Recurrent Estimation of the Information State Vector and the Correlation of Measuring Impact Matrix using a Multi-Agent Model

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Abstract
The information-oriented model is a multi-agent simulation model in which the integrated characteristics of information resources are the result of many local interacting individuals. The information-oriented approach in modeling involves the creation of simulation models that reproduce some criteria of information reliability and their local interaction for the built integrated models of many information resources. Information within this model is considered as a unique, discrete unit in which there is a set of characteristics that change with the introduction of the life cycle.

Keywords
Information, information-oriented model, multi-agent model, measuring impact matrix.

1. Introduction

Building a model at the level of describing a particular information resource provides a number of advantages such as transparency about objective mechanisms, the ability to describe the object under study, with a high degree of detail, to obtain more useful information from the simulation results.

Based on the information-oriented model, we obtain data that fully correspond to the usual state of the data in the information space. To this end, the level of threat is introduced into the model as a result of obtaining and perceiving data. That is, in this case, each cell contains a demand for data and some class of threat. Under the new rules, data is moved to a free cell, where the ratio (demand/threat class) is maximum.

Later modifications of the information-oriented model consider different types of interactions between information, as well as other complications. This makes it possible to analyze a wider range of social processes and procedures.

2. Information-Oriented Model Framework

The following issues are investigated within the framework of the information-oriented model:

- Distribution of the amount of information between the data.
- Distribution of data by significance.
- Data migration.
- Introduction of new properties into the model, such as the demand/threat class ratio, and the corresponding modification of the rules.
- Introduction of new properties of information, such as the impact of information on the individual.

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- Change of rules of the emergence of new data.
- Introduction of inheritance rules, for example, when the amount of information that loses demand is evenly distributed in the new information that has appeared.
- Introduction of multiple units, such as demand.
- Introduction of rules for the exchange of demand between information.

Information-oriented modeling encompasses spatially distributed models in which each unit of information is associated with a specific position in space. Thus, the properties of the model significantly depend on its space-time scale. Models also differ in the amount of information considered. The scope of calculations directly depends on the scale of the problem [1, 2].

It should be noted that the information-oriented model requires more computation than the analytical one. However, in many areas, the development of an information-oriented model is justified due to the fact that:

- Data of real observations of the studied parameters are often not enough to identify the analytical model.
- It is necessary to take into account spatial aspects.
- It is necessary to take into account the mechanisms of the information space.

3. Main Part

To assess the state of information in the information-oriented model, we use partial a priori statistical uncertainty. in which the law of distribution of components-evaluated and measured random processes is known to the nearest certain set of parameters. The parametric description must meet two sometimes conflicting requirements. At first, it must qualitatively and correctly reflect the limited a priori knowledge. At second, the number of parameters should not be too large. The increase in the number of parameters leads to a deterioration in the quality of the main task both due to the complexity of the technical implementation and due to the loss of input to be used to determine parameter values or to exclude unknown and unnecessary parameters. Thus, in the case of parametric a priori uncertainty, instead of a single probability distribution law for random processes, we define a whole class of distributions. The evaluation algorithm must select from a given class of distributions to ensure that the optimization criterion is met. This means that the estimation algorithm must be parametrically adaptive [3].

Under parametrically adaptive estimation algorithm we will understand such algorithm which on the basis of process of the measuring information is capable not only to give an estimation of necessary components of the random process, but also to restore statistical characteristics of the a priori description of the dynamic system.

Consider the construction of a recurrent algorithm for estimating the information state vector xk and the constant correlation matrix R of measuring the impact of a sample of measurements of increasing volume $Y_{k1} = \{y_i,\ i = \overline{(1,k)}\}$ [4]. Consider the case when the linear model of the system and measurements are described by stochastic differential equations, and the measurement model includes the influence of v_k in the form of a definition of $B_k v_k$, where B_k is a time-dependent matrix.

So, given:

1. System model:

$$x_{k+1} = a_0(k) + \Phi(k+1|k)x_k + b(k)w_k. \tag{1}$$

2. Measurement model:

$$y_k = A_0(k) + H_k x_k + B_k v_k. \tag{2}$$

3. A priori data:

$$w_k \sim N(0; Q_R);\ v_k \sim N(0; R); \quad (3)$$

where R is a const; Q_R is an unknown matrix; $x_0 \sim N(x(0|0), P_0;$

$$cov(w_k, w_j)\ cov(v_k, v_j) = cov(x_0, v_k) = cov(x_0, w_k) = 0; k \neq j$$

4. Optimization criterion is the maximum total density of the probability distribution of the estimated and measured parameters

$$\max \pi \left(x_1^k, Y_1^k \middle| R\right) \quad (4)$$

$$X_1^k, R$$

where $X_1^k = \{x_i, i = \overline{1, k}$.

We first convert criterion (4) to an equivalent notation form. First of all, we note that since the natural logarithm is a monotonically increasing function of its argument, instead of (4) we can use an equivalent criterion.

$$max \ln \pi \left(X_1^k, Y_1^k \middle| R\right). \quad (5)$$

$$X_1^k, R$$

Using the formula of total probability, the marking of the process (x_k, y_k), and the independence of measurements $y_i \in Y_1^k, i = \overline{1, k}$, we represent the community of density distribution X_1^k i Y_1^k in two forms:

$$\ln \pi\ (X_1^k\ , Y_1^k|R) = \ln \pi \left(X_1^{k-1}, Y_1^{k-1} \middle| R\right) + +\ \ln \pi \left(x_k, y_k \middle| X_1^{k-1}, Y_1^{k-1}, R\right) ==$$
$$\ln \pi\ (X_1^{k-1}, Y_1^{k-1}, R) + + \ln \pi\ (\ y_k | x_{k-1}, y_{k-1}, R) + \ln \pi\ (x_k | x_{k-1}, y_k, R)\ ; \quad (6)$$

$$\ln \pi \left(X_1^k, Y_1^k \middle| R\right) = \ln \pi \left(X_1^k\right) + \ln \pi \left(Y_1^k \middle| X_1^k, R\right) = \ln \pi \left(X_1^k\right) + + \sum_{i=1}^k \ln \pi \left(y_i \middle| X_1^k, R\right). \quad (7)$$

