Improvement of Grayscale Images in Orthogonal Basis of the Type-2 Membership Function

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Abstract
While analyzing different images, it is very important to identify similar and/or homogeneous areas and boundaries of the objects of interest. Certain ambiguity occurred at this step can be caused by physical characteristics of the used equipment and noise in the process of the image formation, on the one part, and by inaccuracy and fuzziness introduced during digital representation and by processing algorithms, on the other part. It is shown that transition of the features to a fuzzy space, followed by the use of orthogonal transformation and visualization of characteristics synthesized on the basis of their eigenvalues, improves reliability of the objects of interest identification during analyzing of the grayscale images. Informational capabilities of characteristics synthesized with the use of the method of singular decomposition of the type-2 features in a fuzzy space are considered from the aspect of improvement of the grayscale image quality. The obtained experimental results are shown on the example of the real microscopic images.

Keywords 1
image processing, orthogonal transformations, singular decomposition, fuzzy logic, Membership Function Type-2.

1. Introduction

The number of practical problems associated with digital image processing, which are obtained with the help of standard research methods, and which are used, for example, in materials science, or medicine, or flaw detection, is constantly growing. Images formed by various information, tracking or diagnostic systems very often have a quality that is not sufficient for performing a reliable analysis. As a rule, these images contain distortions caused by physical system of their creation and process of their formation (heterogeneity of detectors, lighting, background, dynamic distortions), on the one hand, and by the methods of their representation and displaying in the processing system (discretization and quantization errors, changes in color reproduction, gray ambiguity, etc.), on the other hand.

In order to improve reliability of image analysis performed either by unaided eye or with the help of automated systems, it is desirable, first, to improve brightness characteristics of the images. Their characteristics such as contrast, brightness and resolution are of especial importance for visual perception, for example, in medical applications. For the cases when automated processing systems are used for solving a specific problem and for obtaining quantitative indicators, it is recommended to begin with determining and identifying the necessary value parameters on the basis of initial data for further identifying of the objects of interest [1].

The inherent inconsistency of the image transformation process lies in the fact that it is essential to ensure, on the one hand, maximum sensitivity of the used methods to insignificant local variations in brightness, and, on the other hand, resistance to the effects of the structure- and measurement-

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generated noises. In this case, in addition to randomness, which is described in accordance with the theory of probability, it is necessary to take into consideration an uncertainty present in the images [2, 3, 4], which is an attribute of information [5]. The currently used approach to solving the problem of image analysis is based on the use of fuzzy methods [6, 7, 8] because of inaccuracy and incompleteness of initial data and ambiguity of processing algorithms (for example, when one determines classes, regions/boundaries of the objects).

2. Review of Literature

One of the ways to solve the problem of improving quality of the images and reliability of their analysis assumes transition of informative features formed by the method of spatial transformations [10] to a new space. From the number of methods used for forming informative features, a specific one should be chosen by physical nature and brightness characteristics of initial data and in accordance with the task set [11]. The difficulty in determining an effective solution method is explained by the fact that characteristics of the imaging system, as well as presence and parameters of the object of interest are often unknown a priori.

Fuzzy sets of the type-1 (T1) make it possible to transform an uncertainty into the membership function that has a numerical value within the range $[0,1]$. Nonlinearity of the fuzzy processing methods increases influence of variations in the brightness properties of the analyzed images and eliminates ambiguity of gray. However, the fuzzy sets of the type-1 (T1) do not take into consideration uncertainties in the membership functions, since they are characterized by clear values [12, 13, 14].

Fuzzy logic of the type-2 (T2) was introduced by Zadeh L. as a generalized concept of the theory of ordinary fuzzy sets and make it possible to consider problems with a higher degree of uncertainty, which is typical, in particular, for the methods of image representation and for the algorithms of the image processing [15, 16]. The corresponding membership functions T2 are determined as a generalized fuzzy set by introducing fuzzy intervals; such approach correlates with inaccuracy perception by humans [6]. Fuzzy sets of the T2 feature fuzzy membership functions and are able to simulate such uncertainties [8, 17]. Fuzzy membership functions T2 is characterized by upper $\overline{h(x)}$ and lower $\underline{h(x)}$ boundaries, each of them is determined by the lower (LMF) and upper (UMF) membership functions T1.

