One- and Unidirectional Two-Dimensional Signal Imitation in Complex Basis

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Abstract. Signal imitation is widely used today since it helps to bring the experiment to the virtual domain thus eliminating risks of damaging real equipment. At the same time all signals used in the physical world are limited by the finite band of frequencies rendering bandpass signal studies especially important. The method for imitating bandpass signals in complex basis is favorable in the case of a bandpass signal as it uses resources effectively and provides the desired accuracy.

The author has implemented the method in the form of the PC application generating signals according to the characteristics set by the user. These characteristics are: borders defining the signal's frequency band, the time period, the number of steps for discretization, the spectral density form. The PC application uses the characteristics to generate the signal and its experimental autocorrelation. The application calculates theoretic and algorithmic autocorrelations in order to evaluate the quality of the imitation by computing the error function. The application visualizes all the resulting information via the simple interface.

The application was used to generate two-dimensional signals to highlight the present limitations and to sketch the direction for the future. The application is later to be adapted completely to imitating multidimensional signals.

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Keywords: digital signal processing, DSP, Fourier functions, two-dimensional signals, broadband signal, signal imitation, random signal generation

1 Introduction

The word "signal" today is known to everyone and is used regularly but often we don't even suspect how often this word could be used but wasn't. Temporal changes of some physical value can be represented as time series or as signals. The term "signal processing" is applicable to any processes that change in time [1, 2] including the very large time series data [3]. The problem of forecasting brings these two terms especially close and also links them to events happening in the real world [4]. The fundament of such analysis is derived from the theory of digital signal processing [1].

Increasing volumes of data breed growing amounts of information all of which have to be contained in the form of a signal (or a time series), and as you have to represent more and more linked processes the dimensions of signals being used grow [5]. Multidimensional signals are involved when dealing with visual information: image processing and generation [6] or scanning different sections of a brain [7]. Finance uses one-dimensional time series widely but today multidimensional signals can represent more complex financial phenomena [8]. Thus, digital signal processing provides methods used when analyzing or managing data which nowadays is often multidimensional. Some methods are more effective and work faster which is desirable when data is used intensively.

The complex basis has shown itself to be useful for imitating one-dimensional bandpass signals [9, 10]. The program that uses the complex basis was designed and tested on one-dimensional and two-dimensional unidirectional signals. Since the reviewed works don't consider methods of two-dimensional signal imitation in depth [5], deal with visual methods [6], do not consider the broadband signals separately and do not use complex basis, it is planned then to upgrade the designed program for imitating multidimensional signals with varying numbers of dimensions.

Section 2 shows the results gained by using the program in the case of one-dimensional signals. Section 3 embarks upon settling whether the method of signal imitation in complex basis described in section 2 can be used to generate two-dimensional signals and what changes have to be made to increase quality of such generating. The future plans are described in the conclusion.

2 One-Dimensional Signal Imitation in Complex Basis

2.1 Complex Basis Imitation Algorithm

Bandpass signal's spectrum is constrained within two border frequencies [10]. The spectral density of the bandpass signal is shown on the figure 1.

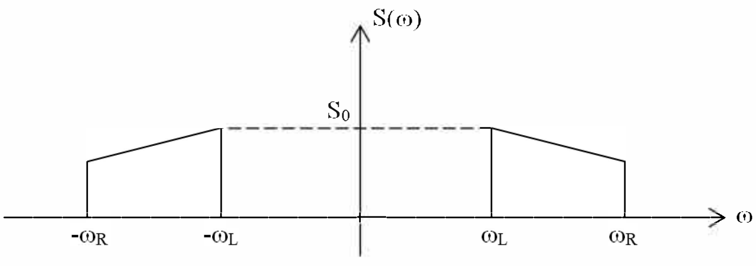

Fig. 1. Bandpass signal's spectral density

The goal of the imitation is to acquire the signal that has such spectrum [11]. User inputs the form of the spectrum, its limiting frequencies ω_L and ω_R, the period T and the number of discretization intervals N [12]. Discretization replaces ω_L and ω_R with discrete borders N_L and N_R. "L" stands for "Left" and "R" stands for "Right". X_{FE} and X_{FO} are even and odd Fourier coefficients.

Formula of the random complex spectrum is as follows:

$$Y_F(k) = \mu_k X_{FE}(k) - j\gamma_k X_{FO}(k),\ k \in [N_L, N_R];$$

$$X_{FE}(k) = X_{FO}(k) = \sqrt{\frac{S_0}{2T}} = \sqrt{\frac{T}{2(N_R - N_L)2T}} = \frac{1}{2\sqrt{(N_R - N_L)}};\ k \in [N_L, N_R];$$

$$Y_F(k) = \frac{1}{2\sqrt{(N_R - N_L)}}(\mu_k + \gamma_k).$$

The formula derived for calculating the resulting signal is presented below:

$$y(i) = \frac{1}{2\sqrt{(N_B - N_H)}}\mu_{NR}\cos(\pi i) +$$
$$+2\sum_{k=N_L}^{N_R-1}\left\{\frac{1}{2\sqrt{(N_R - N_L)}}\left[\mu_k \cos\left(\frac{2\pi}{N}ki\right) + \gamma_k \sin\left(\frac{2\pi}{N}ki\right)\right]\right\},\ i \in [0, N).$$

The spectrum and the signal are connected through the Fourier transform. When imitating random signal, coefficients μ_k and γ_k randomly take on values of "1" or "-1". When imitating determined signal all of them just remain set to "1". The values of Y_F on the borders depending on whether the N is odd or even are to be considered separately which is dropped here in favor of the general method. These formulas to be used in the program were derived by Professor Syusev V. V. [9] and tested experimentally by the author of the current paper.