Given (6) and (7), we convert criterion (5) into a component:

$$\begin{cases} \max\limits_{x_k} \ln \pi\ (x_k | y_k, x(k-1|k-1), \widehat{R_k}) & (8) \\ \max\limits_{R} \sum_{i=1}^k \ln \pi\ (y_i | \widehat{X_1^k}, R) & (9) \end{cases}$$

where $\widehat{X_1^k}$ = {x (i | i), i = $\overline{1, k}$}, x (k | k) i $\widehat{R_k}$ is the estimates x_k and R, obtained from a sample of measurements Y_1^k.

The optimization for criteria (8) for the conditions that R = $\widehat{R_k}$, gives the usual Kalman-type extrapolation and filtering algorithms.

Let us now turn to the optimization for criterion (9). Let us first imagine $\ln \pi\,(y_i|\,\widehat{X_1^k}, R)$ explicitly. To do this, we note that due to the linearity of the transformation $B_i v_i$ and condition (3)

$$B_i v_i \sim N(0, B_i R B_i^\tau). \tag{10}$$

It follows from relation (2)

$$B_i v_i = y_i - H_i x_i - A_0(i) \tag{11}$$

and transformation from the variables $B_i v_i$ to the variables y_i is equal to one, so, given (10) and (11), we can write

$$J(R) = \sum_{i=1}^{k} \ln \pi\,(y_i|\,\widehat{X_1^k}, R) = -(km/2)\ln \widetilde{2\pi} + \frac{1}{2}\sum_{i=1}^{k} \ln |(B_i R B_i^\tau)^{-1} - \ -1/$$
$$2\sum_{i=1}^{k} v_i^\tau\ (\,(B_i R B_i^\tau)^{-1} v_i, \tag{12}$$

where $v_i = y_i - H_i x(i|i-1) - A_0(i)$; $\tilde{\pi} = 3{,}1415 \ldots$.

Using the definition and properties of a pseudo-inverse matrix [5], the following transformations should be performed:

$$(B_i R B_i^\tau)^{-1} = B_i B_i + (B_i R B_i^\tau) + (B_i^\tau) + B_i^\tau = B_i[(B_i^\tau B_i) \times R(B_i^\tau B_i) + B_i^\tau ==$$
$$B_i (B_i^\tau B_i)^{-1} R^{-1} (B_i^\tau B_i)^{-1} B_i^\tau. \tag{13}$$

Also, note that

$$|(B_i R B_i^\tau)^{-1}| = |B_i R B_i^\tau|^{-1} \tag{14}$$

Substituting (13) and (14) into (12), we obtain

$$J(R) = -\left(\frac{km}{2}\right)\ln \widetilde{2\pi} - 1/2\sum_{i=1}^{k} \ln|B_i R B_i^\tau| - 1/\,2\sum_{i=1}^{k} v_i^\tau B_i\,(B_i^\tau B_i)^{-1} R^{-1} (B_i^\tau B_i)^{-1} B_i^\tau v_i \tag{15}$$

The necessary condition for the extremum of the functional (15) is described by equation $\left.\frac{dJ(R)}{dR}\right|_{R=\widehat{R_k}} = 0$
which can be given an explicit look:

$$-k\left(\widehat{R_k}^{-1}\right)^\tau + \left(\widehat{R_k}^{-1}\left(\sum_{i=1}^{k}(B_i^\tau B_i)^{-1} B_i^\tau v_i v_i^\tau\, B_i (B_i^\tau B_i)^{-1}\right)\widehat{R_k}^{-1}\right)^\tau = 0 \tag{16}$$

Transporting (16) and multiplying it left and right by $\widehat{R_k}$, we obtain

$$\widehat{R_k} = \frac{1}{k}\sum_{i=1}^{k}(B_i^\tau B_i)^{-1} B_i^\tau v_i v_i^\tau\, B_i (B_i^\tau B_i)^{-1}. \tag{17}$$

Write (17) in the form

$$\widehat{R_k} = \frac{1}{k}(B_k^\tau B_k)^{-1} B_k^\tau v_k v_k^\tau B_k (B_k^\tau B_k) + + \frac{k-1}{k} \frac{1}{k-1} \sum_{i=1}^{k-1} (B_i^\tau B_i)^{-1} B_i^\tau v_i v_i^\tau B_i (B_i^\tau B_i)^{-1} \quad (18)$$

4. Conclusions

The study of this algorithm using software and hardware allows identifying of some features. If the diagonal elements (R_k) is in the range $\widehat{R_k} \leq (R) \leq 〚15R〛$, i = (1,m) (R is a true correlation matrix of measurements). Then the estimate x (k | k) is not very sensitive to changes in the elements of the matrix C_k. If (R_k) < R, then this can lead to large errors in estimating x (k | k) due to the deterioration of the conditionality of the matrix C_k. To improve the operation of the algorithm, it is advisable to use an iterative procedure for calculating the matrix R_k. It is enough to perform three iterations when calculating the estimate of the matrix R_k, so that the estimate (R_k) almost stopped changing.

5. References

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