The usage of fuzzy logic makes it possible to minimize uncertainties: removing randomness in fuzzy sets T1 leads to unambiguity, and removing uncertainty in T2 leads to the fuzzy sets T1.

Today, image processing algorithms with using T2 are proposed for solving problems of clustering [18], filtering [19], boundary detection [20] and in the image classification [21]. In [22], informational capabilities of the method of segmentation of low contrast grayscale images based on the fuzzy sets T2 were investigated.

3. Problem statement

Purpose of this work was to study a method for improving quality of the grayscale images basing on orthogonalization of fuzzy membership functions of the type-2 with using a method of singular value decomposition and selective filtering of combined noise in conditions of a priori uncertainty of the imaging system characteristics and in the absence of a priori information about the location of a possible objects of interest.

4. Materials and Methods

The ideas of methods of projection into eigen subspaces as one of the tools for mathematical processing of experimental data were presented in [23, 24]. From the theory, it is well known that decorrelation makes it possible to separate information and improve accuracy of estimates, which, from a fundamental point of view, opens up additional possibilities for heightening sensitivity of the analysis procedure.

The Principal Component Analysis (PCA) algorithm for image processing problems representing two-dimensional structures was first applied in practice in the 2000s [25]. Currently, the PCA and other orthogonalization methods are used for solving such problems as compression of visual information and feature extraction during object recognition and video image search.

In the tasks of processing an ensemble of K images represented by a matrix of brightness values I and dimension $[dy \times dx]$ with the help of orthogonalization methods, the main purpose is to transform initial data into a new coordinate system, for which the following condition is satisfied: the sample variance of the data along the values of the *k*-th coordinate should be maximum on the assumption of orthogonality to the first *k-1* coordinates.

In [26], an algorithm for improving sensitivity and reliability of the image segmentation is described, which is based on the process of multi-stage processing, which includes: expansion of space of the input features based on the initial data by using the fuzzy clustering method; orthogonalization of the obtained fuzzy membership functions; and, on their basis, formation and visualization of new informative features.

In [27, 28], informational capabilities of the method of automorphic imaging with implementation of orthogonal decomposition are considered in relation to solving the problems of filtering of low contrast grayscale images. The method is based on the use of dimension-increasing informative features of locally adaptive transformations, and makes it possible to apply methods of multidimensional data processing to the grayscale images.

Processing of images with the help of the fuzzy methods can be considered as a nonlinear type of the initial data transformation, the feature of which is that it is performed for the functions of pixel membership with predefined clusters [2].

One of the most common types of images formed by systems of various physical nature are the grayscale images represented by the level of quantization of their energy characteristics $I(x, y)$, where x and y are the coordinates of a pixel, which usually takes a value within the range $[0,1]$ or $[0,255]$.

An image of the size of $[dy \times dx]$ with gray levels L can be considered as an array of the fuzzy singletons (i.e. fuzzy sets with a single reference point), which display a value of the membership of the fuzzy set T1 $u_{x,y}$ for each point of the image $I(x, y)$ relative to its property to be analyzed (for example, brightness, homogeneity, noise, etc.). Then, in order to describe an ambiguity of the type-2 inherent to the image, one can introduce the fuzzy membership functions of the type-2 (MFT2) basing on the expression:

$$a_{k,i}^{t} = u_{k,i}^{h} - u_{k,i}^{l}, \quad (1)$$

where u^h is the "upper" value of the membership function T1 (MFT1), and u^l is the "lower" value of the MFT1.

In Figure 1 graphical image of the membership functions T2 is shown, which is characterized by upper and lower boundaries, each of them is determined by the lower (LMF) and upper (UMF) membership functions of the type-1. The shaded area in right part of Figure 1 is called a footprint of uncertainty (FOU) and displays uncertainty of the solution made in the range between the upper and the lower boundaries. The distribution form is determined by nature of the problem uncertainty. The membership estimation for each element of the T2 type is a fuzzy set with the value of the T2 within the range $[0,1]$.

