

# Optimal Spatial Configurations Synthesis Using Visualization of the Decision-Making Process

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## Abstract

The paper proposes basic models of the problem of optimal synthesis of spatial configurations of geometric objects. The efficiency of using modern information technologies is substantiated, which allow automatically creating models of optimization problems and their subsequent solution using specialized software packages. It is proposed to use the visualization of geometric information in the decision process to attract the decision maker. A technology is described that allows you to change the current configuration of geometric objects in an interactive mode.

## Keywords

Geometric object, spatial configuration, information technologies, visualization.

## 1. Introduction

At present, there is a sharp increase in interest in the development and implementation of information technologies for support and decision-making, requiring the synthesis of complex technical systems, taking into account the spatial form of their elements. This task is inextricably linked with the tasks of geometric design, where the mapping of geometric information about material objects into Euclidean space is performed. Among such practical areas of application are logistics of transport and warehouse services, the layout of aerospace objects, the safety of fuel and energy complexes, the development and improvement of 3D printing technologies, etc. All of them are necessary taking into account the shape of the corresponding prototypes and the imposition of restrictions on their relative position. The synthesis of such spatial configurations of material objects includes their processing, transformation and storage of geometric information about objects. It is based on the construction and use of information and analytical models of relevant problems, the development and implementation of modern information technologies. An integral feature of such technologies is the constant visualization of design solutions, which allows them to be automatically corrected under the guidance of a decision-maker.

This research is intensively developing all over the world. An example is the Springer Optimization and Applications series [1], which focuses on models, methods, and information technology for assembly synthesis in aerospace engineering. Another illustration is the series "Lecture notes on logistics" [2], devoted to the study of the problems of logistics, transport, warehousing and distribution services. Important results on mathematical modeling and software development for the problem of geometric design are presented in [3] - [13]. To solve the problems of synthesizing optimal configurations of complex spatial objects, it is of interest to develop new information technologies using the automatic creation of mathematical models. In this case, computer modeling involves visual transformation of geometric information.

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Thus, the purpose of this article is, first of all, the development and research of information technologies that allow the formation of an information-analytical model for describing data structures when creating complex spatial configurations. Second, it is important to consolidate the geometric information data structures used in the optimization and visualization of spatial configurations. Third, it is necessary to perform dynamic transformation of data structures in the synthesis and visualization of spatial configurations of complex objects..

## 2. General information

Consider a set of material objects of a given geometric shape. Certain restrictions can be imposed on their relative position, when fulfilled, a certain spatial configuration is formed. Let's introduce the concept of geometric information  $g = (\{s\}, \{m\}, \{p\})$  about an object  $S$ , which includes a spatial shape  $\{s\}$  of  $S$ , metric parameters  $\{m\} = (m_1, \dots, m_k)$  that specify its sizes, and placement parameters  $\{p\} = (p_1, \dots, p_l)$  about the object  $S$  position in space [14]. A class of problems associated with such a display of geometric information are studied in the theory of geometric design. Usually, geometric objects with the shapes of a sphere, cone, parallelepiped, polygon, as well as complex objects composed of them are considered. In this case, the metric characteristics of the geometric objects involved are considered to be fixed. The problem of synthesizing spatial configurations of objects of arbitrary shape have their own characteristics, which requires the development of new mathematical models, as well as further development of methods for processing, transforming and storing geometric information about material objects.

To describe the external structure and type of a set of material objects in papers [14, 15], the concept of a configuration space of geometric objects is introduced, which is formed on the basis of geometric information  $g = (\{s\}, \{m\}, \{p\})$ . Let the object  $S$  has a equation of its boundary

$$f(\xi, m) = 0,$$

where  $\xi = (x, y)$ , if  $S \subset R^2$ , and  $\xi = (x, y, z)$ , if  $S \subset R^3$  underlies determining the components  $\{s\}$  and  $\{m\}$  of geometric information  $g$ . Some approaches to setting boundary equations for 2D and 3D objects are presented in [10].

