Air Navigation: Synthesis of Signal Detectors' Structure in Collaborative Airspace Surveillance Systems in Airport Areas

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Abstract

For fundamental reasons regarding the detailed treatment of airspace control technological processes arising from the need to satisfy a set of basic requirements for the accuracy and timeliness of detecting controlled airspace's violations, the article considers the following points: the process of obtaining data from airborne objects (data collection); transmitting data to airborne objects (sending requests for data or delivering ground data to airborne objects); processing data obtained from airborne objects; distributing and organizing requests for airspace maintenance at the local observation subsystem's level; pro-cessing data obtained from airborne objects, distributing and organizing service requests at the regional level. Besides, proposed is a data model for airspace surveillance. It is demonstrated that the most effective airspace control systems include the primary and secondary surveillance systems, which together provide users with data about "who" is "where" and "when". Presented is a methodology for assessing the quality of such a system.

Keywords [1]

Airborne Vehicles, Air Traffic Control System, Classification of Airborne Vehicles.

1. Introduction

A prerequisite for the successful fulfillment of airspace control tasks in airport areas to prevent unauthorized intrusion is the availability of reliable data support (DS). At its core, data support for airspace control in airport areas consists of the consumer receiving coordinate data of the observed airborne object (AO), additional flight data (FD) about its state and movement parameters, as well as the data on the AO's IFF. It follows from an analysis of practical experience that the most comprehensive and detailed picture in the airspace control system is provided by the collaborative use of primary and secondary surveillance systems (SS). In certain cases, however, the use of automatic dependent surveillance can significantly improve the quality of data support.

At present, secondary SS is one of the main data sources about airborne objects in airport areas. This is because the data received from the AO transponders are more complete and reliable in comparison with the data received from the reflected signal during the operation of the primary SS [1]. However, the existing secondary SS has some specific features of construction and functioning, which do not allow these systems to be attributed to noise-immune systems [1].

Nowadays, various technological processes' operation is based on the use of data systems including sources of data, means of transmission, processing, display, storage, general and special software. Personnel also plays an important role in all data technological processes, as well as in operational processes. The technological process of the airspace control system is impossible without the intervention of an operator, who implements the most responsible process - decision-making.

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The airspace control system's data support is carried out by different SS. The process of data collection and processing has become increasingly automated over the years. A large number of different technical and software-technical means are used. Automated decision support systems are being introduced.

All the data obtained from the SS (including the above-mentioned) are processed, transmitted through communication channels, and displayed in the required form for operators or stored in special databases. The effective implementation of all airspace control systems depends on the efficiency of data processes in them and the degree of their security.

The airspace control process is defined as a form of timely detection of AO and determining their location (and, if applicable, obtaining additional data about AO), as well as the timely provision of the data to users to support safe airspace operation based on the airspace control services' concernment. In most cases, SS provides the user with the data of "who", "where", and "when" types. The data on the horizontal and vertical flight speed of the AO can also be provided. The applicable data and parameters of SS' technical characteristics depending on the tasks they complete.

The fundamental reasons regarding the detailed treatment of airspace control technological processes arise from the need to satisfy the following set of basic requirements:

- receiving data from AO (data collection);
- transmitting data about the AO (sending requests for data or delivering ground data to AO);
- processing data from the AO, distributing and organizing requests for servicing the AO's flight at the local observation subsystem's level;
- processing data obtained from airborne objects, distributing and organizing service requests at the regional level.

The data model for airspace control is discussed in [2]. This model reflects the interaction of the airspace control data exchange infrastructure with the modules of SS' ground receiving and transmitting parts. The main object of surveillance in the model is AO and its attributes, such as the AO's four-dimensional location, AO's type, AO's identification, and any other attributes considered to be operationally significant. The infrastructure of an airspace control system can be influenced by numerous factors, especially interference (intentional and unintentional).

Primary SS form the basis of data support for airspace control. These SS allow for determining the coordinates of AO violating the airspace or which transponders do not respond. As a rule, this system functions in collaboration with the system of identification based on "friend or foe". At the data level, these SS are united into a local data surveillance network, the data being fused at the coordinate level. Active-passive multi-position SS are also applied using both their probing signals and those emitted by AO.

When operating on AO, airspace control means transmit data from secondary surveillance SS, Mode S systems, wide-area multi-positional surveillance (WAM), and broadcast dependent automatic surveillance (ADS-B) in one direction, while data to air traffic information services (TIS-B) after their on-ground compilation are transmitted in the opposite direction. In addition, data on AO can be received using satellite data transmission channels (ADS-C).

