Model for the Production Capacity Structure Optimizing in the Context of Digital Transformation

Georgiy Kolesnik[1[0000-0003-0695-5038]] and Mikhail Rybakov[2[0000-0001-8924-0366]]

[1] Plekhanov Russian University of Economics, Stremyanny lane 36, Moscow, 117997, Russia
[2] JSC Russian Helicopters, Bolshaya Pionerskaya, 1, Moscow, 115054, Russia
gvkolesn@inbox.ru

Abstract. An important trend of digital transformation of industrial enterprises is the increase in the efficiency of the use of their production capacity. This issue is particularly relevant for holding structures that have emerged in the process of reforming of high-tech engineering industries.
The use of digital technologies that provide dynamic forecasting and optimization of the state of such systems allows to achieve a number of improvements, including reducing the time and cost of production. Higher production capacity utilization helps to increase profits and the financial stability of the company.
In this paper, a mathematical model is considered that allows estimation of efficiency of strategies for development of production capacities parks. Based on this model a computing algorithm is developed that determines the optimum financing of the production capacity for a wide range of stakeholders' criteria.
The analysis of the mathematical model allowed us to characterize the type of optimal financing strategy for systems with additional requirements for reliability of operation with the efficiency criterion determined by the minimum number of functional production assets in the planning period.

Keywords: Capital Assets, Production Capacity, Digital Transformation, Dynamic Model, Optimization.

1 Principles of Managing the Structure of the Company's Production Capacity

One of the important directions of digital transformation of industrial enterprises is to increase the efficiency of the use of their production assets. This issue is particularly relevant for holding structures that have emerged in the process of reforming the high-tech engineering industries (aircraft, shipbuilding, defense industry) and include enterprises, which possess a significant variety of production assets.

The use of digital technologies that provide dynamic forecasting and optimization of the production capacity of such systems allows achieving a number of improvements, including production time and cost reduction and higher assets utilization. This helps to increase profits and the financial stability of the enterprise.

Proceedings of the of the XXIII International Conference "Enterprise Engineering and Knowledge Management" (EEKM 2020), Moscow, Russia, December 8-9, 2020.

Dynamic optimization of the production capacity use should take into account the issues of operation and maintenance of objects at different stages of the life cycle, determined by their individual characteristics and regulation [1, 2].

The additional peculiarities in the operation of the production assets arise with the increase in their variety due to the technological development. They include the need for joint operation of equipment of different generations and, as a result, the adaptation of existing resources to the use of more advanced means in order to prevent a decrease in their efficiency [3].

The combination of different types of capital assets used in the enterprise's production process forms its production facilities park. While the issues of modelling the life cycle of a single object of capital assets are well covered in the scientific literature (see, for example, [4 - 6]), the life cycle of a production facilities park is much less studied. Its modelling is usually carried out empirically, without sufficient theoretical justification.

In general, the life cycle duration of a production facilities park is a random variable, since it largely depends on the properties of its components, their cost, durations of their individual life cycles, the capabilities of their manufacturers and a number of other factors. The following stages can be highlighted in the production facilities park life cycle:

- park formation, that begins with the development of serial production of the corresponding equipment by industry;
- dynamic equilibrium, within which the natural loss of production capacity is fully compensated by the supply of new objects;
- aging and re-equipment, when the natural loss is not compensated due to the termination of production of this type of equipment, the remaining objects in operation are removed from service and replaced with new types of equipment as their life cycle is completed.

The structure of the life cycle of the production facilities park becomes more complicated when the element base of the equipment changes. Using a new element base and increasing the complexity of new devices leads to the need to significantly adjust the maintenance technology and adapt it to the conditions of joint operation of modern and outdated facilities [7].

In this paper, we formulate a mathematical model that allows estimation of efficiency of strategies for development of production capacities parks. Based on this model a computing algorithm is developed that determines the optimum financing of the production capacity for a wide range of stakeholders' criteria. The algorithm can be used for decision-making on financing the development of industrial organizations in the conditions of digitalization, since it takes into account specific non-financial criteria for their functioning.

