Derivative Pricing: Predictive Analytics Methods and Models

Ivan Burtnyak [1[0000-0002-9440-1467]], Anna Malytska[2[0000-0003-5811-9288]]

[1,2] Vasyl Stefanyk Precarpathian National University, Ukraine, bvanya@meta.ua, hanna.malytska@pnu.edu.ua,

Abstract. The article deals with the study of pricing and calculating the volatility of European options with general local-stochastic volatility applying Taylor series methods for degenerate diffusion processes, in particular for diffusion with inertia. The application of this idea requires new approaches caused by degradation difficulties. Price approximation is obtained by solving the Cauchy problem of partial differential equations diffusion with inertia, and the volatility approximation is completely explicit, that is, it does not require special functions. If the payoff of options is a function of only x, then the Taylor series expansion does not depend on t and an analytical expression of the fundamental solution is considerably simplified. We have applied an approach to the pricing of derivative securities on the basis of classical Taylor series expansion, when the stochastic process is described by the diffusion equation with inertia (degenerate parabolic equation). Thus, the approximate value of options can be calculated as effectively as the Black-Scholes pricing of derivative securities.

Keywords: stochastic volatility, European options, degenerate diffusion processes, Kolmogorov equation.

1. Introduction

Derivative prices tend to change and predict their behavior is becoming more and more complicated. Stochastic processes that are described by the equation of diffusion with inertia are widely used in the theory of mass service, in particular in the theory of queues. Often, such processes occur on financial markets with the pricing of European and Asian options. the theory of pricing derivatives and the study of the behavior of volatility for the analysis of profitability are necessary for flexibility in the adoption of managerial strategic decisions by managers. The validity of strategic decisions allows managers to make step-by-step additional investments in order to maintain strategic positions of the company in the stock market. Typically, a high level of volatility gives the manager more opportunities to change their decisions in the future. Volatility is important for traders when pricing several different series of options with different execution rates and maturities. Since is stable in the Black-

Scholes model [6], speculative price changes can occur in these cases, which can not be verified without the presence of a certain kind of specialized knowledge. This is one example of how volatility pricing options are related to the principles of modeling and risk management in the financial market. Therefore, it is necessary to use models with variable volatility.

The purpose of this article is to introduce a unified approach to pricing and find the implied volatility of European options, which are described by degenerate diffusion processes of diffusion type with inertia, based on classical Taylor series approximation. Therefore, the approximate value of options can be calculated as effectively as the price of Black-Scholes [6] for European options.

2. Literature Review

Modeling and forecasting of pricing dynamics, using financial instrument is an important element of investment activity, as they have a high level of financial leverage and fulfillment of obligations is realized in the future. Derivative prices tend to change and predict their behavior is becoming more and more complicated. Various operations with financial products suitable for selling and buying are carried out using derivative financial instruments [1]. The effective investment of decision making while operating in the Ukrainian stock market is the major factor of investors' interest in it. Securities market participants should have a good understanding of derivative pricing to achieve successful financial results. Derivative securities transactions occupy an important place in the stock market financial activity, since each participant must hedge the risks to obtain extra profit based on stock market speculations [2]. Therefore, the derivatives are one of the major instruments in the securities market. An important task is to study the state and dynamics of the domestic stock market in close interrelation with other countries' stock markets and analyze the volatility of financial instruments to increase the efficiency of investment operations. Nowadays many approaches have been developed to calculate local and stochastic volatility that describe the overall dynamics of underlying price using CEV models [4, 12], JDCEV [7]., Heston-model [11], *SABR*-model [16], but the application of these models requires the use of special functions and numerous integrations of complex functions. Hence, this may lead to miscalculations, but considering the time law (dependency), a considerable number of the models become unstable. Other methods for pricing derivative securities are required to get direct calculations [9]. The use of Taylor series expansion depends on the model structure, specifically, on function properties which are used in the model and model ability to maintain the stability at time change.

We consider models without default of diffusion process with inertia, with coefficients depending on the variables (t, x, y), for derivatives pricing we use Taylor series expansion for degenerate diffusion processes. In particular, complete correlation matrices are non-degenerate in the following works [4, 13, 14], and in our work they are degenerate, so the application of this idea requires new approaches caused by degradation difficulties

The purpose of this article is to introduce a unified approach to pricing and find the implied volatility based on classical Taylor series approximation. Therefore, the approximate value of options can be calculated as effectively as the price of Black-Scholes [6] for derivative securities.