The SS data streams are divided into primary (primary processing of SS data) and improved ones, i.e. an air situation map (data produced by the data processor, i.e. secondary and tertiary processing of SS data). It is noteworthy that, as a rule, SS includes a signal processor performing all signal processing procedures. The data from SS goes to the data processor in which the subsequent data processing is performed. The data from the data processor in the form of an AO record is transmitted to users using data and communication networks.

AO's records must include the following points:

- location (the current state vector and precision matrix);
- flight data;
- identification based on "friend or foe";
- time of data acquisition.

It is mathematically possible to present AO's records as follows:

$$W,\ C_p,\ \text{«friend – foe»},\ T_i, \qquad (1)$$

in which: W is the AO's current state vector, Cp is the correlation matrix of the state vector's measurement errors.

Therefore, modern SS systems consist of numerous sources and consumers of surveillance data, both at the level of a separate system and at the level of mutual exchange between systems. To ensure the protection of the surveillance data distribution system's effective and reliable data exchange, the most important task was to study possible threats to the surveillance data distribution system as a set of conditions and factors that can lead to a violation of data's integrity, availability, and confidentiality.

The data received from SS are unstable since they are meaningful only if they are timely received at the place of processing. This allows us to propose the following requirements for data transmission, which are presented in the descending order of priority:

- limited time delay of surveillance data's transmission (real-time transmission);
- transmission without data distortion;
- transmission without data loss.

An essential requirement for SS characteristics is the minimization of the time delay caused by data transmission. Considering that data's operational value is more dependent on time, the delay from the moment an AO is detected to the moment the data about its location is displayed on the air situation visualization means is the key parameter of the system's performance. The delay is considered acceptable if it is assumed that all the data within the system is time-stamped. In fact, for airspace control data processing systems, it is better to receive time-lagged plots containing an accurate time stamp than prompt plots with an undefined time stamp.

To equate messages identifying the same AO but arriving at the corresponding node by different routes, and therefore at different points in time, a developed time control system is needed. Moreover, accurate synchronization is essential as all data are logged for legal reasons, so timestamps in this context make it easier to track down logical cause-and-effect relationships. The main issue now is to provide multicast (group) data distribution from one source to several consumers. Thus, multicast distribution and routing are essential basic surveillance data exchange functions.

The following additional functions can be performed during the exchange of surveillance data:

- data collection from various (ground and air) sources;
- local and global distribution of surveillance data;
- checking the accuracy of the obtained data;
- routing change;
- conversion of application protocols' data formats;
- transformation of surveillance data (depending on the finite systems, application, and expected level of service quality);
- coordinate systems' transformation;
- support for several time determination systems;
- ensuring the ability to recover quickly;
- ensuring the functionality of system control (including network load operation);
- accumulation of statistical data (it can be carried out outside the distribution of surveillance data and other functions);
- transportation of system operation and control data.

All this allows for formulating the requirements for the SS data network:

- the network must ensure the required quality of data streams' transmission;
- the network must ensure all the required functionality;
- the fallback network must provide data redundancy.
- The quality requirements for the surveillance data exchange network include the following features:
- safe, reliable, and timely delivery of surveillance data;
- safe and reliable delivery of control and operation data;
- continuous availability;
- minimal mutual influences between the network's nodes.

It should be noted that the current state of development of SS connection infrastructure is characterized by the increasing use of advanced network technologies for the distribution of surveillance data. The development of airspace control systems is characterized by the following features:

- • high level of process automation;
- • deep integration of IT applications;
- • increasing complexity of IT products;
- • an increase in the volume of their introduction into the system.

The research into the data model of airspace control shows that secondary SS should become more widespread in the airspace control system. They should be used both in identifying AO and transmitting flight data from an AO to the control points.

2. Main Part

It can be deduced from the above-mentioned information that SS is the main data source for airspace control systems. The surveillance system presents data on AO's detection, determination of their coordinates, assessment of movement parameters, and IFF. In most cases, SS provides the user with the data about "who" is "where" and "when". Horizontal and vertical velocity data may also be provided, which identifies AO's characteristics or intentions. The required data and performance values for a particular SS depend on the specific application. The minimum criterion for the airspace control system is the provision of data about the AO at a specified time.