2 Modeling of production facilities park operation processes

Let us study the process of a production facilities park creation and operation. Formally, it can be considered as a finite set of objects, each of which can be in one of the following states at any given time:

1. arrival of the object;
2. operation;
3. current repairs;
4. major repairs;
5. brand repair;
6. modernization;
7. utilization.

The moments of transition of objects from one state to another are generally random and are described by a Poisson distribution with intensities λ_{ij} determined by the properties of the system of operation, maintenance and repairs at the enterprise (Fig. 1).

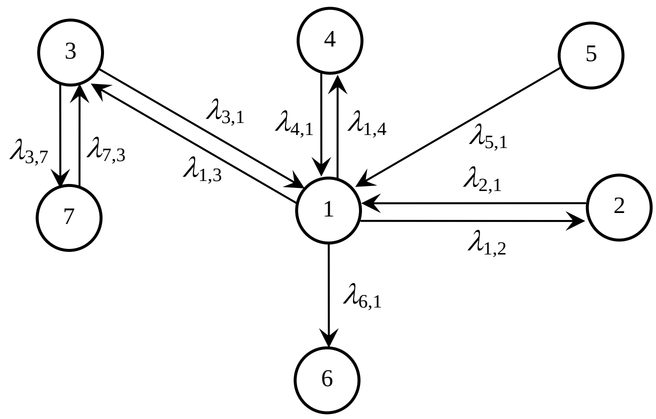

Fig. 1. Graph of the states of production facilities park objects

The arrival of new assets not only restores the system's resource, but also contributes to its modernization and acquisition of a new level of quality. A large number of factors that influence the production process and the adoption of new assets make this process stochastic.

The probability of arrival of a new assets to the park at any time can be determined by the normal distribution of time intervals between regular deliveries using the formula

$$\lambda_{5,1}(t)=\frac{\sqrt{\frac{2}{\pi}}\exp\left[-\frac{\left(\tau^{*}+vt-\tau T_{\varPi}\right)^{2}}{2\sigma_{\tau}^{2}}\right]}{\sigma_{\tau}\left[1+\Phi\left(\frac{\tau^{*}+vt-T_{\varPi}}{\sigma\sqrt{2}}\right)\right]}, \tag{1}$$

where T_P is duration of the manufacturing and delivery of a new product.

When studying the planned repair subsystem, let us assume that a repair is performed when a certain resource is running out, which is set for each type of planned repair. At the initial moment, each object has a random amount of this resource, distributed according to the normal law with the distribution density

$$P(\tau)=\frac{c_{1}}{\sqrt{2\pi}\sigma_{\tau}}\exp\left[-\frac{\left(\tau-\tau_{p}\right)^{2}}{2\sigma_{\tau}^{2}}\right], \tag{2}$$

where τ_p is mathematical expectation of the amount of time corresponding to the object's resource; c_1 - truncation coefficient; σ_τ^2 - variance.

In (2) the probability of an object running out of a resource is a function of τ. Taking into account the rate v of resource consumption, the density of the distribution of the time interval to the corresponding type of repair is

$$P(t)=\frac{v}{\sigma_{\tau}\sqrt{2\pi}}\exp\left[-\frac{\left(vt-\tau_{p}\right)^{2}}{2\sigma_{\tau}^{2}}\right], \tag{3}$$

where τ is the random value of the product's operating time, equal to

$$\tau=\tau^{*}+\tau_{u},$$

τ^* - initial resource consumption τ_u - resource consumption planned for the period t.