3. Methodology and Data

We consider market without arbitration, zero interest rate and no dividends. Without losing generality these considerations may be extended to deterministic interest rate. We consider probabilistic space with martingale measure Ξ, with filtering $\{\mathcal{F}_t, t \geq 0\}$ defining market history. Let asset S represent such phenomena as stocks, price index, a reliable short-term interest rate i.e. and where $S_t = \mathbb{I}_{\{\tau > t\}}\, \mathrm{e}^{X_t}$, and processes X_t and Y_t are set by such system of equations

$$\mathrm{d}X_t = \mu(t, X_t, Y_t)dt + \sigma(t, X_t, Y_t)dB_t, X_0 = x \in R,$$

$$\mathrm{d}Y_t = \alpha(t, X_t, Y_t)dt, Y_0 = y \in R, \tag{1}$$

where τ is the stopping time $\tau = \inf\left\{t \geq 0 \colon \int_0^t r(s, X_s, Y_s)ds \geq \varepsilon\right\}$, with the exponential distribution ε, which does not depend on X, the drift function μ has the form

$$\mu(t, X_t, Y_t) = -\tfrac{1}{2}\sigma^2(t, X_t, Y_t) + r(t, X_t, Y_t),$$

Let U be a non-arbitrary price of the European option, which at time T is a gain $\mathcal{K}(S_T)$ [3], that

$$U_t = K + \mathbb{I}_{\{\tau > t\}}\mathbb{E}\left\{\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{T}} r(s, X_s, Y_s)ds}(k(X_t) - \mathrm{K})|X_t, Y_t\right\}, t < T.$$

$K = \mathcal{K}(0), k(x) = \mathcal{K}(e^x)$. To calculate the price of the European option, we must calculate the mathematical expectation from $e^{-\int_{\mathrm{t}}^{\mathrm{T}} r(s, X_s, Y_s)ds}(\mathrm{k}(X_t) - \mathrm{K})$ in particular

$$w(t, x, y) = \mathbb{E}\left\{e^{-\int_{\mathrm{t}}^{\mathrm{T}} r(s, X_s, Y_s)ds}(\mathrm{k}(X_t) - \mathrm{K})|X_t = x, Y_t = y\right\} \tag{2}$$

The function $w(t, x, y)$ satisfies the Kolmogorov equation of diffusion with inertia

$$-\partial_t\big(x\partial_y + \mathrm{P}\big)w = 0,\ w(T, x, y)\big|_{t=T} = k(x, y),$$
(3)

where operator P has the form

$$P = a(t, x, y)(\partial_x^2 - \partial_x) + r(t, x, y)(\partial_x - 1)$$
(4)

where $a(t, x, y)$ is equal to

$$a(t, x, y) \coloneqq \frac{1}{2}\sigma^2(t, x, y).$$

If we consider deterministic interest rates then we need to calculate the mathematical expectation of this form

$$\widetilde{w}(t,\tilde{x},y) = \mathbb{E}\left\{\mathrm{e}^{-\int_t^T r(s,\tilde{X}_s,Y_s)ds}\left(\mathrm{k}(\tilde{X}_t) - \mathrm{K}\right)\middle|\tilde{X}_t = \tilde{x}, Y_t = y\right\},$$

$$d\tilde{X}_t = dX_t + \gamma(t)dt$$

A direct verification proves that $w(t,x(t,\tilde{x}),y)$, satisfies (3).

An approximate solution to the Cauchy problem for diffusion equation with inertia (3) is obtained by adapting the ideas of [5,10] which are applied to non-degenerate diffusion processes. We consider degenerate diffusion processes with inertia on which singular integro-differential pricing operators of Levy type are applied. We have introduced a unified approach to pricing and estimation of implicit volatility using Taylor series expansion. We consider that a and r are infinitely differentiable functions of variables (x,y), continuous on t and bounded $\forall(t,x,y) \in [0,T] \times R^2$

Let $(\bar{x},\bar{y}) \in \mathrm{R}^2$ be a fixed point then any analytical function $h = h(t,x,y)$ we may write down Taylor series expansion

$$h(t,x,y) = \sum_{n=0}^{\infty}\sum_{l=0}^{n} h_{n-l,l}(t)\,(x-\bar{x})^{n-l}(y-\bar{y})^l,$$

$$h_{n-l,l}(t) := \frac{1}{(n-l)!\,l!}\partial_x^{n-l}\partial_y^l h(t,\bar{x},\bar{y}).$$

