fig. 2.** The flowchart of the modeling algorithm for the continuous Markov process on the example of simulating the operation of the aircraft engine TV3-117 as part of the helicopter power plant

Since it is known that a helicopter engine in flight test conditions can be in four states: $S_0$ – engine idle, $S_1$ – engine runs in rated mode, $S_2$ – serviceable engine, reconfiguration is in progress, $S_3$ – engine defective. According to these data, one can obtain a discrete Markov network based on the following possible transition states: $S_0$ – $S_1$, $S_1$ – $S_0$, $S_0$ – $S_3$, $S_1$ – $S_3$, $S_2$ – $S_0$, $S_2$ – $S_1$, $S_2$ – $S_3$, $S_3$ – $S_2$, in this case, it is necessary to take into account the probability of the engine staying in each of the above states (fig. 3).

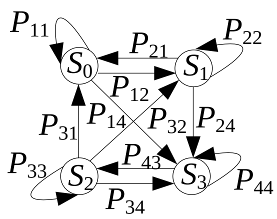


**Fig. 3.** Markov discrete network showing all kinds of changes in the state of TV3-117aircraft engine as a part of a helicopter power plant during flight tests

According to fig. 3, to find the probabilities of a Markov chain staying in certain states as $n \to \infty$ (final probabilities), we solve a system of equations that will have the form:

$$\begin{cases} P_{11}\pi_1 + P_{21}\pi_2 + P_{31}\pi_3 = \pi_1; \\ P_{12}\pi_1 + P_{22}\pi_2 + P_{32}\pi_3 = \pi_2; \\ P_{33}\pi_3 + P_{43}\pi_4 = \pi_3; \\ P_{14}\pi_1 + P_{24}\pi_2 + P_{34}\pi_3 + P_{44}\pi_4 = \pi_4. \end{cases} \tag{14}$$

Deciding which, we get:

$$\pi_1 = \frac{-P_{21}P_{32}P_{43} - P_{31}P_{43}\left(P_{22} - 1\right)}{\left(P_{11} - 1\right)\left(\left(P_{22} - 1\right)\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) - P_{21}\left(P_{12}\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) + P_{31}\left(-P_{12}P_{43} + P_{43}\left(P_{22} - 1\right)\right)};$$

$$\pi_2 = \frac{P_{31}P_{43}\left(P_{11} - 1\right) - P_{12}P_{31}P_{43}}{\left(P_{11} - 1\right)\left(\left(P_{22} - 1\right)\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) - P_{21}\left(P_{12}\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) + P_{31}\left(-P_{12}P_{43} + P_{43}\left(P_{22} - 1\right)\right)};$$

$$\pi_3 = \frac{-P_{43}\left(P_{11} - 1\right)\left(P_{22} - 1\right) + P_{12}P_{21}P_{43}}{\left(P_{11} - 1\right)\left(\left(P_{22} - 1\right)\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) - P_{21}\left(P_{12}\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) + P_{31}\left(-P_{12}P_{43} + P_{43}\left(P_{22} - 1\right)\right)};$$

$$\pi_4 = \frac{\left(P_{11} - 1\right)\left(P_{22} - 1\right)\left(P_{33} - 1\right) - P_{12}P_{21}\left(P_{33} - 1\right)}{\left(P_{11} - 1\right)\left(\left(P_{22} - 1\right)\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) - P_{21}\left(P_{12}\left(P_{33} - P_{43} - 1 + P_{32}P_{43}\right)\right) + P_{31}\left(-P_{12}P_{43} + P_{43}\left(P_{22} - 1\right)\right)}.$$

As a result of solution (14), expressions are obtained for determining the elements of the state probability vector $\pi(\pi_1, \pi_2, \pi_3, \pi_4)$ (shows the probability that the engine will be in the $i$-th state), which allows, if there are transition probabilities $P_{11}\ldots P_{33}$ to obtain prognostic indicators of changes in the state of TV3-117aircraft engine as a part of a helicopter power plant. As can be seen from the system (14), to determine the probability of engine failure $\pi_4$, it is enough to know the transition probabilities for the other three states, as well as the transition probabilities $P_{34}$, $P_{43}$, which can be obtained as a result of technical diagnostics of the helicopter and other statistical data.