Given that $\tau_u = vt$, the transition intensities will be:

- for brand repairs:

$$\lambda_{1,3}(t)=\frac{\sqrt{\frac{2}{\pi}}\exp\left[-\frac{\left(\tau^{*}+vt-\tau_{p}\right)^{2}}{2\sigma_{\tau}^{2}}\right]}{\sigma_{\tau}\left[1+\Phi\left(\frac{\tau^{*}+vt-\tau_{p}}{\sigma\sqrt{2}}\right)\right]}, \tag{4}$$

- for capital repairs $\lambda_{1,4}(t)$ is determined by (4) with τ_p corresponding to the standards of capital repairs.

Assuming a relatively constant duration of the repair cycle, the total duration of the planned repair will be random due to a random delivery time, acceptance time, and other reasons. It will be distributed according to the truncated normal law with density:

$$f(t) = \frac{\alpha}{\sigma_t\sqrt{2\pi}} \exp\left[-\frac{(t-t_0)^2}{2\sigma_t^2}\right], \tag{5}$$

where σ_t^2 is the variance of the repair time; t_0 – the expected duration of repair; α – truncation coefficient.

Then the intensity of the flow of objects from the repair is

- for brand repairs

$$\lambda_{3,1}(t) = \frac{\sqrt{\frac{2}{\pi}} \exp\left[-\frac{(t-t_0)^2}{2\sigma_t^2}\right]}{\sigma_t\left[1-\Phi\frac{(t-t_0)}{\sigma_t\sqrt{2}}\right]}. \tag{6}$$

- for capital repairs, the intensity $\lambda_{4,1}(t)$ is determined by (6), where t_0 is the mathematical expectation of the duration of capital repair.

Modernization of production capacity is carried out in the following cases:

- to eliminate their obsolescence and improve their performance;
- to replace components that are not supplied anymore by the manufacturer.

Assume that there are N objects in the park, each of which consists of K groups of m_j elements in each group. The faulty element is sent for repair or replacement to the manufacturer and then it is returned to restore the object. The average recovery time for an element at the manufacturer is t_{1j}. The average recovery time through upgrades or improvements is t_{2j}. Then the average recovery time for a single spare part is

$$t_{3j} = t_{1j} + t_{2j}.$$

Then the intensity of the transition to the state of modernization is determined by the formula:

$$\lambda_{3,7}(t) = \sum_{j=1}^{k}\left\{m_j\left[\frac{\tau}{t}\cdot\lambda_{1j} + \left(1-\frac{\tau}{t}\right)\cdot\lambda_{2j} + n_{1j}\cdot\lambda_j^2\right]\right\}, \tag{7}$$

where n_{1j} – the number of elements of the j-th group, λ_{1j}, λ_{2j} - respectively, the failure rate of the element of the j-th group directly in the equipment and in a set of spare parts.

The length of stay of the object in this state depends on the duration of the production cycle of the upgrade kit, the duration of the operations, the configuration and commissioning of the modified product. When performing modernization at manufacturing or repair plants, the duration of the modernization cycle increases by the duration of delivery of the object to the place of modernization and back.

Assuming that the delivery time intervals are distributed according to the exponential law, the intensity of the objects' exit from the state of modernization can be determined by the formula:

$$\lambda_{7,3}(t) = \lambda_e(t) + \lambda_i(t) + \lambda_m(t), \tag{8}$$

where $\lambda_e(t)$, $\lambda_i(t)$, $\lambda_m(t)$ are the rates of delivery of the spare parts from the enterprise's warehouse, from the intermediate warehouse and from the manufacturer, respectively.

The intensity of write-offs and withdrawals of objects from the park is determined by the formula:

$$\lambda_{1,6}(t) = \frac{\sqrt{\frac{2}{\pi}} \exp\left[-\frac{\left(\tau^{\bullet} + vt - \tau_{\text{п}}\right)^2}{2\delta_{\tau \text{П}}^2} \right]}{\sigma_{\tau \text{п}} \left[1 - \Phi \frac{\left(\tau^{*} + vt - \tau_{\text{п}}\right)}{\sigma_t \sqrt{2}} \right]}. \tag{9}$$

where τ^* is the available resource consumption; v - the rate of resource consumption per unit of time; $\delta^2_{\tau\text{П}}$ - variance; $\tau_{\text{П}}$ – maximum allowable resource consumption.