Consider a coordinate system  $Oxyz(Oxy)$  in space  $R^3(R^2)$ , respectively, further referred to it as a fixed one. With an object  $S$ , we associate their own coordinate system, which origin is called a pole. A relative position of these coordinate systems characterizes the placement parameters  $p = (p_1, \dots, p_\beta) = (v, \theta)$  of the object. The general position equation of a geometric object (GO) relative to a fixed coordinate system can be specified as follows

$$F(\xi, m, p) = f[A(\xi - u), p] = 0,$$

where  $A$  is an orthogonal operator defined through the angular parameters  $v$ .

This equation is the basis for the formation of the configuration space of a geometric object and determines a set of values of geometric variables, which are called generalized coordinates. Generalized coordinates specify the position of a set of geometric objects in space both relative to each other and relative to a fixed coordinate system. In what follows, we will assume that the configuration space  $\Xi(S)$  of GO  $S$  is induced by the generalized variables  $g = (m, p)$ .

## 3. Configuration Space of Geometric Objects

Consider a set of objects  $\Omega = \{S_1, \dots, S_n\}$ . By  $\Xi(S_i)$ , we denote a configuration space of an object  $S_i$ , which the generalized variables are  $g^i = (m^i, p^i)$ ,  $i \in J_n$ , where  $J_n = \{1, \dots, n\}$ . To each point

$g^i \in \Xi(S_i)$ , a parameterized object  $S_i(g^i) \subset R^3(R^2)$  will be associated. Let us form configuration space

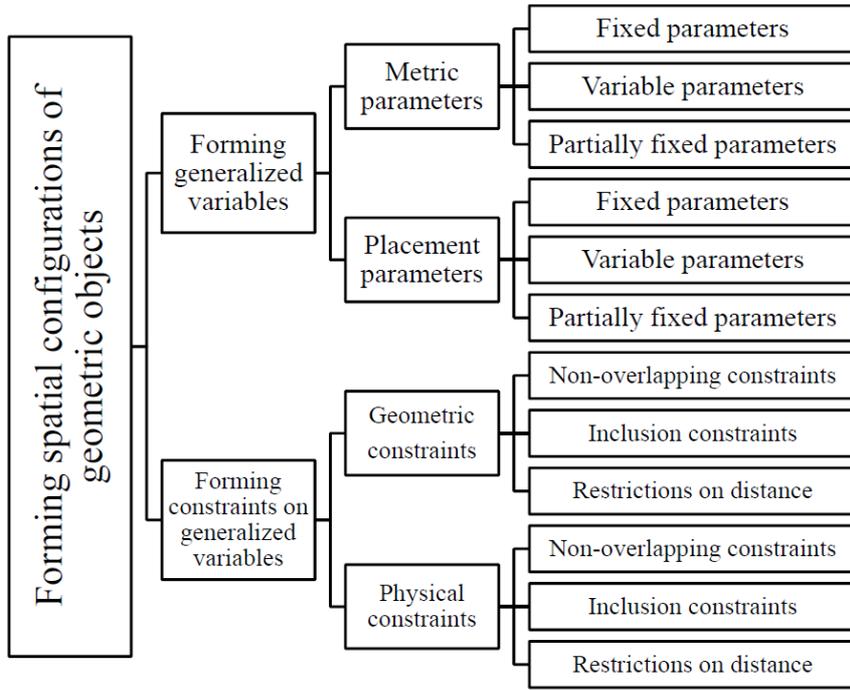
$$\Xi(\Omega) = \Xi(S_1) \times \dots \times \Xi(S_n)$$

of a set of basic GOs given by the generalized variables  $g = (g^1, \dots, g^n)$ .

A mapping  $\xi: \Omega \rightarrow \Xi(\Omega)$  satisfying a given set of constraints  $\Lambda$  determines a spatial configuration of GOs  $S_i, i \in J_n$  [14, 15].

Presence of the constraints  $\Lambda$  allows offering a typology of spatial configurations depending on relations GOs. Note that approaches of formalizing the relations are completely determined by choice of the generalized variables of CS, by constraints on a mutual location of GOs, and by their physicommechanical characteristics (see Fig. 1).