2.2 Applying the Method

Three the signals and also three experimental autocorrelations generated by the program are presented on the figure 2.

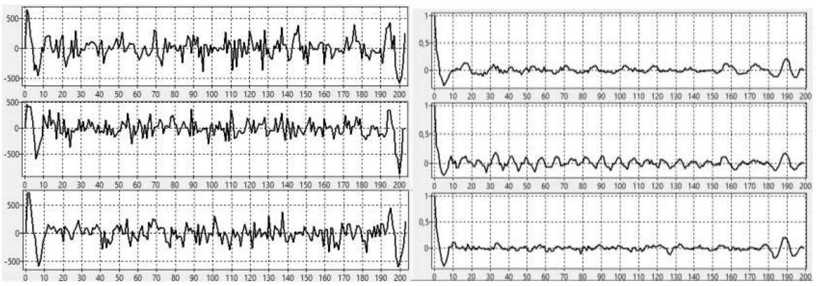

Fig. 2. Three random signals (on the left) and three resulting autocorrelations (on the right) based on the same spectral density

The resulting experimental autocorrelation is calculated as follows:

$$R_{\mathrm{E}}(m) = \frac{1}{N-m} \sum_{i=0}^{N-1-m} \mathrm{y(i)y(i+m)}, \quad m \in [0, M).$$

The program calculates different autocorrelations (figure 3). The first one (figure 3, a) is an a priori theoretical autocorrelation derived directly from the spectral density. The second one (figure 3, b) is an algorithmic autocorrelation that uses Fourier coefficients. The third one (figure 3, c) substitutes Fourier coefficients with their complex basis versions, this is the resulting experiment autocorrelation that is compared to the other two in order to estimate the quality of imitation.

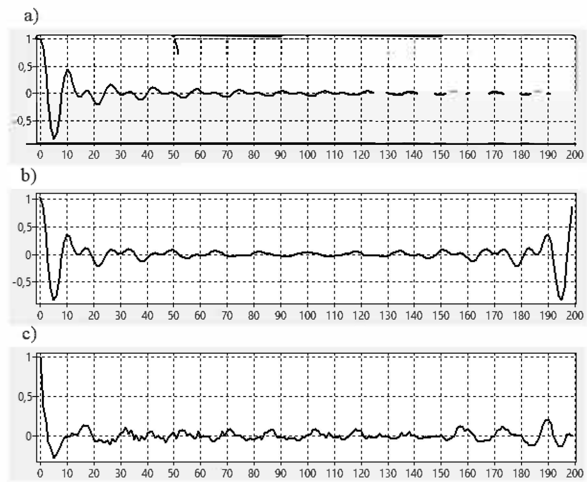

Fig. 3. Comparison of different autocorrelations: a) theoretical a-priori autocorrelation, b) algorithmic Fourier autocorrelation, c) resulting experiment autocorrelation

The error function and the mean error are computed by finding the difference between the two autocorrelation being compared. An example of the error function calculated is presented on the figure 4. Due to the symmetry of the digital spectrum the right half of the error function plot with the peak on the very right could be ignored.

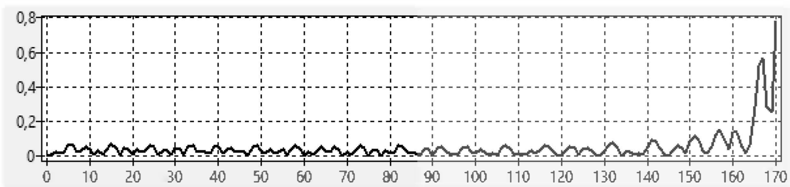

Fig. 4. Error function shown by the program

When generating determined signals comparison is done between the resulting autocorrelation and the theoretical autocorrelation that is derived a priori. The random signals are qualified on the difference between the experimental autocorrelation and the algorithmic autocorrelation.

3 Two-Dimensional Signal Imitation in Complex Basis

3.1 The Specifics of Two-Dimensional Signal Processing

The structure of multidimensional signals presents the certain level of difficulty when it comes to both representing and processing [5]. Figure 5 shows the two-dimensional signal $S(\omega_1, \omega_2) = \sin(\omega_1^2 + \omega_2^2)$.

