Numerical Optimization of Non-Conflict Aircraft Flow Merging *

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Abstract. Nowadays, aircraft move along routes consisting of horizontal tunnels and vertical flight levels. With that, the routes can split or join. At the point of route joining, a problem of aircraft flows merging appears. Such a problem is highly important near airports, where the air traffic is very dense. The main demand for aircraft flows merge is the presence of the minimal safe time interval between arrival instants at the merge point. There are two main tools for changing arrival instant of an aircraft to a checkpoint. The first of them is control of the aircraft velocity, which allows to obtain relatively small changes of the arrival instant both to earlier or later times. To get larger delays one uses the second tool, delay schemes. As a result of designing system of delay schemes for a certain airport, one has information about possible acceleration and deceleration of aircraft moving along each route. Further on the basis of this information, it is necessary to study capabilities of the constructed system for formation of safe aircraft flows merge. In the paper, a formalization is set forth for the problem of optimal formation of aircraft arrival schedule under the present delay scheme system as a finite-dimensional optimization problem. Also, the authors consider applicability of different methods for search of multivariable function extrema to this problem. Results of numerical computations are discussed.

Keywords: Aircraft flows merging · Arrival instant variation · Safety interval · Discrete optimization · Piecewise-linear criteria · Non-linear criteria · Linear programming · Breadth-first search · Gradient descent · Newton method · Hooke-Jeeves algorithm · Successive linear programming method

1 Problem Statement

The problem of non-conflict aircraft flows merging is considered. We are given the following input data:

– an ascending ordered nominal arrival times collection $\{t_i^{\mathrm{nom}}\}_{i=1}^N$;

– a variation interval $t_i^{\mathrm{nom}} \to [t_i^{\mathrm{nom}} - t_i^{\mathrm{acc}}, t_i^{\mathrm{nom}} + t_i^{\mathrm{dec}}]$ for each aircraft in the queue. The values t_i^{acc}, t_i^{dec} show how long the ith aircraft can be accelerated or decelerated according to its specifications and the delay schemes allocated along the airway of this aircraft;
– a minimal safe time interval $\tau_{i,j}^{\mathrm{safe}}$ between the arrival instants with the indices i, j in the merged queue.

The objective is to obtain a new collection $\{t_i\}$ of instants of aircraft arrival to the merge point such that $t_i \in [t_i^{\mathrm{nom}} - t_i^{\mathrm{acc}}, t_i^{\mathrm{nom}} + t_i^{\mathrm{dec}}]$. The new collection should obey the safety demands: $\forall\, 1 \le i, j \le N,\;\; t_j - t_i \ge \tau_{i,j}^{\mathrm{safe}}$ (assuming that the ith aircraft arrives earlier than the jth one). With that, one should minimize some criterion $F\big(\{t_i\}, \{t_i^{\mathrm{nom}}\}\big)$, which describes the optimality of the obtained schedule from the point of view of air-traffic control dispatchers and airport services.

Thus, the following optimization problem arises:

$$\begin{aligned}
&F\big(\{t_i\}, \{t_i^{\mathrm{nom}}\}\big) = \sum_{i=1}^{N} f(t_i, t_i^{\mathrm{nom}}) \to \min \\
&\text{subject to} \\
&t_i \in [t_i^{\mathrm{nom}} - t_i^{\mathrm{acc}}, t_i^{\mathrm{nom}} + t_i^{\mathrm{dec}}], \\
&\forall\, 1 \le i, j \le N \;\; (t_j > t_i \;\Rightarrow\; t_j - t_i \ge \tau_{i,j}^{\mathrm{safe}}).
\end{aligned} \tag{1}$$

2 Aircraft with the Same Type

2.1 Convex Piecewise-Linear Criteria

Simplest Criterion. Let us consider the simplest convex piecewise-linear penalty criterion of the following form:

$$F\big(\{t_i\}, \{t_i^{\mathrm{nom}}\}\big) = \sum_{i=1}^{N} (t_i - t_i^{\mathrm{nom}}) = \sum_{i=1}^{N} t_i - \sum_{i=1}^{N} t_i^{\mathrm{nom}} \tag{2}$$

If to take into account that the values t_i^{nom} are constant, then the subtrahend in the right-hand of the equality is constant too. Therefore, this criterion minimizes the sum of actual instants of aircraft arrivals to the merging point, or minimizes the instants themselves. Its sense is “to let an aircraft pass as earlier as possible”. Therefore, it is enough to apply the greedy algorithm for minimization of this criterion, which assigns the earliest possible arrival instant for each aircraft.