Thus, the analysis of an object in the production facilities park shows that it can be represented by a system S, which at each time can be in one of the states A_1, A_2, ..., A_7. The probability of transition to any state A_i, $i = 1, ..., 7$ at the time t_s depends only on its previous state. Therefore, it is Markov process described by the Kolmogorov system of differential equations [8].

Assume that each object in the park can be in one of the states at each moment t. It is obvious that the sum of the numbers of objects in all states is equal to the total number of objects, i.e. if we denote by $X_i(t)$ the number of objects that are in the i-th state at the moment t, then

$$\sum_{i=1}^{n} X_i(t) = N, \tag{10}$$

where N is the total number of production assets in the park.

The value $X_i(t)$ is a random function of time. By defining for any t its mathematical expectation $m_i(t)$ and the variance $D_i(t)$, the average value of the number of objects in each state can be found, as well as the spread of the actual number around the average.

Merging the above relationships into a single system, we get the following model of the state dynamics of the production assets in the park:

$$\frac{dm_1}{dt} = -m_1(\lambda_{1,3} + \lambda_{1,4} + \lambda_{1,2} + \lambda_{1,6}) + \lambda_{2,1}m_2 + \lambda_{3,1}m_3 +$$

$$+\lambda_{4,1}m_4+\lambda_{5,1}m_5;$$

$$\frac{dm_2}{dt}=\lambda_{1,2}m_1-\lambda_{2,1}m_2;$$

$$\frac{dm_3}{dt}=-m_3(\lambda_{3,1}+\lambda_{3,7})+\lambda_{7,3}m_7+\lambda_{1,3}m_1;$$

$$\frac{dm_4}{dt}=\lambda_{1,4}m_1-\lambda_{4,1}m_4; \tag{11}$$

$$\frac{dm_5}{dt}=\lambda_{5,1}m_5;$$

$$\frac{dm_6}{dt}=\lambda_{1,6}m_1;$$

$$\frac{dm_7}{dt}=\lambda_{3,7}m_3-\lambda_{7,3}m_7.$$

For the known intensities of event flows, the expectation and variance of the i-th state number will be

$$m_i(t) = NP_i(t),$$

$$D_i(t) = NP_i(t)(1 - P_i(t)),$$

where $P_i(t)$ is the probability of the i-th state of the object.

Based on these results, the most rational parameters of maintenance and current repairs are determined, as well as requirements for reliability, maintainability, and durability at the life cycle of the production facilities park.

3 Modeling of financial and economic aspects of production capacity development

The efficiency of production organizations is largely determined by financial, economic and social factors that characterize the ability of markets and the state to meet their needs for various types of resources [9]. These factors, on the one hand, act as the material basis for the functioning of industrial enterprises, and on the other hand, as constraints limiting the maximum permissible level of diversion of the resources from other sectors of the economy. Thus the resource and economic justification of an enterprise's production capacities development strategy becomes of great importance in modern conditions.

The above-described model of production facilities park life cycle reflects only the technological aspects of this process, leaving behind their dependence on funding.

Ignoring the economic aspects of the process might result in unreliable estimates, since the intensities of the event flows in the model depend on the volume of financing allocated for the corresponding activities.

Let us study the impact of financial constraints on the properties of the optimal mode of development of the production facilities park. To this end, we enhance the model (11) with a description of the financial and economic aspects of this process.

For a given enterprise, consider a project of financing the development of production capacity in the form of cash flow $\{X_t\}$, $t = 0,..., T$, where X_t is the funds allocated for financing in the time t. The following representation of total expenses holds

$$X_t = X_t^0 + X_t^1 + X_t^2, \tag{12}$$

where X_t^0, X_t^1, X_t^2 denote the funds allocated for the repair, modernization, and for the purchase of the new assets, correspondingly.