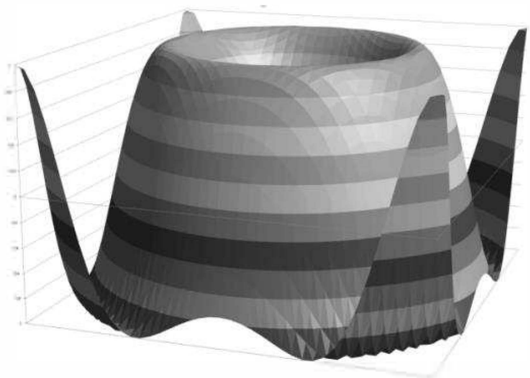

Fig. 5. Two-dimensional spectral density

The Fourier transform is different when it comes to multidimensional signals as the Fourier functions are defined in the $\mathbb{R}^n$ space. But discrete Fourier transform exchanges the $\mathbb{R}^n$ space for the n-dimensional arrays of numbers. The direct discrete Fourier transform:

$$Fx(K_1, \ldots, K_n) = \sum_{a_1=0}^{N_1-1} \ldots \sum_{a_n=0}^{N_n-1} fx(a_1, \ldots, a_n) e^{-i\frac{2\pi}{A_1}a_1K_1 - \cdots - i\frac{2\pi}{A_n}a_nK_n},$$

where $0 \leq K_i \leq A_i - 1, i = 1, 2, \ldots, n$. Inverse transform:

$$fx(a_1, \ldots, a_n) = \frac{1}{A_1 \ldots A_n} \sum_{K_1=0}^{A_1-1} \ldots \sum_{K_n=0}^{A_n-1} Fx(K_1, \ldots, K_n) e^{i\frac{2\pi}{A_1}a_1K_1 + \cdots + i\frac{2\pi}{A_n}a_nK_n},$$

where $0 \leq a_1, \ldots, a_n \leq A_{(1,\ldots,n)} - 1$.

However, before advancing into two-dimensional domain it was decided to study the specifics of the "quasi-two-dimensional" signals that are obtained by stacking together random broadband one-dimensional signals generated earlier.

3.2 Applying One-Dimensional Algorithm to Two-Dimensional Signals

Despite the need of readjusting the method for two-dimensional signals this method can already be used. To do so we just have to transform the two-dimensional spectral density into an array of one-dimensional broadband ones stacked together. Resulting two-dimensional spectral density is presented on figure 6.

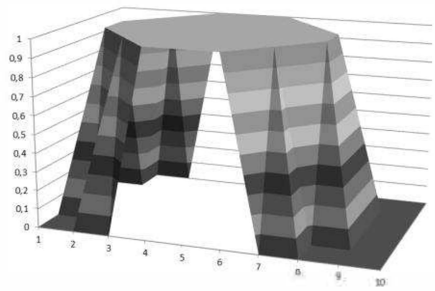

Fig. 6. Two-dimensional spectral density

Then the one-dimensional signals comprising the two-dimensional one can be generated separately and stacking them together side by side provides us with a two-dimensional signal (figure 7). This signal inherits the quality of either being determinate or random by the virtue of its coefficients.

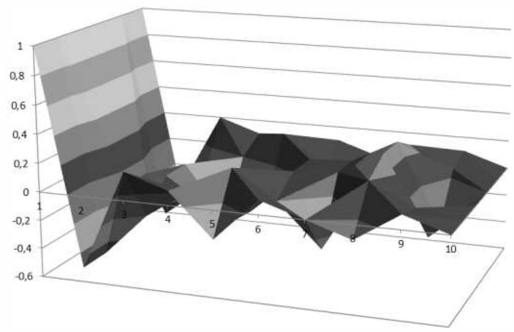

Fig. 7. Two-dimensional signal imitated

Signals generated while being two-dimensional are unidirectional as clear from the figure 7 – the most obvious trends are visible on the main horizontal axis so the so called waterfall plot appears. Waterfall plots are encountered in medicine [13], in physics [14] and in other fields where one-dimensional signals that follow the same trend are analyzed [15], therefore the need for generating arrays of such codirected signals is also present.

4 Conclusion

This paper is a part of a new development for high-dimensional signal simulation that is presented in the conference by the paper where the author was involved too. The method of imitation developed earlier for one-dimensional imitation was used to imitate two-dimensional signals. Further research and adaptation of this method is to be performed in due course.

The method of imitation in complex basis reduces algorithmization to the execution of pre-derived mathematical equations, which reduces the computational complexity and resource intensity of the algorithm, and the use of linear data structures positively affects the scalability of the developed solution.

The software solution was implemented in the Lazarus IDE which allowed to meet all the accuracy criteria and to create the interface. Free Pascal language used in Lazarus IDE is very clear as it was designed by mathematicians to be understood by their colleagues. This language is also widely used in education field in Russia so the program developed could be studied by the future students during their digital signal processing course.

Since the in-box work with two-dimensional signals is not supported yet and to be added later the results in section 3 were obtained by putting the one-dimensional signals comprising the two-dimensional one through the software and later stacking the results back together for the visualization through MS Excel 2010.

The first test of the one-dimensional algorithm being expanded to imitate two-dimensional signals highlighted the direction for future development: the algorithm should be adopted to allow for signals with different numbers of dimensions, the visualization facilities should be expanded. The method as it is can be used for modeling the unidirectional two-dimensional data in the form of a waterfall plot.

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