Two-Zone Symmetric Piecewise-Linear Criterion. Next criterion allows to penalize both accelerations and decelerations due to existing costs on acceleration and deceleration maneuvers (fuel consumption, for example):

$$F\big(\{t_i\}, \{t_i^{\mathrm{nom}}\}\big) = \sum_{i=1}^{N} |t_i - t_i^{\mathrm{nom}}|. \tag{3}$$

It punishes both delays and accelerations of aircraft with respect to their nominal arrival instants.

Two-Zone Asymmetric Piecewise-Linear Criterion.

$$F\big(\{t_i\},\{t_i^{\mathrm{nom}}\}\big) = \sum_{i=1}^{N} K \cdot (t_i - t_i^{\mathrm{nom}}), \tag{4}$$

$$K = \begin{cases} K_+, & t_i > t_i^{\mathrm{nom}}, \\ -K_-, & t_i < t_i^{\mathrm{nom}}. \end{cases}$$

Consideration of such a criterion is stipulated by potentially different expenditures for accelerating and delaying an aircraft. Two cases are evident: $K_- < K_+$ when to accelerate an aircraft is cheaper than to delay it, and the opposite case $K_- > K_+$.

Three-Zone Piecewise-Linear Criterion. Criterion (4) can be made more precise. Namely, there are two variants of an aircraft delay: when the aircraft decreases its velocity or when it moves along some delay schemes with normal velocity. Fuel expenditures are greater in the second variant, therefore, long delays should be punished more than short ones.

In this situation, one can choose the optimality criterion for one aircraft as it is shown in Fig. 1. The parameter δ defines the maximal delay value, which can be achieved by decreasing velocity only. The coefficients K_-, K_+^1, K_+^2 describe the values of specific fine for each of the motion regimes: acceleration, delay with decreasing velocity only, delay with usage of delay schemes. The continuity of the function f at the point $t_i^{\mathrm{nom}} + \delta$ is questionable, but we assume it, because we need it to pass to a linear programming problem.

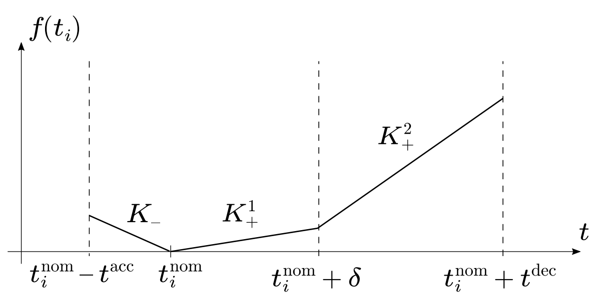

Fig. 1. Three-zone piecewise-linear criterion.

On the basis of the values δ, K_-, K_+^1, K_+^2 and continuity of the function f, one can compute coefficients a, b, c, d of the following representation of f:

$$\begin{gathered} f(t_i, t_i^{\mathrm{nom}}) = a|t - t_i^{\mathrm{nom}}| + b\big|t_i - (t_i^{\mathrm{nom}} + \delta)\big| + ct_i + d, \\ F\big(\{t_i\},\{t_i^{\mathrm{nom}}\}\big) = \sum_{i=1}^{N} f(t_i, t_i^{\mathrm{nom}}). \end{gathered} \tag{5}$$

Minimization of all convex piecewise-linear criteria described above can be represented as a problem of linear programming by introducing additional variables denoting the absolute values involved in the criteria. To solve the linear programming problem we have used the simplex-method fulfilled in the computer library `GLPK` [4].

2.2 Non-Linear Criteria

All the objective functions considered above are linear in the sense that the penalty for a deviation from the nominal arriving time grew in proportion to the deviation. Also it would be reasonable to use criteria, in which deviations of different magnitudes are penalized in different ways (non-proportionally). Studying of such a situation naturally leads to nonlinear penalty functions.