For the objects  $S_i, i \in J_n$ , let us introduce a binary relation  $\{*\}$  of a pairwise non-intersection. We will use a notation  $S' * S''$  if  $\text{int } S' \cap \text{int } S'' = \emptyset$ , i.e., if objects  $S'$  and  $S''$  have no common internal points. If  $S_i(g^i) * S_j(g^j)$  for all  $i, j \in J_n, i \neq j$ , then the spatial configuration is called a packing configuration. In most of the packing problems, one more object  $S_0$  called a container is involved. In this case, all objects  $S_i, i \in J_n$  must be placed into the container  $S_0$ .



**Figure 1:** The process of forming spatial configurations of geometrical objects

Now, on the set of GOs, let us introduce one more binary relation – an inclusion relation  $\{\circ\}$ , where a denotation  $S' \circ S''$  is used to reflect that  $\text{int } S' \subset S''$ .

Assume that an object  $S_0$  has the generalized variables  $g^0$  in configuration space  $\Xi(S_0)$ . Let us form a new configuration space  $\Xi(S_0, \Omega) = \Xi(S_0) \times \Xi(\Omega)$ . Then a set of the generalized variables will define a layout configuration, if  $S_j(g^j) \circ S_0(g^0)$  and  $S_i(g^i) * S_j(g^j)$  for any  $i, j \in J_n, i < j$ .

The generalized variables  $g^0, g^1, \dots, g^n$  of configuration space  $\Xi(S_0, \Omega)$  may also be subject to additional constraints. This induces special classes of packing configurations, such as a layout

configuration [15, 16] when constraints on the minimum and maximum distances between objects are given. If the objects  $S_i, i \in J_n$  are solid bodies with given masses, then a balanced system of such bodies yields a balanced packing configuration [17]. When the parameters of the models are discrete, so-called Euclidean combinatorial configurations can be considered [18].

Let a functional  $\xi: \Xi(S_0, \Omega) \rightarrow R^1$  be given in configuration space  $\Xi(S_0, \Omega)$ . Now, we can formulate a problem of finding an optimal configuration of GOs  $S_i, i \in J_n$ . So, it is required to find

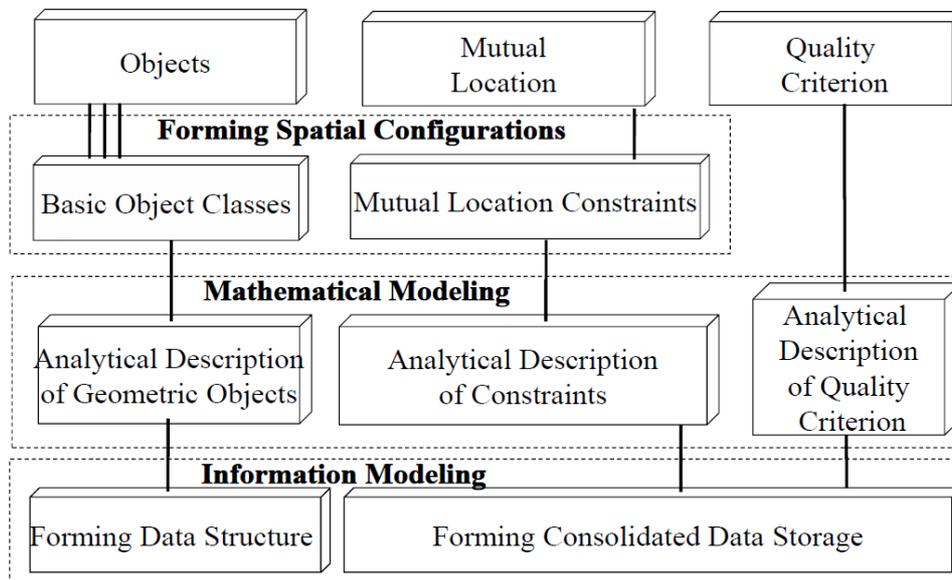
$$g^* = \arg \min_{g \in W \subseteq \Xi(S_0, \Omega)} \xi(g), \quad (1)$$

where  $W$  is a set of admissible configurations satisfying certain constraints  $\Lambda$ .