In this case, the intensities of new assets inflow to the park ($\lambda_{5,1}$), as well as their return from repair ($\lambda_{4,1}$) and from modernization ($\lambda_{3,7}$) become increasing functions of the corresponding expenditures (X_t^0, X_t^1, X_t^2):

$$\begin{aligned} &\lambda_{4,1}(t) = G_{4,1}(t, X_1^0, \ldots, X_{t-1}^0). \\ &\lambda_{3,7}(t) = G_{3,7}(t, X_1^1, \ldots, X_{t-1}^1); \\ &\lambda_{5,1}(t) = G_{5,1}(t, X_1^2, \ldots, X_{t-1}^2); \end{aligned} \tag{13}$$

If we assume that the allocated funds are fully spent within a single period, then the functions $G_{i,j}$ will depend only on the amount of funding in the period t-1:

$$\lambda_{4,1}(t) = G_{4,1}(t, X_{t-1}^0), \quad \lambda_{3,7}(t) = G_{3,7}(t, X_{t-1}^1), \quad \lambda_{5,1}(t) = G_{5,1}(t, X_{t-1}^2).$$

Using these dependencies, the model (11) takes the following form

$$\begin{aligned} &\frac{dm_1}{dt} = -m_1(\lambda_{1,3} + \lambda_{1,4} + \lambda_{1,2} + \lambda_{1,6}) + \lambda_{2,1}m_2 + \lambda_{3,1}m_3 + \\ &\qquad + G_{4,1}(t, X_{t-1}^0)m_4 + G_{5,1}(t, X_{t-1}^2)m_5; \\ &\frac{dm_3}{dt} = -m_3(\lambda_{3,1} + G_{3,7}(t, X_{t-1}^1)) + \lambda_{7,3}m_7 + \lambda_{1,3}m_1; \\ &\frac{dm_4}{dt} = \lambda_{1,4}m_1 - G_{4,1}(t, X_{t-1}^0)m_4; \\ &\frac{dm_5}{dt} = G_{5,1}(t, X_{t-1}^2)m_5; \\ &\frac{dm_7}{dt} = G_{3,7}(t, X_{t-1}^1)m_3 - \lambda_{7,3}m_7. \end{aligned} \tag{14}$$

In contrast to the basic model, the dynamic system (14) is controllable. Indeed, by choosing a specific flow of financing $\{X_t\}$, the enterprise's management can influence the intensity of the transition between the states of the system, and consequently, the quantitative and qualitative composition of the production capacity.

Using this relationship, it is possible to consider the process of production facilities park development as an investment project of specific type. Then the problem of choosing its optimal mode can be formulated in the following form.

Consider a set of investment projects A. The implementation of each of them $a \in A$ is associated with the cost $\{X_t^a\}$ and yields a profit $\{P_t\}$, $t = 0, ..., T$, where T is the planning horizon. The problem is to determine the project that will be optimal for the investor.

In market conditions, the standard criterion for the optimality of an investment project is its net present value (NPV) [10]

$$NPV(a) = \sum_{t=0}^{T} \beta^t (\Pi_t^a - X_t^a) \, , \tag{15}$$

The peculiarity of the system considered here is that in addition to a profit it is also characterized by other efficiency criteria [11, 12, 13]. A promising approach to their accounting is to formulate the problem as a multi-criteria one. To do this, we introduce the reliability of the system W as the additional criterion of effectiveness. It will be considered as a monotonic function of the number of assets in the park

$$W = W(R).$$

In each moment t the park size R depends on the funds allocated for its development in previous periods:

$$R_t = F_t(X_0, ..., X_{t-1}). \tag{16}$$

The model above allows to implicitly restore the structure of mappings $\{F_t\}$ for a given investment flow $\{X_t\}$. Then the problem of the production facilities park optimization can be presented as a multi-criteria optimization problem:

$$V(X) = \sum_{t=0}^{T} \beta^t X_t \to \min \tag{17}$$

$$C(R) = \min\{W(R_0), ..., W(R_T)\}. \tag{18}$$

under conditions (16).