For the case of a gray-scale image, this approach allows forming a multidimensional matrix X with dimensions $[dy \times dx \times K]$, which makes it possible to use an orthogonal singular transformation during its processing.

From mathematical point of view, matrix X is decomposed into the product of three matrices:

$$X = U \cdot S \cdot V^{T}. \quad (2)$$

Here, U (left singular vectors) is a matrix formed by the orthonormal eigenvectors u_r of the matrix $X \cdot X^{T}$, which correspond to the values λ_r:

$$X \cdot X^T \cdot u_r = \lambda_r \cdot u_r, \tag{3}$$

V (right singular vectors) is a matrix formed by the orthonormal eigenvectors v_r of the matrix $X^T \cdot X$:

$$X^T \cdot X \cdot v_r = \lambda_r \cdot v_r, \tag{4}$$

and S (eigenvalues) is a positively-defined diagonal matrix whose elements are singular values $\sigma_1 \geq ... \geq \sigma_r \geq 0$ equal to the square roots of the eigenvalues λ_r.

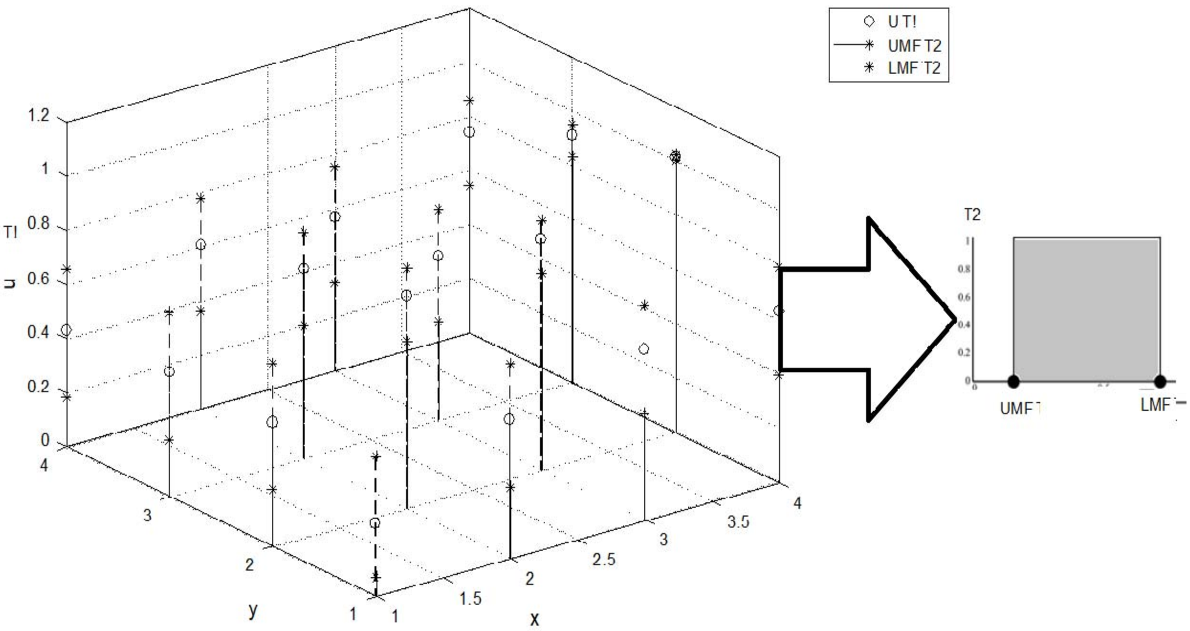


Figure 1: Type-2 fuzzy set: lower (LMF) and upper (UMF) membership functions and its interval a footprint of uncertainty (FOU).

As a result, this transformation provides a transition of informative features to the new orthogonal space. The first singular vector is low-frequency and has a constant component not equal to zero. The rest of the singular vectors can be interpreted as noise components.

The combination of singular eigenvectors obtained in the space of membership functions T2 for gray levels of the grayscale image allows performing selective filtering of the noise.

Therefore, the proposed in this work algorithm for improving quality of the grayscale images consists of the following steps.