Quadratic Criterion. The simplest non-linear criterion is the quadratic one:

$$F\big(\{t_i\},\{t_i^{\mathrm{nom}}\}\big) = \sum_{i=1}^{N} (t_i - t_i^{\mathrm{nom}})^2. \tag{6}$$

Large deviations are punished quadratically larger than small ones.

Criterion with a Restriction on the Minimum Variation Value. The need to use non-linear criteria was due to avoiding of small variations of aircraft arrival time at the merge point, which are not technological. The following fine function has been proposed for deviations of the assigned arrival instant of aircraft from the nominal instant, which has minimums at zero (no variation) and at some value δ, which corresponds to the minimum feasible non-zero variation of the arrival instant (see Fig. 2).

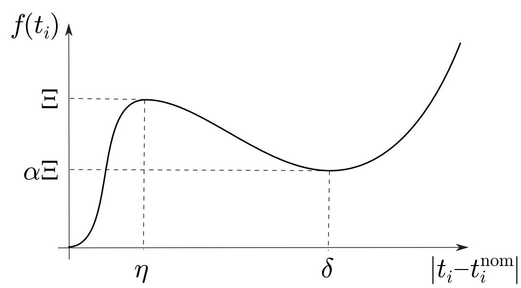

Fig. 2. Criterion with a restriction on the minimum variation value.

The function depends on the deviation of the assigned arrival time from the nominal (that is, on the absolute value of the difference of these instants) and is determined by four parameters: the value η of variation that can be neglected (3-5 seconds), the minimum value δ of nonzero variation (30-40 seconds), the parameter Ξ of the depth of the ravine, the parameter α of the ratio of the values of the auxiliary minimum and the intermediate maximum.

Non-Linear Optimization Approaches. A typical approach during numerical studying nonlinear extreme problems with constraints is to pass to unconditional minimization of a function obtained from the original one by adding some terms reflecting the constraints. Unconditional minimization is carried out by means one or another variant of the gradient descent (see, for example, [2]) if the obtained function is differentiable or by a direct search method if the differentiability is absent. As the initial point for the method, one should find a point, which strictly obeys all constraints of the original problem. Due to this, methods of this family are often called *internal point methods* (see, for example, [1, 2]).

In the problem under consideration, we have two series of constraints. The first of them are the constraints for the maximal accelerations/decelerations of aircraft: $t_i \in [t_i^{\mathrm{nom}} - t^{\mathrm{acc}}, t_i^{\mathrm{nom}} + t^{\mathrm{dec}}]$. Note that for these constraints the point $\{t_i^{\mathrm{nom}}\}$ is internal and can be taken as the initial one. Therefore, these constraints have been taken into account in a standard way, by terms with logarithms:

$$-\alpha \cdot \Big(\sum_{i=1}^{N} \ln\big(t_i - (t_i^{\mathrm{nom}} - t^{\mathrm{acc}})\big) + \sum_{i=1}^{N} \ln\big((t_i^{\mathrm{nom}} + t^{\mathrm{dec}}) - t_i\big)\Big).$$

Inside the parallelepiped $\prod_{i=1}^{N} [t_i^{\mathrm{nom}} - t^{\mathrm{acc}}, t_i^{\mathrm{nom}} + t^{\mathrm{dec}}]$, the arguments of the logarithms are positive, at the boundary and outside it they are non-positive and, therefore, these terms are defined only inside the parallelepiped. With approach to the boundary from inside due to the sign minus, the value of these terms tends to $+\infty$, what under application of a gradient descent prevents leaving the parallelepiped. Often, constraints taken into account through terms of such a type are called *severe*: they principally prevents violating the corresponding constraints of the original problem. The coefficient α regulates the severity of the constraints: the less it is, the closer point can go to the boundary of the parallelepiped.