The generalized variables  $g = (g^0, g^1, \dots, g^n)$  of configuration space  $\Xi(S_0, \Omega)$  take real values and allow equivalent formulating the problem (1) as a mathematical programming problem. To formalize constraints on relative positions of GOs,  $\Phi$ -functions [19] and  $\omega$ -functions [20] may be used. In this study, existing methods for constructing  $\Phi$ -functions of basic 2D and 3D objects were summarised and generalized. They concern placement and packing problems with variable metric parameters of GOs and are utilized when the BaseObjects database was formed.

We propose an object-oriented model of a GO consisting of an abstract class GeometryObjectBase, which implements a collection of virtual methods with common operations for all geometric objects (GOs). Examples of such operations are reading information from a file or database, saving data, visualizing configurations, etc. A GeometryObjectBase class' descendants have two implementations depending on the GO-dimension – 2D or 3D. Each of these classes contains fields and virtual methods for affine transformations of the objects' motion in the corresponding space. Such an implementation allows creating and processing spacial GOs of highly complex shapes.

Based on the peculiarities of the GO-object-oriented model and the specifics of mathematical modelling of GOs' relations, an information-analytical model of the problem of spatial configurations synthesis is designed (Fig. 2).



**Figure 2:** The process of forming information-analytical model

Note that, in the model, analytical and informative components can be singled out. The analytical component is associated with a choice of the generalized variables in the problem's mathematical model, as well as with a way of formalization of constraints on the relative position of GOs and present quality criteria.

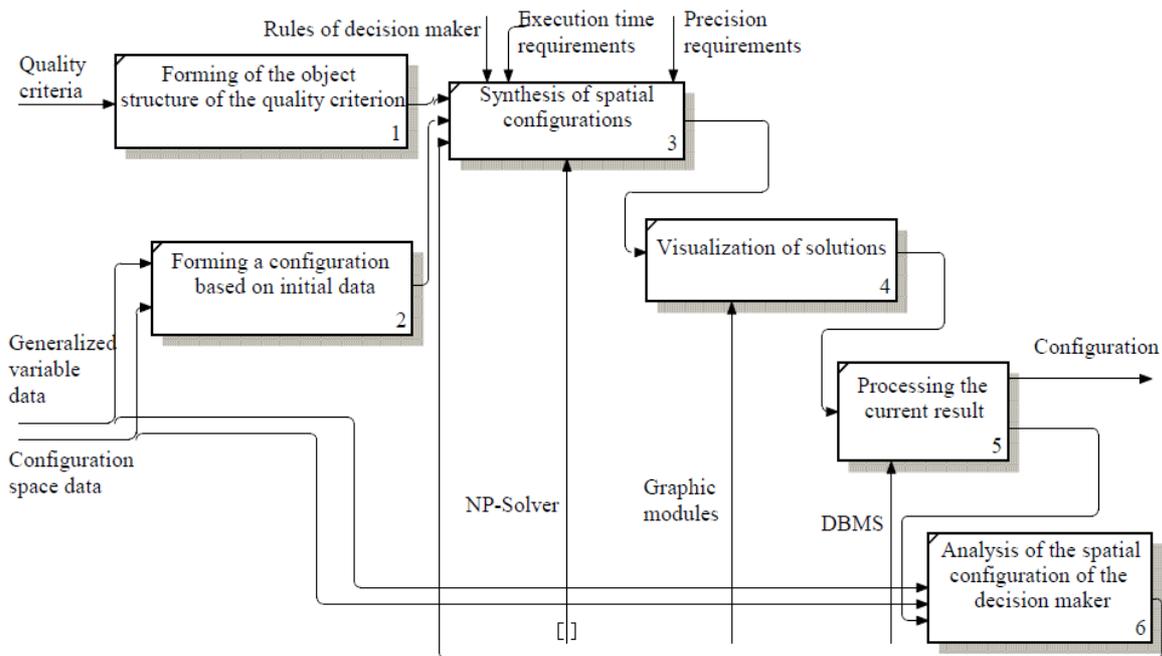
The model's information component describes the formation of a spatial objects' data structure and creating consolidated storage of the spatial configuration data. A choice of a structure of input data is

caused by the specifics of utilizing available optimization solvers and visualization software in the process of solving the problem. For instance, we focused on the usage of COIN-OR, which is a popular open-source software package.