1. Scaling of initial range of the input image brightness to the range $[0,1]$.
2. Preprocessing of initial image The need for this step arises due to the possible irregularity of the background component in the image, which, in particular, leads to the appearance of light-stuck or darkened areas. In this research, we performed this procedure by the methods of adaptive power-law correction of brightness and local background subtraction [29]. In the first method, the initial grayscale image I undergoes the following transformations:

$$I^1_{x,y} = I_{x,y} - 1/256, I_{x,y} > \left(\bar{I} + 0.5\right)/2, x \in [1, dy], y \in [1, dx], \tag{5}$$

where $\bar{I}$ is a mean of the brightness of image I. The need of applying the expression (5) is explained by necessity to use the power transformations. As a result, a slight decrease in brightness occurs, and values equal to 1 are deleted. Then, the image I^1 is subjected to the following transformations:

$$I^2_{x,y} = \left(I^1_{x,y}\right)^{1+\mathrm{sgn}\left(I^1_{x,y}-\bar{I}\right)\cdot\left(I^1_{x,y}\right)^{\left(1-\mathrm{sgn}\left(I^1_{x,y}-\bar{I}\right)\cdot I^1_{x,y}\right)\cdot\left(\frac{\left\{\bar{I}+0.5\right\}}{2}-I^1_{x,y}\right)}}, \tag{6}$$

$$I^3_{x,y} = \left(I^2_{x,y}\right)^{1+\left(I^2_{x,y}\right)^{1-I^2_{x,y}/2}}. \quad (7)$$

Transformation (6) proportionally reduces brightness of the light-stuck areas and increases brightness of the dark areas. Transformation (7) proportionally reduces brightness of the entire image.

For the case when brightness correction is made by the method of local background subtraction [29], transformation of the initial image is performed with the help of non-overlapping windows (in our experiments window size was $[15\times15]$), and brightness of all pixels in each window is determined by the expression:

$$w^1_{x,y} = \left(\overline{w} + \overline{I}\right)/2. \quad (8)$$

So, this is the way of how values of the image brightness I^1. On its basis, the image I^3 is formed by the formula:

$$I^3_{x,y} = I_{x,y} - I^1_{x,y}. \quad (9)$$

After applying expression (9) the image I^3 is scaled to the range $[0,1]$ and adaptive histogram equalization is applied to the obtained image.

3. The image I^3 is interpreted as a fuzzy membership function. And for it, the MFT2 (I^4)is formed on the basis of the "upper" (I^3_h) and "lower" (I^3_l) values of the MFT1 according to the formula (1). In this case, I^3_h и I^3_l are calculated by the method of power transformations with using the following formulas:

$$\left(I^3_{x,y}\right)_h = \left(I^3_{x,y}\right)^{1-\left(I^3_{x,y}\right)^{1-I^3_{x,y}/2}}, \quad (10)$$

$$\left(I^3_{x,y}\right)_l = \left(I^3_{x,y}\right)^{1+\left(I^3_{x,y}\right)^{1-I^3_{x,y}/2}}. \quad (11)$$

This step is necessary for the subsequent application of the singular value decomposition.

4. An ensemble of images is formed (I^3, I^4, I^3_l when performing the step 2 with the use of power-law correction, or I^4, I^3_l, I^3_h in the case of preprocessing by the method of local background subtraction). Various methods are required for forming an ensemble of images because when the processed images are light-stuck the preprocessing method affects the brightness level I^3_h. In case of the use of adaptive power-law correction, brightness level I^3_h слишком высок, is too high, which makes this image uninformative in comparison with the preprocessed initial picture.

For this ensemble, singular value decomposition is applied. The resulting matrix of the left singular vectors U is interpreted as a multidimensional image I^5 with dimensions $[dy \times dx \times K]$, each spectral component of which is scaled to the range $[0,1]$.

5. On the basis of the matrix of right singular vectors V , a vector of coefficients C is calculated (it is used for estimating significance of the components of the matrix of left singular vectors) by the following formula:

$$C_i = \left(\left|\sum_{j=1}^{K} V_{i,j}\right| + \left|\sum_{j=1}^{K} V_{j,i}\right|\right)/2, i \in [1, K]. \quad (12)$$

This vector is ordered in descending order, and its elements are normalized so that their sum is equal to 1.