Since the criterion in the form (18) cannot be measured in monetary terms, it seems appropriate to use methods of multi-criteria optimization. The general principle of optimality underlying these methods is Pareto efficiency of the solution, which consists in the impossibility of improving it for all criteria at the same time.

For the problem considered here, this principle is as follows: the investment flow $X = \{X_t\}$ is Pareto efficient if there is no other investment flow $X' = \{X_t'\}$, such that the pair (X', R'), where R' is determined from the (16), satisfies the conditions:

$$V(X') \geq V(X), \quad C(R) \geq C(R'),$$

and at least one of these inequalities is strict.

We will call the investment flow $X = \{X_t\}$ as rational if it satisfies the restrictions on the minimum acceptable level of efficiency C_0 and the maximum possible investment V_0:

$$V(X') \geq V_0, \tag{19}$$

$$C(R) \leq C_0. \tag{20}$$

Thus, the choice of the optimal variant of the production capacities park development can be reduced to the problem of finding an acceptable and effective point (V, C) from the set of possible solutions.

One method for solving such problems is the constraint method, which consists in reducing the original multi-criteria problem (16) - (18) to a single-criteria problem solved by standard optimization methods.

This reduction is made by introducing additional constraints that reflect the desired values of the criteria and in the subsequent optimization on a new, narrower set of alternatives.

Let us find the optimal solution to this problem in the class of stationary modes with a constant amount of the production assets in the park over time.

4 An optimal mode of production capacity development

We illustrate the application of this computational procedure using the following example of a production system. Assume that the intensity of production assets inflow from repair and modernization is constant and does not depend on the funding, and the intensity of new production assets inflow $G_{5,1}(t, Z)$ has the form

$$G_{5,1}(t, Z) = AZ^{\beta}, \tag{21}$$

where A is a normalizing factor, Z is the amount of financing, β is a scale factor.

The optimal financing of production facilities park development under given budget is shown in Fig. 2. It can be seen that in the initial period, the supply of new assets is not being financed. As a result of this, the dynamics of the number of production assets in this period is described by the transition mode.

Further, the supply of new assets in the system is financed with a constant intensity, such that their number in the system does not change. The dynamics of the number of assets in the system, as well as capital and brand repairs are shown in Fig. 3.

This mode corresponds to the maximum level of efficiency under given budget. If the system performance requirements exceed this value, the set of acceptable alternatives in the corresponding decision-making task is empty.