Applying the above considerations to the coefficients a and $,\gamma$, we obtain that operator P in (4) has the form

$$P = \sum_{n=0}^{\infty} P_n\,, \qquad P_n := \sum_{l=0}^{n}(x-\bar{x})^{n-l}(y-\bar{y})^l P_{n-l,l}, \qquad (5)$$

where $\{P_{n-l,l}\}$ are differential operators of the

$$P_{n-l,l} := a_{n-l,l}(t)(\partial_x^2 - \partial_x) + r_{n-l,l}(t)(\partial_x - 1).$$

Operator P of a parabolic type x and has a degenerate parabolicity on y (since there is no second order derivative on y), which usually operates in financial spaces, that is, in the financial markets. Based on Taylor series expansion for P, the pricing function w has the form [8]

$$w = \sum_{n=0}^{\infty} w_n\,, \qquad (6)$$

Substituting (5) and (6) into (3), we obtain the Cauchy problems for diffusion equations with inertia.

$$\left(-\partial_t + x\partial_y + P_0\right)w_0 = 0, \qquad w_0(T,x,y) = k(x,y), \qquad (7)$$

$$\left(-\partial_t + x\partial_y + P_0\right)w_n = -\sum_{k=1}^{n} P_k w_{n-k}, \quad w_n(T,x,y) = 0, \tag{8}$$

We construct a fundamental solution of the homogeneous Cauchy problem (7) and use the properties of the fundamental solution (7) to find $w_n \ \forall\, n$ as a solution of the problem (8), while we use only the properties of the distribution function for the diffusion equation with inertia expressed by the classical Chapman-Kolmogorov-Planck equations and the Duhamel's principle are applied to partial derivatives equations.

Let's consider the Cauchy problem (7). Operator P_0 is a degenerate parabolic diffusion operator with inertia, or a Kolmogorov operator with time-dependent coefficient t. Thus, the solution w_0 has the form

$$w_0(t,x,y) = e^{\int_t^T r_0(s)ds} \int_{R^2} \mathcal{E}_0(t,x,y;T,\xi,\eta)k(\xi,\eta)d\xi d\eta, \tag{9}$$

where $\mathcal{E}_0(t,x,y;T,\xi,\eta)$ is a two-dimensional Gaussian density of diffusion process with inertia.

We find $\mathcal{E}_0(t,x,y;T,\xi,\eta)$, so we solve the Cauchy problem for the diffusion equation with inertia and variable coefficients depending on t

$$\partial_t w - x\partial_y w = a_0(t)(\partial_x^2 w - \partial_x w) + r_0(t)(\partial_x - 1)u \tag{10}$$

$$w(t,x,y)|_{t=T} = k(x,y). \tag{11}$$

We apply Fourier transform to solve the Cauchy problem (10), (11) assuming that $w, w_t, w_x, w_{x^2}, w_y$ are completely integrable on (x,y).

$$F\big(w(t,x,y)\big) = \frac{1}{2\pi}\int_{R^2} \exp\{ix\xi + i\eta y\}u(t,x,y)dxdy = v(t,\xi,\eta),$$
$$(\xi,\eta) \in R^2, 0 < t \le T.$$

Since

$$F\big(\partial_t w(t,x,y)\big) = \partial_t v(t,\xi,\eta),$$
$$F\left(x\partial_y w(t,x,y)\right) = -\eta\partial_\xi v(t,\xi,\eta),$$
$$F\big(\partial_x w(t,x,y)\big) = -i\xi v(t,\xi,\eta),$$
$$F\big(\partial_x^2 w(t,x,y)\big) = -\xi^2 v(t,\xi,\eta),$$

then we will have

$$\left(\partial_t + \eta\partial_\xi\right)v(t,\xi,\eta) = \{a_0(t)[(-i\xi)^2 + i\xi] + r_0(t) - i\xi - 1\}v(t,\xi,\eta), \tag{12}$$

$$v(t,\xi,\eta)|_{t=T} = \tilde{k}(\xi,\eta). \tag{13}$$

where $\tilde{k}(\xi,\eta) = F\big(k(x,y)\big)$.

The problems (10), (11) are reduced to (12), (13) so these are the Cauchy problem for a linear differential equation with partial first order derivatives [10].