6. The vector dC is formed, which contains differences for each pair of adjacent elements of the vector C.

7. The value dC_a is calculated by the formula:

$$dC_a = \left(\left(\sum_{j=1}^{K-1} dC_j / (K-1)\right) + \left(dC_{\min} + dC_{\max}\right)/2\right)/2, \quad (13)$$

where $dC_{\min}$ and $dC_{\max}$ are the minimum and maximum elements of the vector dC, respectively.

8. On the basis of the value dC_a, when scanning the elements of the vector dC starting from the end such index $i_{\max}$, is selected, for which the following condition should be satisfied:

$$dC_{i_{\max}} \geq dC_a . \tag{14}$$

Then elements of the vector dC with indices from 1 to $i_{\max}+1$ are normalized so that their sum is equal to 1.

9. The final image I^6 is formed as a weighted sum of the most significant components of the matrix U according to the following formulas:

$$I^6_{y,x} = \sum_{j=1}^{i_{\max}+1} I^5_{x,y,j} \cdot dC_j \cdot S_j , \tag{15}$$

$$S_j = \operatorname{sgn}\left(\sum_{j=1}^{n_i} V_{i,j} \cdot \sum_{j=1}^{n_i} V_{j,i} \right). \tag{16}$$

For the final image, histogram equalization is applied when its preprocessing is performed by the method of adaptive power-law correction, and inversion, which is followed by adaptive histogram equalization, is applied when the preprocessing is based on the local background subtraction method.

5. Results and Discussion

Quantitative metallography is widely used for specifying characteristics of the alloy microstructure, namely: volumetric content of phases, grain size, specific surface area of grain boundaries, distance between similar particles or phases, and others.

To obtain high quality materials, experimental alloys are smelted and mechanism of forming their structure and morphology, which determine their properties, is researched. For achieving this goal, among others, methods that allow to form images of the test samples are used (for example, micro X-ray or X-ray structural, metallographic analyzes, microscopy). Based on analysis of the images brightness characteristics it is possible to determine such quantitative parameters as the average size of phases, geometrical value of the external specific surface of the phase, and statistical data.

However, used digital images often feature an inadequate quality due to irregular background, noise, aberration artifacts, poor contrast, etc. In order to obtain reliable quantitative information from the pixel intensity values, it is necessary to apply correction methods to ensure good accuracy of photometry and to eliminate common defects of the images.

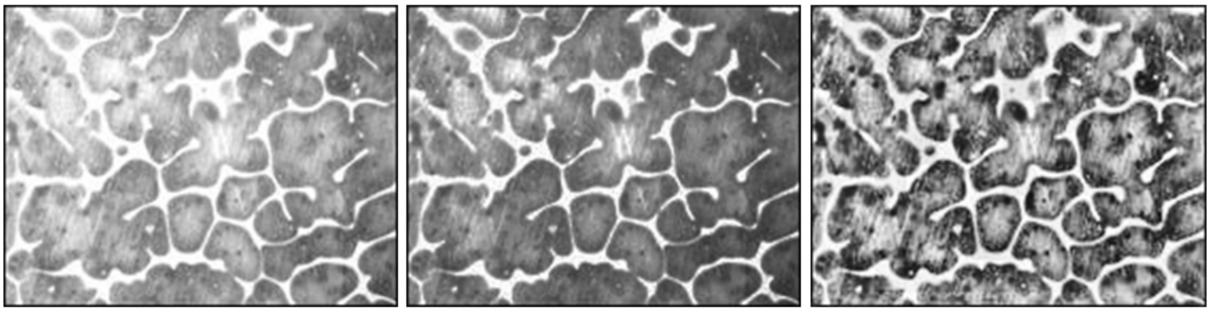
In Figure 2a, an example of grayscale image of the phosphorus-containing alloy Fe–2%P–0,042%C is shown. This image was obtained with magnification of x250 on a metallographic microscope GX51 with a digital image analysis system of the company "Olympus". To separate the individual phases of the alloy during determining its structure and properties, chemical etching was used at first. After that thermal etching at temperatures of 400-600 ^{o}C with natural air circulation was performed. The atoms that make up the phases interact with oxygen; therefore, an oxide membrane of different thickness is formed in different phases. When observing through an optical microscope and, accordingly, in the image, the phases have different levels of intensity [30].