The second group of constraints is connected explicitly to the objective of the original problem, to the safe intervals between aircraft at the merge point: $|t_i - t_j| \geq \tau_{i,j}^{\mathrm{safe}}$, $1 \leq i, j \leq N$. Taking into account the keeping the order of aircraft, one can reduce number of constraints: $t_{i+1} - t_i \geq \tau_{i,j}^{\mathrm{safe}}$, $i = \overline{1, N-1}$. Unfortunately, the point $\{t_i^{\mathrm{nom}}\}$, generally speaking, does not obey these constraints (otherwise, the problem itself would be absent). So, we tested two ways to take into account them.

The first way is the way of consideration of them as *strict*. That is to the function to be minimized we add the following terms:

$$-\alpha \cdot \ln\big((t_i - t_j)^2 - (\tau_{i,j}^{\mathrm{safe}})^2\big).$$

The difference of the instants t_i and t_j is powered by 2, because, in the general case (not for the quadratic criterion), there is no guarantee that the initial order of the nominal instants would be kept. The coefficient α regulates the severity of the constraints.

In this case, the minimized function has the following form:

$$\begin{aligned}\mathcal{F}(\{t_i\},\{t_i^{\text{nom}}\}) = F\big(\{t_i\},\{t_i^{\text{nom}}\}\big)& \\ -\,\alpha\sum_{i=1}^{N}\Big(\ln(t_i - t_i^{\text{nom}} + t^{\text{acc}}) + \ln(-t_i - t_i^{\text{nom}} + t^{\text{dec}})\Big)& \\ -\,\alpha\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\ln\big((t_i - t_j)^2 - (\tau_{i,j}^{\text{safe}})^2\big).&\end{aligned} \tag{7}$$

However, we need some initial point for the algorithm of gradient descent, which obeys the strict constraints $t_{i+1}^0 - t_i^0 > \tau_{i,i+1}^{\text{safe}}$. It has been performed in the following way. At first, the greedy algorithm (see Section 2.1) works. If it fails, then it means that the incoming flows are too dense and there is no any safe schedule for all aircraft; the entire algorithm stops in this case. If the greedy algorithm finishes successfully, then the obtained collection $\{t_i^g\}$ is checked. Namely, if there is a number i^* such that $t_{i^*}^g = t_{i^*}^{\text{nom}} + t^{\text{dec}}$, it means that there is a group of aircraft such that the first aircraft in the group is accelerated maximally, the last aircraft (with the number i^*) in the group is delayed maximally, and between each two neighboring aircraft in the the group there is exactly the minimal safe interval $\tau_{i,j}^{\text{safe}}$. So, such a group cannot be "slid apart" to obtain a collection of instants, for which all constraints are fulfilled strictly. That is one cannot obtain the internal initial point for the chosen non-linear optimization method. In this situation, the algorithm ceases too.

Finally, if the check is passed successfully, then on the basis of the obtained set $\{t_i^g\}$ the internal point $\{t_i^0\}$ is constructed. The "slide reserve" is computed: $\eta = \min_i(t_i^{\text{nom}} + t^{\text{dec}} - t_i^g)$. Indeed, the computation of η is made simultaneously with the check of slide possibility: if $\eta = 0$, it just means that for some i^* the equality $t_{i^*}^g = t_{i^*}^{\text{nom}} + t^{\text{dec}}$ occurs. Then the computation of the initial internal point is carried out: $t_i^0 = t_i^g + (i-1)\cdot\eta/N$. According this formula each consecutive pair of aircraft is slid apart to the initial value plus the value η/N; with that, the boundaries of the segments $[t_i^{\text{nom}} - t^{\text{acc}}, t_i^{\text{nom}} + t^{\text{dec}}]$ are not reached. Therefore, the collection $\{t_i^0\}$ computed in the shown way indeed is an internal point.

After that, the chosen non-linear optimization method is started from the computed point.

The second way for taking into account these constraints is in representing them as *soft constraints*: the fine criterion is chosen in such a way that it is defined even for $t_{i+1} - t_i \le \tau_{i,i+1}^{\text{safe}}$ in the contrast with the logarithms in the severe constraints. So, one needs that the function has a plateau when $t_{i+1} > t_i + \tau_{i,i+1}^{\text{safe}}$, which turns fast to a step when t_{i+1} tends to $t_i + \tau_{i,i+1}^{\text{safe}}$. When $t_{i+1} < t_i + \tau_{i,i+1}^{\text{safe}}$, the function should be inclined enough to "push out" the argument point to the area $t_{i+1} > t_i + \tau_{i,i+1}^{\text{safe}}$.