We formulate the corresponding characteristic equation

$$dt = \frac{d\xi}{\eta} = \frac{dv}{v\big(-\xi^2 a_0(t) + i\xi a_0(t) - i\xi r_0(t) - r_0(t)\big)}.$$

Equation of characteristics

$$dt = \frac{d\xi}{\eta};\ \ dt = \frac{dv}{v\left(-\xi^2 a_0(t) + i\xi\big(a_0(t) - r_0(t)\big) - r_0(t)\right)}.$$

So, we have the first integral $\xi = \eta t + C_1;$ from

$$\frac{dv}{v} = \big(-\xi^2 a_0(t) + i\xi a_0(t) - i\xi r_0(t) - r_0(t)\big)dt,$$

we have

$$\ln v = \int_t^T [(-\xi^2 + i\xi)a_0(\beta) + r_0(\beta)(-i\xi - 1)]\, d\beta + \ln C_2\,, \qquad C_2 > 0,$$

taking into consideration that $\xi = \eta t + C_1$, we have

$$v(t, \eta t + C_1, \eta) = C_2 \exp\left\{\int_t^T \{[-(\eta\beta + C_1)^2 + i(\eta\beta + C_1)]a_0(\beta) + r_0(\beta)(-i(\eta\beta + C_1) - 1)\right\} d\beta\},$$

providing that

$$t = T, \quad v(T, \eta T + C_1, \eta) = \tilde{k}(\eta T + C_1, \eta) = C_2, \quad C_2 = \tilde{k}(\eta T + C_1, \eta).$$

$$v(t, \eta t + C_1, \eta) = \tilde{k}(\eta T + C_1, \eta) \exp\left\{\int_t^T \{[-(\eta\beta + C_1)^2 + i(\eta\beta + C_1)]a_0(\beta) + r_0(\beta)(-i(\eta\beta + C_1) - 1)\right\} d\beta\},$$

since $C_1 = \xi - \eta t$, then

$$v(t, \xi, \eta) = \tilde{k}(\eta(T - t) + \xi, \eta) \exp\left\{\int_t^T \{[-(\eta(\beta - t) + \xi)^2 + i(\eta(\beta - t) + \xi)]a_0(\beta) + r_0(\beta)(-i(\beta - t)\eta - \xi i - 1)\right\} d\beta\},$$

We take an inverse Fourier transform from v (t, ξ, η)

$$F^{-1}\big(v(t, \xi, \eta)\big) = \frac{1}{2\pi} \int\limits_{R^2} \exp\{-ix\xi - i\eta y\}\, v(t, \xi, \eta) d\xi d\eta = w(t, x, y),$$

$$w(t, x, y) = \frac{1}{2\pi} \int_{R^2} \tilde{k}(\xi + \eta(T - t), \eta) \exp\left\{\int_t^T \{[-(\eta(\beta - t) + \xi)^2 + i(\eta(\beta - t) + \xi)]a_0(\beta) + r_0(\beta)[-i(\beta - t)\eta - \xi i - 1]\}d\beta\, \{-ix\xi - i\eta y\}\, d\xi d\eta. \right. \tag{14}$$

In (14) we will substitute the variables

$$\begin{cases} \xi + \eta(T-t) = \gamma, \ \xi = \gamma - \eta(T-t), \\ \eta = \eta, \end{cases}$$

then

$$w(t,x,y) = \frac{1}{2\pi}\int_{R^2} \tilde{k}(\gamma,\eta)\exp\left\{-ix\big(\gamma-\eta(T-t)\big) - iy\eta + \int_t^T \{[-(\eta(\beta - T)+\gamma)^2 + i\eta(\beta-T) + ir]a_0(\beta) + r_0(\beta)[-i\eta(\beta-T) - ir - 1]\}d\beta\right\}d\gamma d\eta \quad (15)$$

by substituting the value $\tilde{k}(\gamma,\eta) = Fk(x,y)$ in (15) we have

$$w(t,x,y) = \int_{R^2} k(x',y')\, e^{\int_t^T r_0(\beta)d\beta}\mathcal{E}_0(t,x,y;T,x',y')dx'dy'. \quad (16)$$

where $\mathcal{E}_0(t,x,y;T,\xi,\eta)$is the density of the two-dimensional diffusion process with inertia and with coefficients dependent on a time variable.

$$\mathcal{E}_0(t,x,y;T,x',y') = (4\pi c^*)^{-1}\left(\int_t^T a_0(\beta)d\beta\right)^{-\frac{1}{2}}\exp\left\{-\left(4\int_t^T a_0(\beta)d\beta\right)^{-1}\left(x'-x-\int_t^T\big(a_0(\beta)-r_0(\beta)\big)d\beta\right)^2 - (2c^*)^{-2}\left[y'-y+x(T-t)-\left(x'-x-\int_t^T\big(a_0(\beta)-r_0(\beta)\big)d\beta\right)\int_t^T(\beta-T)a_0(\beta)d\beta\left(\int_t^T a_0(\beta)d\beta\right)^{-1} - \int_t^T(\beta-T)\big(a_0(\beta)-r_0(\beta)\big)d\beta\right]^2\right\}, 0 \le t < T,$$

$(x,y) \in R^2, \ (x',y') \in R^2$.