The airspace control system consists of several elements, the use of which is determined by the requirements for specific types of SS use. Neither applications nor end-users are part of the airspace control system.

Primary SS presents the data basis of the airspace control system determining AO's spatial coordinates, i.e. answering the question "where", while secondary SS identify AO to be "friend or foe", i.e. answer the question "who". It should be noted that the consumer, as a rule, needs three more AO's spatial coordinates. A typical data model of such an airspace control system includes two local and one remote surveillance subsystems and a data processor [3]. SS's boundary is the application interface, i.e. the point where the SS provides surveillance data for use and the system's robot is evaluated.

The requirements for SS's technical characteristics should be independent of the technology and architecture used in the SS which should provide a specific type of airspace control. Typically, a set of quality criteria is used for SS performance, such as the probability of detection, accuracy, update rate, integrity, and availability. Such characteristics have the following disadvantages:

- as a rule, the listed criteria are oriented towards data sensors. For instance, the accuracy parameter expressed in polar coordinates (range and azimuth) is logical for a radar set, but not necessarily for any other surveillance methods;
- certain requirements are focused only on the use of advanced technology (e.g., the accuracy of 15 m, which is set for modern S-mode radars);
- certain basic requirements can be omitted as they relate to only one specific technology.

In this respect, it seems essential to define the objective performance requirements that would apply to SS using different methods or their combinations to support different types of applications. The development of new applications possibly requiring special characteristics should also be considered. Moreover, some of these new applications may introduce additional requirements for SS's end beam. Therefore, it would be desirable to develop a common approach to defining the characteristics of airspace control. The most important function of the airspace control system is AO's accurate positioning and identification at a given moment in time. Depending on the type of application and operating conditions, other requirements may appear, e.g., the need for data on AO's speed and acceleration. The main elements of the characteristics are the type of surveillance data and their quality. As a rule, the format for presenting data to the user is not included in the technical requirements for the characteristics of an airspace control system.

The filtered signals y1 (t), y2 (t), and y3 (t) of primary, secondary, and identification SS, respectively, arrive at the inputs of users' data support structure based on a complex SS, which includes primary, secondary and identification SS. The structure contains signal detectors, from the output of which a sequence of random zeros and one's *xi* is taken. Thus, the signal is detected according to the required F0i and D0i quality indicators. The sequence of zeros and ones from the output of the signal detector undergoes time sampling and goes further to the inputs of the detectors and AO's coordinates meters. The task of AO detectors is to decide optimally based on the analysis of the received sequence of zeros and ones, whether the received sample is a group of signals or can be classified as interference.

To solve the above-mentioned problem, AO detectors must process the incoming signals according to a certain algorithm. The algorithm for AO identification is reduced to testing the *H0* hypothesis about the AO's absence and the alternative *H1* hypothesis about its presence, i.e. to the formation of a likelihood ratio and comparing this ratio with some deliberately specified number based on the permissible probability of AO's erroneous detection. The decision to identify an object with F1i and *D1i* quality indicators are sent to the AO coordinates' meter. The evaluation of the coordinates of the AO's instantaneous position is conducted simultaneously with the AO's identification. The task of the AO's coordinates meter is to estimate the AO's coordinates optimally based on the analysis of the obtained sequence of zeros and ones.

Consequently, when the signal about the AO's detection is generated, the output of the AO's coordinates meter of each channel of the joint SS forms an estimate of the coordinate measurement vector αi, which is characterized by the correlation matrix of errors *Cp*. The primary data processing ends with the generation of the AO's records.

It should also be noted that the current vector of AO's state W with the corresponding matrix of accuracy Cp compiled based on measuring AO's coordinates from primary SS contains AO's spatial coordinates. As a rule, the speed of AO's movement is not estimated according to the results of processing the received signals. However, it should be noted that the speed can be reached from a "friend" AO with high accuracy from secondary SS.

The intermediate results of joint SS's signal processing are transmitted to the consumer for the implementation of data network processing with data unification at different levels. The data is transmitted to consumers after the primary processing and must indicate the time of its receipt {Tik(t)} thereafter. This significantly simplifies data combination procedures.

The probabilistic indicator can be an integral indicator of the data quality support for airspace control when using IT technologies. In particular, for the primary data processing, private indicators of the data quality support can be the probabilities of AO's correct detection by each channel of the joint SS Pi = *D1i*, which are functions:

$$D_{1i} = f\left(D_{0i}, F_{0i}, k_i, P_0\right) = f\left(q_{0i}, z_{0i}, k_i P_0\right) \quad (2)$$

in which *z0(ki) is the signal detection threshold*, P0 is SS's availability factor.