The analysis of image in Figure 2a is difficult, in particular, due to the nonuniformity of the background and the presence of the light-stuck area.

To obtain experimental results we used Matlab 6.1. Source code for proposed methods was written in internal Matlab language (except for standard functions such as svd, histeq, adapthisteq, mat2gray).

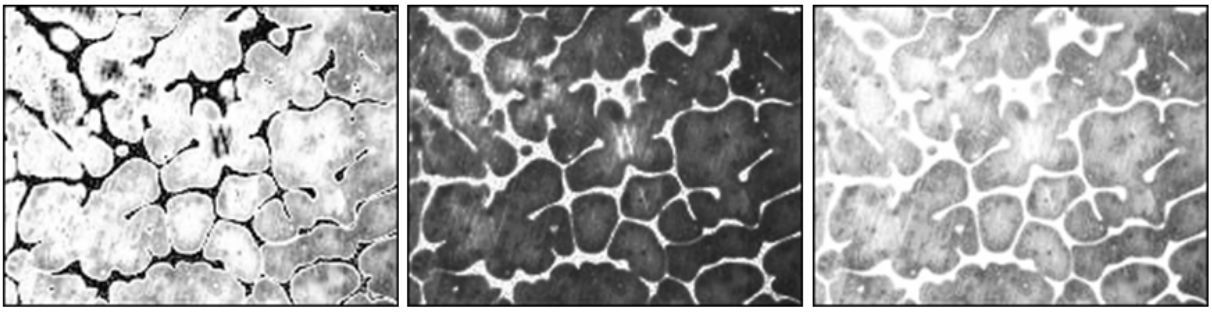
In Figure 2b and Figure 2c, the results of preprocessing of the initial picture by means of the adaptive power-law correction and by method of local background subtraction are shown, respectively. In both cases, we managed to reduce brightness of the light-stuck area and to preserve the overall intensity level at the level acceptable for visual analysis. In the latter case, the image is more detailed, but at the same time it contains a blur effect.

In Figure 3 and Figure 4, the results of MFT2 formation are shown, as well as the "lower" and "upper" values of the MFT1 interpreted as grayscale images, for both preprocessing methods. It can be easily seen that preprocessing based on the local background subtraction method gives higher brightness in the generated images.



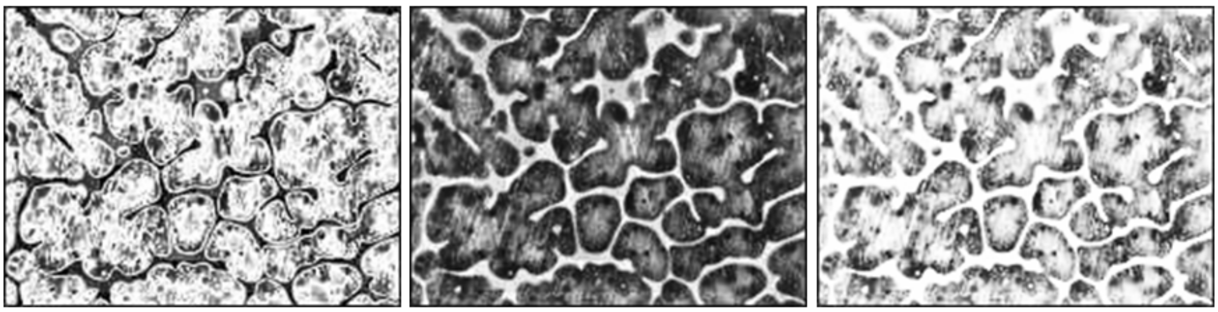
a) b) c)

Figure 2: Image of the phosphorus-containing alloy Fe–2%P–0,042%C: a – initial grayscale image (142x186); preprocessing based on the b – adaptive power-law brightness correction; c – method of local background subtraction.



a) b) c)

Figure 3: Calculation of the MFT2 for preprocessing based on the adaptive power-law brightness correction: a – MFT2; b – "lower" value of the MFT1; c – "upper" value of the MFT1.



a) b) c)

Figure 4: Calculation of the MFT2 for preprocessing based on the method pf local background subtraction: a – MFT2; b – "lower" value of the MFT1; c – "upper" value of the MFT1.