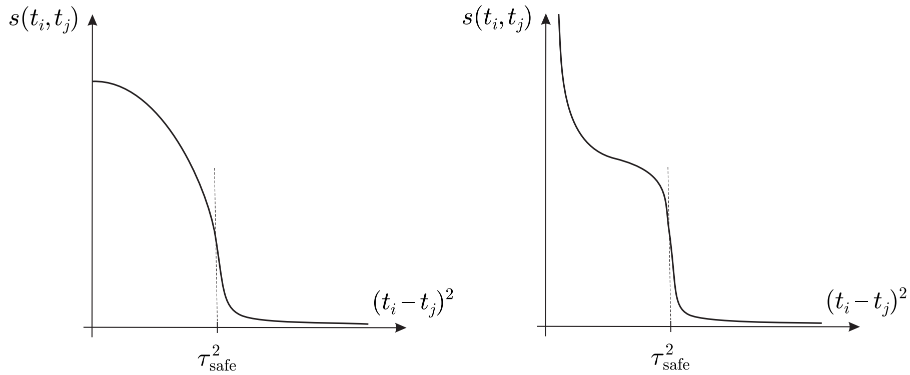

Fig. 3. Possible forms of representing the soft constraints functions connected with the minimal safe time interval between the arrival instants; *on the left*: coincidence of the arrival instants t_j and t_j is allowed; *on the right*: coincidence of the arrival instants t_j and t_j is not allowed.

Two types of the soft constraints $s(t_i, t_j)$ were considered. The first type of soft constraints (see Fig. 3, on the left) potentially allows the coincidence of the current arrival instants t_i and t_j, particularly, the coincidence of the nominal arrival instants. The constraint function $s(t_i, t_j)$ of the second type (see Fig. 3, on the right) tends to infinity when the values t_i and t_j come closer and is undefined when t_i and t_j are equal. In this situation, it is necessary to slide apart slightly each group of coinciding nominal arrival instants. Now, the function to be minimized looks like

$$\begin{aligned}\mathcal{F}(\{t_i\}, \{t_i^{\text{nom}}\}) = F\big(\{t_i\}, \{t_i^{\text{nom}}\}\big) \\ - \alpha \sum_{i=1}^{N} \Big(\ln(t_i - t_i^{\text{nom}} + t^{\text{acc}}) + \ln(-t_i - t_i^{\text{nom}} + t^{\text{dec}}) \Big) \\ + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} s(t_i, t_j).\end{aligned} \tag{8}$$

The sense of the last term is that it iterates over all pairs of aircraft and fines the result if the minimal safe time interval is not maintained between this pair of aircraft.

As the initial point for minimization method in the case of soft constraints, the collection of the nominal instants $\{t_i^{\text{nom}}\}$ is taken with groups of coinciding instants slightly slid apart. After that, the chosen minimization method is started.

To optimize the criteria with soft and strict constraints, the Newton method and the Hooke – Jeeves direct search method have been used. It is worthy to note that Newton method is changed to the gradient descent method when the point is inside the unsafety area, because the fine function is non-convex there and the Newton method is not applicable.

Unfortunately, optimization was successful only for the convex quadratic fine function (6). But substantially non-linear fine function (see Fig. 2) generates a multiextrema total criterion $F\big(\{t_i\}, \{t_i^{\mathrm{nom}}\}\big)$ and, therefore, criteria (7) and (8). Of course, multiextrema criteria almost can not be successfully minimized, and neither Newton, nor Hooke – Jeeves methods give anything similar to the desired minimum.

There is another approach to minimizing nonlinear functions with constraints such as equalities / inequalities using linear programming methods called Successive Linear Programming [3].

The main idea of the SLP method is that the nonlinear problem (7) and its constraints are locally linearized. The linearized target function together with constraints, which are already linear, gives a linear programming problem, which is solved by means of linear programming methods.

SLP successfully works in more situations than Newton and Hooke – Jeeves methods, nevertheless, quite often does not reach the minimum point.

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