(t,x,y) is the initial point, (T,x',y') is the end point (current),

$$c^{*2} = \int_t^T (\beta-T)^2 a_0(\beta)d\beta - \left(\int_t^T (\beta-T)a_0(\beta)d\beta\right)^2\left(\int_t^T a_0(\beta)d\beta\right)^{-1}.$$

First, we solve the Cauchy problem (8) with $n=1$ with payoff $=\sigma(X,Y)$, $w(t,x,y) = \mathcal{E}_0(t,x,y;T,X,Y)$. We have

$$w_1(t,x,y)e^{\int_t^T r_0(s)ds} = \int_t^T ds\int_{R^2} d\xi d\eta\mathcal{E}_0(t,x,y;s,\xi,\eta)P_1\,\mathcal{E}_0(s,\xi,\eta;T,X,Y) = \int_t^T dsB_1^{(x,y)}(t,s)\,\mathcal{E}_0(s,\xi,\eta;T,X,Y).$$

We multiply both sides by $e^{\int_t^T r_0(s)ds}$ and using (9) we have

$$w_1(t,x,y) = A_1\,w_0(t,x,y), \ \ A_1 := \int_t^T dsB_1\ (t,s),$$

Using method of mathematical induction and taking into consideration properties of the fundamental solution, we have

$$w_n(t,x,y) = A_n\,w_0(t,x,y),$$

$A_n := \sum_{k=1}^{n} \int_t^T ds_1 \dots \int_{s_{n-1}}^T ds_n \ \ \sum_{i \in I_{n,k}} B_{i_1}(t, s_1)\big(B(t, s_2) + B(s_1, s_2, 0) + (1 + s_2 - s_1)B(t, s_1, 0) - B(t, s_2, 0)\big)_{i_2} \dots \big(B(t, s_n) + B(s_{n-1}, s_n, 0) + (1 + s_{k-1} - s_k)B(t, s_{k-1}, 0) - B(t, s_k, 0)\big)_{i_k}$,

$$I_{n,k} = \{i = (i_1, \dots i_k) \in N^k \ \ |i_1 + \dots + i_k = n\}, \ \ 1 \le k \le n,$$
$$B_{i_k}(t, s) := B_{i_k}(t, s, x, y, \bar{x}, \bar{y}), \ \ B_{i_k}(t, s)_{|(x,y)=(\bar{x},\bar{y})} = B(t, s, 0),$$

$$B_n(t, s) := \sum_{k=1}^{n} \mathcal{M}_{n-k,k}(t, s) P_{n-k,k}(s), \qquad \mathcal{M}_{k,l} = \big(\mathcal{M}_1(t, s)\big)^k \big(\mathcal{M}_2(t, s)\big)^l,$$

where

$$M_1(t, s) := \Bigg(x - X + \int_t^s \big(a_0(\beta) - r_0(\beta)\big)\, d\beta + 2\int_t^s a_0(\beta)\, d\beta \partial_x + 2\left(\int_t^s (\beta - s) a_0(\beta)\, d\beta + (s - t)\int_t^s a_0(\beta)\, d\beta\right)\partial_y\Bigg)^n,$$

$M_2(t, s) := \Big(y - Y - \int_t^s (\beta - s) a_0(\beta)\, d\beta \big(\int_t^s a_0(\beta)\, d\beta\big)^{-1} \int_t^s \big(a_0(\beta) - r_0(\beta)\big)\, d\beta + \int_t^s (\beta - s)\big(a_0(\beta) - r_0(\beta)\big)\, d\beta + 2\int_t^s (\beta - s) a_0(\beta)\, d\beta \partial_x + 2c^{*2}\partial_y + \Big(2\int_t^s (\beta - s) a_0(\beta)\, d\beta \big(\int_t^s a_0(\beta)\, d\beta\big)^{-1}\Big)\big(\int_t^s (\beta - s) a_0(\beta)\, d\beta + (s - t)\int_t^s a_0(\beta)\, d\beta\big)\partial_y 2c^{*2}\Big)^n$,

The results of the asymptotic approximation are proved in [13,15]. These formulas are considerably simplified when the coefficients do not depend on time.