Depending on a chose of quality criteria and additional constraints, problems of optimal configurations' synthesis can be included in different classes of mathematical programming problems. To find a local extremum, it is sufficient to formalize the objective function, functional constraints, Jacobian and Hessian. In this case, developing new information technologies for optimal configurations' synthesis in complex systems requires constructing their information-analytical models in automatic mode. With this regard, the advantage of our approach is the possibility of implementing methods for solving such problems disregarding a research domain. Also, note that the approach is applicable for solving NP-hard problems ignoring their high dimensionality and multiextremality.

#### 4. Information Technology Model for the Synthesis of Spatial Configurations

The information technology of spatial configuration synthesis consists of six interconnected blocks and is depicted in Figure 3 in the form of an IFEFO chart. In the figure, it is shown how the system functional blocks interact with each other. Also, the required steps to obtain a new configuration are shown.



**Figure 3:** Information technology for the synthesis of spatial configurations

Block 1 depicts a process of forming the model's analytical component with respect to a quality criterion and additional constraints.

In Block 2, depending on initial data, a spatial configuration data structure is formed using the generalized variables and configuration space.

Block 3 is responsible for the synthesis of the locally optimal spatial configurations with respect to the chosen quality criterion and generalized variables. Then, the designed mathematical model is analyzed, and a choice of the software COIN-OR is justified. After that, the geometric information data structure is adapted for utilizing the solver. Finally, limitations on computing resources (memory size and runtime) are established, and recommendations on a relevant solution accuracy are developed. An output of the block is a spatial configuration meeting all the constraints.

In Block 5, processing and storage of results of spatial configurations' synthesis in a data warehouse "Configuration Repository" using DBMS are carried out.

Finally, Block 6 describes the possibilities of involving a decision-maker (DM) in an analysis of the found spatial configuration. Depending on the quality of a decision found so far, DM can either leave the decision unchangeable or proceed in order to improve it. In the latter, DM sets the new spatial configuration's generalized variables and repeat the above process again. Thus, parameters of the optimization model may be changed at the following iterations. Note the initial spatial configuration may be infeasible, but, according to the technology proposed, a locally optimal configuration are automatically synthesized, which satisfy all available constraints.

## 5. The Software Package Description

In the scope of models and methods of information technologies of layout synthesis of spatial configurations, a software applications has been developed, including [21]:

- ✓ software application – a solver for implementation of methods of spatial configurations' optimization;
- ✓ repository program – a consolidated repository for storing information on the process of solving the problems;
- ✓ software applications for interactive visualization of the solutions obtained.

The software complex has a hierarchical structure and consists of three functional parts. The first component is the software application (SA) GeneralSolver that integrates a chosen Solver with necessary computational software components. GeneralSolver is used to dynamically transform the data structure of geometric information in the process of synthesizing spatial configurations of complex objects. In turn, this SA is based on the .NET Framework. The program validates the data entered by a user and verifies their correctness and compliance with the format. If incorrect data are entered, an error warning with clarification is displayed on the screen. If a critical error occurs while executing a program, the program automatically terminates.

The second component of the complex is SA for storing results of the GeneralSolver calculations. In addition to the evaluation results, the repository store full information about spatial configurations of underlying objects, values of configuration space generalized variables involved in the synthesis, and information on a mathematical formulation of problems (quality criteria and constraints).

The third component is SA for 2D or 3D visualization. For 3D visualization, one can use different visualization packages: 3Ds Max Studio, Revit, VRED, Stingray, etc.

The described software package was developed in Visual Studio 2017 using different programming languages (C #, C ++, and ECMA Script).