In data comparison and combination needed for AO's records automatic compilation the criterion is the quality of AO's position indication using the probabilities of actions, the following being among them:

- the loss of correct flight data;
- the distortion of flight data;
- combining coordinate data and primary data of secondary SS;
- comparing coordinate data of the primary and identification SS;
- combining coordinate data and flight data in a joint SS.

When processing flight data by the scheme according to the "signal/noise" criterion, the correct flight data in the processing device is likely to be lost:

$$P_v = 1 - P_{p.i}^k,$$

in which Pp. i is the possibility of issuing flight data from the output of secondary SS in the first *m* informational responses.

When the confirmation schemes are used in the device for processing flight information according to the *k/m* criterion, the possibility of flight data's distortion is estimated by the formula:

$$P_{ic.p.i} = \sum_{i=k}^{m} C_m^i P_{ic}^i \left(1 - P_{ic}\right)^{m-i},$$

in which Pic is the possibility of a secondary SS issuing erroneous flight data.

The flight data from secondary SS can arrive with time delays in comparison to the coordinate data. Consequently, the discrete number of flight data acquisition is as follows:

$$N'_d = N_d + \frac{T(KI)}{r_d},$$

in which *Nd* is the discrete number of flight data acquisition; T(KI) is the time delay for the secondary SS, corresponding to the KI code; *rd is the discrete value of the range*.

The possibility of coordinate and flight data combination Is as follows:

$$P_o = \left(1 - P_{v.p.i}\right)\left(1 - P_{ic.p.i}\right)P\begin{Bmatrix} +N_0' \\ -N_0' \end{Bmatrix},$$

in which $P\begin{Bmatrix} +N_0' \\ -N_0' \end{Bmatrix}$ is the conditional probability of flight data acquisition in the strobe from *+N0* to *–N0* regarding the coordinate data about the AO.

The algorithm for combining data in the processing device is set in such a way that single marks of joint SS channels are combined if the azimuth angle between the centers of the packages does not exceed Δβ and the difference in their ranges does not exceed Δr. Provided that the deviation of the packages' centers in the primary and secondary channels of the joint SS are independent and obey the normal distribution law, the probability of combining packets can be determined from the following relationship:

$$P_p = 0{,}25\left[1 + \Phi\left(\frac{\Delta\beta}{\sqrt{2\left(\sigma_{\beta 1}^2 + \sigma_{\beta 2}^2\right)}}\right)\right]\left[1 + \Phi\left(\frac{\Delta r}{\sqrt{2\left(\sigma_{r1}^2 + \sigma_{r2}^2\right)}}\right)\right]$$

in which σβ1, σβ2; σr1, σr2 are the mean square deviations of azimuths (ranges) of packages' centers of joint SS primary and secondary channels.

Thus, an integral indicator of the quality of data support for users can be a possibility for the primary data processing, which can be represented as follows:

$$P_f = D_{11} D_{12} D_{13} P_o P_{p1} P_{p2} \quad (3)$$

Therefore, the proposed approach facilitates the combination of the processing efficiency criteria of both signals and SS data based on the signal detection threshold, i.e., the analog threshold's value can be used as a parameter for joint optimization of the characteristics of primary, secondary, and tertiary processing.

The proposed integral indicator of the data support's quality for airspace control makes it possible to improve the characteristics of the stages of SS's data processing along with the widespread use of data technologies and to carry out the distributed processing of the relevant data from individual SS by users.

The modern format of airspace control provides for the joint operation of primary and secondary SS, which can be used to improve the quality of consumers' data support. In modern surveillance systems, consumers are given an estimate of the measurement vector α, which is characterized by the correlation matrix of errors C obtained from the results of measuring the primary SS. Secondary SS data are used to obtain AO's onboard data, which is also transmitted to the consumer. It should be noted that to combine the assessment of the AO measurement vector obtained from the primary SS and the flight data received by the secondary SS, the same detection and measurement procedures are carried out both by the secondary SS and the primary SS. This circumstance can be taken into account in data processing, which will improve the quality of data transmitted to consumers.