It is assumed that in perfect condition (the engine is flawless), the probability of its operation in idle mode and in nominal mode is 1.0, that is, $P_{11} = P_{22} = 1$. The probability of transition from idle mode to nominal mode and vice versa is also 1, 0, since the operator can at any moment of time freely change its operating modes, that is, $P_{12} = P_{21} = 1$. Reconfiguring the engine does not make operational sense, however, point changes can be corrected, that is, $P_{33} = 0.1$. Consequently, the transition after reconfiguration to both idle and nominal mode will be 0.9, that is, $P_{31} = P_{32} = 1 – 0.1 = 0.9$. The probability of engine failure is assumed to be 0.1% – this is due to production fault or a professional mistake of engineers when assembling the engine structure ($P_{44} = 0.001$) [6], and, consequently, the transition probabilities associated with reconfiguring the engine after detecting its failure or malfunction is $P_{43} = 1 – 0.001 = 0.999$. The transition probabilities associated with the transition to the engine idling and / or to the nominal operating mode are zero, since after a malfunction which led to engine failure, is detected the engine is sent for repair, reconfiguration, etc., i.e. $P_{41} = P_{42} = 0$.

The transient probabilities associated with engine failure during its operation both in idle mode and in nominal mode are 99.9 % (the defect described above is taken into account when assembling the engine), i.e. $P_{14} = P_{24} = 1 – 0.999 = 0.001$.

Given the human factor in the maintenance of aircraft engines [10], it is accepted that after reconfiguring the engine due to professional mistakes by the maintenance personnel, the probability of failure can be 1 % $P_{34} = 0{,}01$.

Absolute and relative errors of the obtained values of the state probability vector $\pi(\pi_1, \pi_2, \pi_3, \pi_4)$ in numerical modeling of the Markov chain and analytical solution amounted to:

$$\Delta\pi_1 = |\,0{,}9196 - 0{,}9208| = 0{,}0012;\ \delta\pi_1 = \frac{|0{,}9196 - 0{,}9208|}{0{,}9196}\cdot 100\% = 0{,}13\%;$$

$$\Delta\pi_2 = |\,0{,}9196 - 0{,}9208| = 0{,}0012;\ \delta\pi_2 = \frac{|0{,}9196 - 0{,}9208|}{0{,}9196}\cdot 100\% = 0{,}13\%;$$

$$\Delta\pi_3 = |\,0{,}0107 - 0{,}0108| = 0{,}0001;\ \delta\pi_3 = \frac{|0{,}0107 - 0{,}0108|}{0{,}0107}\cdot 100\% = 0{,}93\%;$$

$$\Delta\pi_4 = |\,0{,}0029 - 0{,}00292| = 0{,}00002;\ \delta\pi_4 = \frac{|0{,}0029 - 0{,}00293|}{0{,}0029}\cdot 100\% = 0{,}69\%.$$

As can be seen from the obtained results, the relative error of the obtained values of the state probability vector during numerical simulation of the Markov chain and the analytical solution did not exceed 1 %, which confirms the adequacy of the results.

## 4 Conclusion

1. In this work, the equations for describing the failure of one of the TV3-117 aircraft engines, which are part of the helicopter power plant, are obtained, which makes

it possible to use standard methods for constructing an algorithm for determining continuously Markov process when simulating the operation of the TV3-117 aircraft engine as a part of the helicopter power plant.

2. In this work, we obtain a graph of the standard deviations of the state probability vector $\pi(\pi_1, \pi_2, \pi_3, \pi_4)$ for the ideal state of the engine (a new engine exited from the factory), which is an information model for determining failure when comparing it with the graph obtained as a result of flight tests.

3. Both the algorithm for constructing this graph and the algorithm for determining the state probability vector $\pi(\pi_1, \pi_2, \pi_3, \pi_4)$ are implemented in the MatLAB software package, it shows the probability that the engine will be in the *i*-th state. The following value of this vector π(0.9196 0.9196 0.0101 0.0029) was obtained in the work, the value of the element $\pi_3$ is explained by a possible point reconfiguration, and $\pi_4$ due to the human factor during assembly and maintenance of the engine.

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