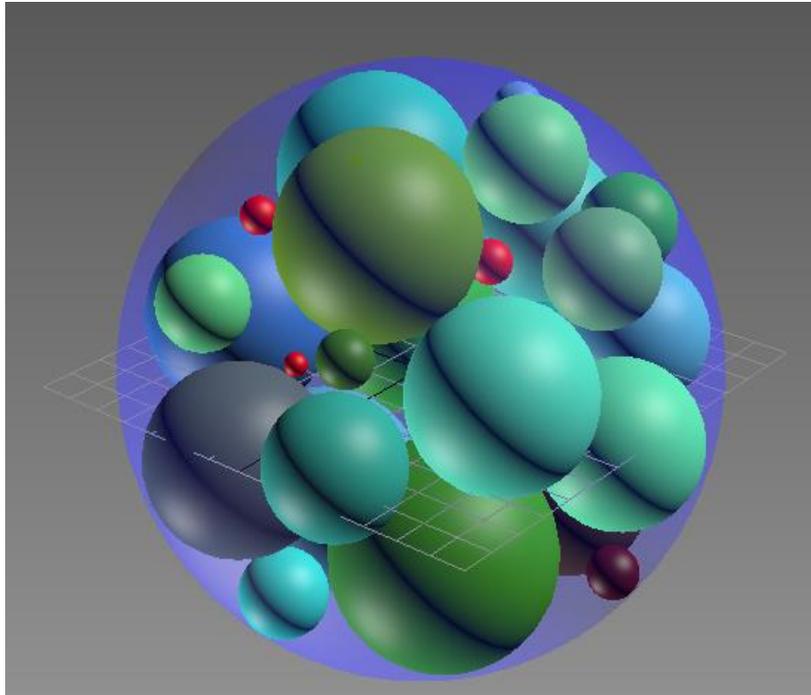
## 6. An Example of Implementing the Results

As an example of implementing the results, consider the following packing problem for spherical objects. It is required to place a set of spheres of a given radius into a sphere of minimum radius. Generalized variables in this problem are the coordinates of the centers of spheres (placement parameters) and radii (metric parameters). The mathematical model as a nonlinear optimization problem is given in [22-25]. NLP-Solver IPOPT (<https://tomopt.com/tomlab/>) was used to find local extrema. The interactive environment 3Ds Max Studio was used to display the results and interact with DM.

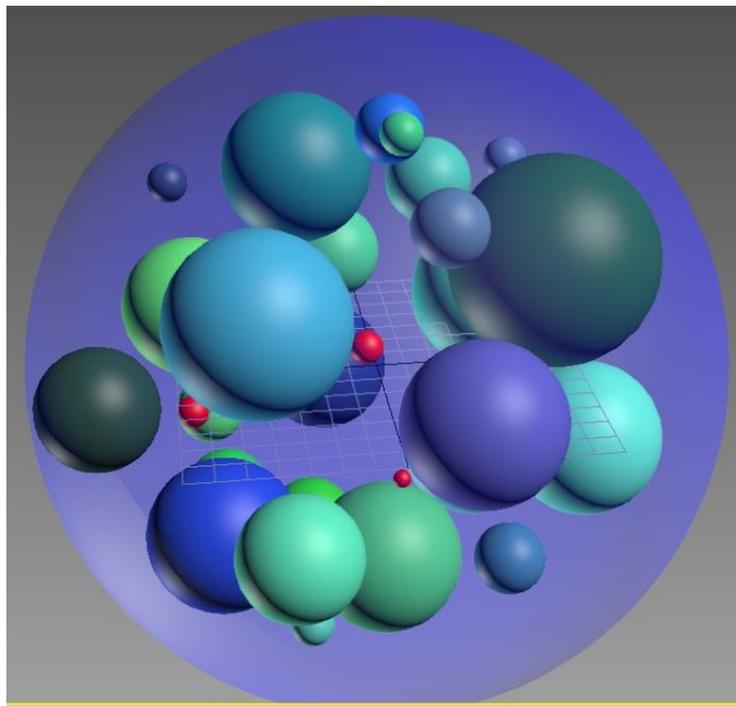
First, the format of the initial data of the mathematical model is converted into the form of the Solver representation. Then the initial values of the generalized variables are set and the optimization process is carried out using IPOPT. The result (local extremum) is stored in a special format in the consolidated database and converted to the 3Ds Max Studio data format. As a result of the analysis of solution, the decision maker can make any changes using the interactive graphical environment and (or) the associated numeric parameter values. Taking into account the changes made, we get a new starting point, which is again fed to the Solver. We have an iterative process that is repeated until a solution is obtained that satisfies the DM.