In joint SS, the structure of the primary data processing is two-channel and forms consolidated data for consumers based on the weight consolidation of the channel detection's results and the estimates of vectors of channel measurements of AO's flight parameters with the simultaneous inclusion of the data block and flight data received through the channels of secondary SS. The estimates of the measurement vectors obtained in each of the channels, along with the correlation matrices of the measurement errors, are fed to the estimates' consolidation device. The estimate of the resulting vector of measurements and the resulting correlation matrix of errors are calculated in the device for combining estimates based on the analysis of estimates of measurement vectors and correlation matrices of measurement errors. The resulting measurement vector αp along with the resulting error correlation matrix Cp are provided to consumers.

If we assume that the measurement vector α is simultaneously estimated in M processing channels, set the normal distribution law for each of the components of the vector α, and take into account the non-correlatedness of measurements in the processing channels, then the logarithm of the likelihood ratio up to a constant value can be written as follows:

$$\ln l = \sum_{k=1}^{M} \left[-0{,}5 (\alpha - \alpha_k)^T C_k (\alpha - \alpha_k) \right],$$

in which αk and Ck are the estimates and matrices of the estimation accuracy for the signals of each SS channel.

This expression is reduced to the following form up to the constant value:

$$\ln l = -0{,}5 (\alpha - \alpha_p)^T C_p (\alpha - \alpha_p),$$

in which αp is the final grade, Cp is *the* final precision matrix which can be determined by the following expressions:

$$\alpha_p = C_p \sum_{k=1}^{M} C_k \alpha_k, \quad C_p = \sum_{k=1}^{M} C_k.$$

Following on from the above-mentioned expressions, it is possible to estimate the resulting measurement vector and the final precision matrix, and thus the final correlation matrix of measurement errors with the general use of the joint SS channels' measurement results.

We synthesize the structure of the optimal AO detector with distributed data processing at the signal level in the joint SS network consisting of primary and secondary SS, i.e. we assume that R SS are consisting of k primary and (R - k) secondary ones. Signals are identified in each SS, the results of which are transmitted to consumers with the indicators of the quality of detection. In each of the SS, the signals received after optimal linear processing and detection are compared in threshold devices.

After the threshold devices, the implementation matrix X = ||xi.n|| comes for further processing, in which *xi.n* = 1, when the threshold has been surpassed in the temporal resolution element corresponding to the analyzed spatial resolution; when the threshold has not been surpassed, then *xi.n* = 0, in which N is the number of signals in the group.

To decide on identifying an AO, a set of zeros and ones *xi.n* is subjected to joint processing. With this setting, it is obvious that *xi.n* is a random variable obeying the Bernoulli distribution:

$$P(x_{i.n}) = P_{i.n}^{x_{i.n}} (1 - P_{i.n})^{1 - x_{i.n}} \tag{4}$$

in which *Pi.n* is the probability of exceeding the threshold in the (i.n) processing channel. In the absence of a signal *Pi.n = Fi.n*, an erroneous alarm is probable, and when the signal is influenced by *Pi.n = Di.n*, the detection of a signal is probable.

The problem of optimal signal processing in the described data model can be considered in different settings. In our case, we shall consider the problem of optimal signal processing in a "narrow" setting, which consists of the fact that the quality indicators of detection in the processing channels are specified. If we assume that a set of random variables *xi.n*, arrives at the device's input for joint processing of the received signals, then the total probability distribution of all possible combinations of *xi.n*, both in the absence and in the presence of a signal (hypotheses *H0* и *H1*), is *P(xi.n|H0)* and *P(xi.n|H1)*. These combinations are arbitrary but known.

For each specific set *xi.n*, we shall form the likelihood ratio:

$$\Xi = \frac{P(x_{i.n} \mid H_1)}{P(x_{i.n} \mid H_0)} \tag{5}$$

Comparison of Ξ with a threshold determined by the admissible probability of an erroneous alarm provides an optimal decision on the presence or absence of a signal according to the Neumann-Pearson criterion. Through the independence of the noise in different processing channels, it can be formulated as follows:

$$\begin{gathered} P(x_{11}, \ldots, x_{i.n} \mid H_0) = \prod_{i,n=1}^{R} P(x_{i.n} \mid H_0) = \\ = \prod_{i,n=1}^{R} F_{i.n}^{x_{i.n}} \left(1 - F_{i.n}^{x_{i.n}}\right)^{1 - x_{i.n}}. \end{gathered} \tag{6}$$