In Figure 5, formation of the resulting image for both preprocessing methods is shown. It should be noted about different levels of detailing in the results. Use of the adaptive power-law correction led to the lower level of the resulting image brightness, but, at the same time, gives better image definition, which is preferable when determining quantitative parameters and highlighting the contours of the objects and various areas within one object. Preprocessing based on the method of local background subtraction resulted in a brighter and more contrasting image, which simplifies its visual analysis, but there the blur effect is seen.

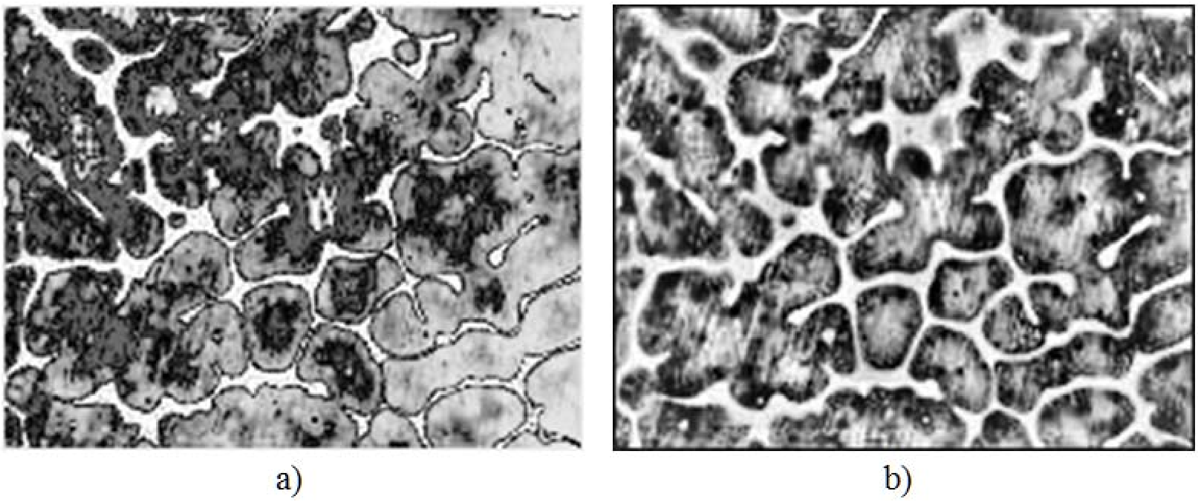


Figure 5: The resulting image after preprocessing by the: a – adaptive power-law intensity correction; b – local background subtraction method.

6. Conclusions

On the basis of analysis of the experimental results obtained, the following conclusions are made:

- the usage of fuzzy sets of the T2 type allows to synthesize additional parameters on the basis of the grayscale initial image with using of nonlinear functions based on the local transformation of the brightness levels, and to obtain an ensemble of data, to which methods of multidimensional information processing can be applied;
- orthogonalization, the method of singular value decomposition in particular, applied to the ensemble of images with taking into account components of the T2 set, makes it possible to take into consideration original ambiguity and uncertainty of the initial data, to analyze the ensemble as a whole and, at the same time, to interpret each new component as a result of anisotropic filtering in two-dimensional plane of the spatial frequencies;
- visualization of the parameters synthesized on the basis of the eigenvalues of the singular value decomposition, makes it possible to increase the level of detailing, contrast and resolution of the resulting image and, hence, to improve reliability of visual and automated analysis;
- promising areas for further researches are the usage of different orthogonal transformations; and applying of various methods for preprocessing of initial data and formation of sets of the T2 type.

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