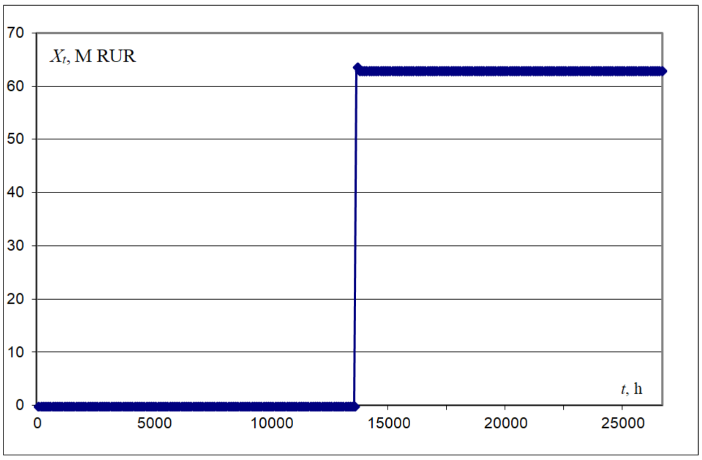

Fig. 2. The optimal financing of production facilities park development

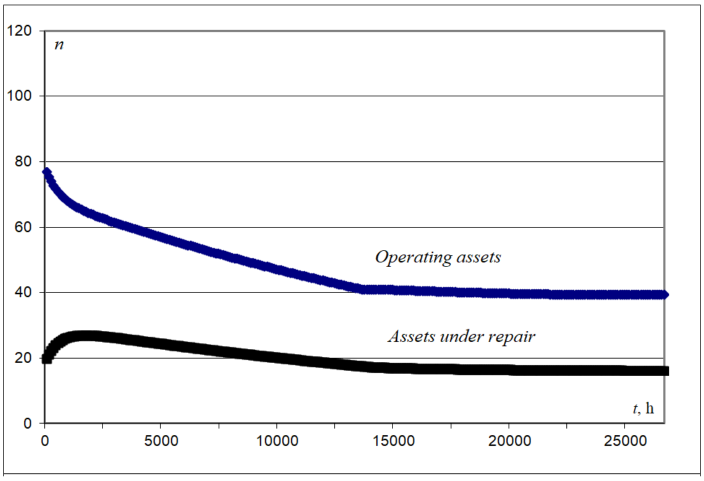

Fig. 3. Dynamics of the number of the assets in the park

In the example above, the transition mode occurred at the beginning of the planning interval. If the decision-maker has requirements for the final state of the system that are set by the boundary condition at time T, transient mode may occur on the final section of the trajectory. The example of such behavior is shown in Fig. 4.

In this example the financing is no longer piecewise constant, but increases by the end of the planning period due to the "forced" funding in order to satisfy the boundary condition. However, with a fixed total budget, this increase is compensated by underfunding of the system in previous periods, that leads to a decrease in its efficiency.

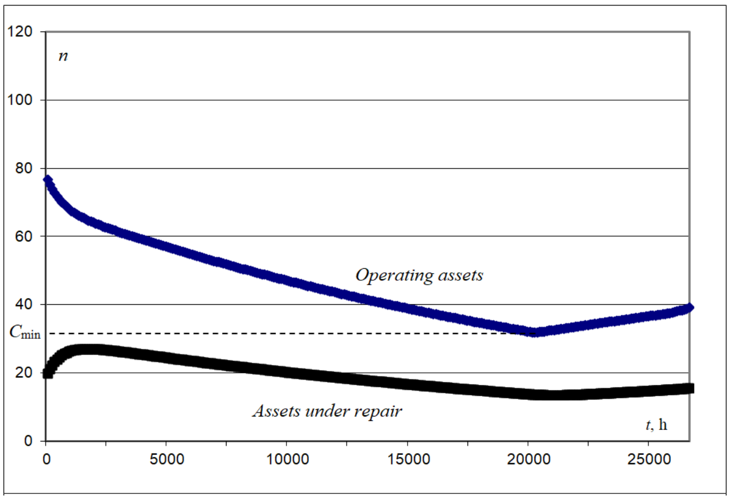

Fig. 4. Transient mode with a boundary condition

5 Conclusion

Currently the requirement of efficient use of enterprises production capacity is one of the key trends that determine its strategic development. In this regard, considerable attention is paid to optimizing production programs and investment strategies in the context of multiple performance criteria, some of which are non-economic in nature.

In this paper, a mathematical model is considered that allows estimation of efficiency of the strategies for development of production capacities parks. Based on this model a computing algorithm is developed that determines the optimum financing of the production capacity for a wide range of stakeholders' criteria. The algorithm can be used for decision-making on financing the development of industrial enterprises in the conditions of digitalization, since it takes into account specific non-financial criteria for their functioning.

The analysis of the mathematical model allows to characterize the optimal financing strategy for systems with additional requirements for reliability of operation with the efficiency criterion determined by the minimum number of functional production assets in the planning period.

The resulting mode of operation of the system is stationary, with a constant number of its elements, while non-stationary modes occur when it is necessary to satisfy the initial or terminal conditions.

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