Below we will illustrate the visualization of the described process. Let's randomly generate the initial configuration of spherical objects. Note that the initial configuration may not satisfy the constraints of the problem, that is, the spheres may intersect, be outside the placement area and at large distances from each other. However, using IPOPT ensures that all the constraints are met, as shown in Fig. 4. The DM

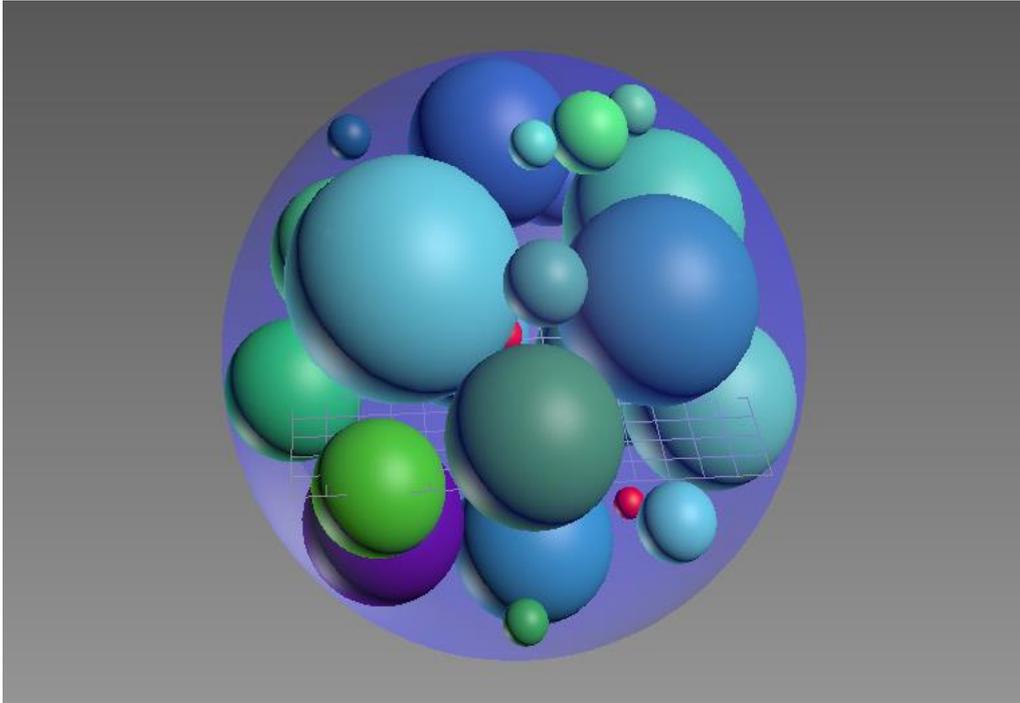
can modified the obtained solution. As a result, using the export script, a new spatial configuration is formed, which is used as the starting point (Fig. 5). The improved locally optimal configuration is shown in Fig. 6. Other results of numerical experiments are described in [26, 27]. The results are saved in text files, which allows you to return to any stage of the calculation at any stage. If the spatial configuration satisfies the DM, then it is saved in the form of graphic data, files with initial, intermediate and resulting data.



**Figure 4:** The locally optimal solution



**Figure 5:** The decision, modified by the decision-maker



**Figure 6:** The improved configuration

## 7. Conclusion

Thus, the paper describes the models and methods for solving the optimization problems of the synthesis of spatial configurations of geometric objects. It is based on the definition and formalization of geometric information and the concept of configuration space. This made it possible to propose new information technologies for interactive problem solving, using both specially developed methods and modified known approaches. The presented approach is applied to solve a wide class of problems of geometric design, in particular, problems of packing, layout and coverage.

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