4. Results and analysis

The degenerate diffusion process that describes price dynamics and implicit volatility, depending on time and (t, x, y), indexes of base assets, stocks, options of financial flows in the method of calculating the company's rating based on the available quotations of securities, is considered in the article. The research was conducted on the basis of Taylor series. The density distribution of the probabilities of passing this process is constructed. With density distribution, you can find $\mathcal{E}_0$, you can find the price $w(t, x, y)$ at any given time. The method of successive finding of the price is developed when the coefficients depend on (t, x, y). Using approximate approximations using the Black-Scholes price function as an initial approximation, we obtain explicit formulas for finding the initial approximations of implicit volatility. We note that they are in the form of the record match the formulas [13], but in this case they are more complicated by the degeneration of the equation and the formula for the approximation of the price.

The obtained implications of implicit volatility and consistent price approximations make it possible to analyze the process of passage in the financial market. Make corrections and concrete steps to improve the situation for optimizing financial strategies.

A Taylor series expansion method for the Kolmogorov equation (degenerate Heston model) is used.

$$\partial_t w - x\partial_y w + \frac{e^y}{2}(\partial_x^2 w - \partial_x w) = 0,$$

with the initial function, which is the Black-Scholes value-dependent volatility based on S&P500 options data in the time interval from November 1 to December 31, 2020. The exact values of changes in volatility and profitability are found, which allows managers to predict the process of forming a portfolio, financial flows, and changes in pricing.

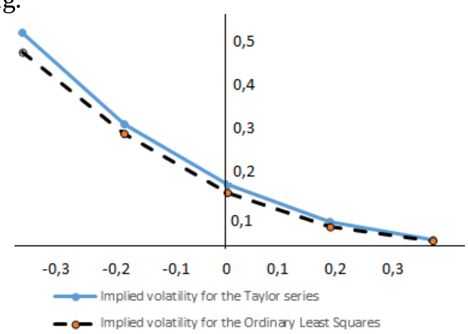

Figure 1: The implied volatility obtained by the Ordinary Least Squares and our second order approximation the Taylor series for the degenerate Heston model. $e^y = 0{,}249^2, T = 0.125, t = 0.$

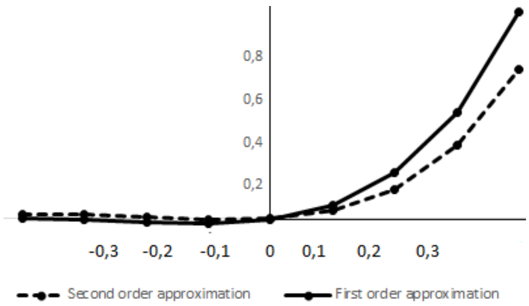

Figure 2: The yield curve first and second order approximation the Taylor series for the degenerate Heston model. $e^y = 0{,}249^2, T = 0.125, t = 0.$

In the example of the degenerate Heston model, which describes the dynamics of pricing and the development of implicit volatility, the initial approximation of the Black-Scholes price function made volatility and yield calculations based on the second Taylor approximation and the the Ordinary Least Squares. The results obtained are almost identical, indicating a high accuracy of approximation.

Knowledge of the approximate price and implied volatility at each step at a fixed time gives an opportunity to develop a strategy for managing the dynamics of derivative prices in financial markets and to avoid speculative changes in pricing.

The density of distribution for degenerate diffusion processes is constructed when the coefficients depend only on time (7).

The method of constructing the density of distribution in the case of coefficients dependence on time and spatial variables is developed (3), when the correlation matrix is not strictly positive definite.

Knowing the density of distribution, one can always find the price by the formula (13) where the role $\mathcal{E}_0$ is played by the density of distribution of probabilities of the investigated economic process.

5. Conclusions

This paper expands the methodology of approximate pricing for a wide range of derivative assets. Price approximation is obtained by solving the Cauchy problem for differential equations in partial derivatives of diffusions with inertia. If the payoff of options is a function of only x, then the Taylor series expansion does not depend on t and an analytical expression of the fundamental solution is considerably simplified. We have applied an approach to the pricing of derivative securities on the basis of classical Taylor series expansion, when the stochastic process is described by the diffusion equation with inertia (degenerate parabolic equation). For a degenerate parabolic equation, the approximate price of options is fairly simple since it uses only estimates of derivatives of the distribution density of diffusion with inertia, we obtained explicit formulas for derivatives price approximations.

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