Taking into account the breakdown of data sources, it can be formulated as follows:

$$P(x_{11},\ldots,x_{i.n} \mid H_0) =$$
$$= \prod_{i,n=1}^{k} F_{i.n}^{x_{i.n}} \left(1 - F_{i.n}^{x_{i.n}}\right)^{1-x_{i.n}} \prod_{i,n=1}^{R-k} F_{i.n}^{x_{i.n}} \left(1 - F_{i.n}^{x_{i.n}}\right)^{1-x_{i.n}}. \tag{7}$$

It is easy to notice that when the signal is active, the exceeding of the thresholds in different processing channels forms independent events. Then, taking into account the breakdown of processing channels, it can be formulated as follows:

$$P(X \mid H_1) = \prod_{i=1}^{R} \prod_{n=1}^{N} P(x_{i.n} \mid H_1) =$$
$$= \prod_{i=1}^{k} \prod_{n=1}^{N} D_{i.n}^{x_{in}} (1 - D_{in})^{1-x_{i.n}} \cdot \tag{8}$$
$$\cdot \prod_{j=1}^{R-k} \prod_{n=1}^{N} P_{0j} D_{j.n}^{x_{j.n}} (1 - P_{0j} D_{j.n})^{1-x_{j.n}}.$$

Expression (5), taking into account (7) и (8), can be formulated as follows:

$$\Lambda = \prod_{i=1}^{R} \frac{D_i^{x_i} (1 - D_i)^{1-x_i}}{F_i^{x_i} (1 - F_i)^{1-x_i}} =$$
$$= \prod_{i=1}^{k} \frac{D_i^{x_i} (1 - D_i)^{1-x_i}}{F_i^{x_i} (1 - F_i)^{1-x_i}} \prod_{j=1}^{R-k} P_{0j} \frac{D_j^{x_j} \left(1 - D_j\right)^{1-x_j}}{F_j^{x_j} \left(1 - F_j\right)^{1-x_j}}. \tag{9}$$

Taking the logarithm of this expression, we obtain the following:

$$L = \ln \Lambda = \sum_{i=1}^{k} x_i (\ln D_i - \ln F_i) + (1 - x_i) \cdot$$
$$\cdot \left[\ln(1 - D_i) - \ln(1 - F_i)\right] +$$
$$+ \sum_{j=1}^{R-k} x_j \left(\ln P_{0j} \ln D_j - \ln F_j\right) + \left(1 - x_j\right) \cdot \tag{10}$$
$$\cdot \left[\ln\left(1 - P_{0j} D_j\right) - \ln\left(1 - F_j\right)\right].$$

We shall denote the factors *xi* and *xj* as Qi and Qj respectively:

$$Q_i = \ln D_i - \ln F_i - \ln(1 - D_i) + \ln(1 - F_i) =$$
$$= \ln\left(\frac{D_i}{1 - D_i} \frac{1 - F_i}{F_i}\right),$$
$$Q_j = \ln P_{0j} D_j - \ln F_j - \ln\left(1 - P_{0j} D_j\right) + \ln\left(1 - F_j\right) =$$
$$= \ln\left(\frac{P_{0j} D_j}{1 - D_j} \frac{1 - F_j}{F_j}\right). \tag{11}$$

Discarding the terms that do not depend on *xi.n*, we obtain the optimal algorithm for AO detection (according to the Neumann-Pearson criterion) while combining the previous solutions of all processing channels:

$$L = \sum_{i=1}^{k} Q_i x_i + \sum_{j=1}^{R-k} Q_j x_j \gtrless z_0 \tag{12}$$

in which z0 is the threshold determined by the specified initial alarm probability F.

The optimal general processing of signals from joint SS is reduced to the weighted summation of ones and zeros *xi.n* reflecting the previous decisions made in all processing channels. Weight coefficients (11) increase the role of those channels where the correct previous decision is more likely, i.e. where the probability of detecting Di or *Dj* (taking into account P0) is higher and the probability of an erroneous alarm *Fi.n* or Fj.n is lower.

Thus, when implementing optimal algorithms for identifying signals from joint SS, the weight coefficients depend both on the signal-to-noise ratio and the noise level in different processing channels, and on the data reliability from secondary SS. This is the specificity of the structure of processing devices generally using signals from joint SS. Since *xi* and *xj* are equal to 0 or 1, the left side of (12) is the sum n < M of the weight coefficients *Qi* and *Qj*, which means that it can only take certain discrete values. In this case, the value of the threshold z0 may be within the limits

$$0 < z_0 < \sum_{i=1}^{k} Q_i + \sum_{j=1}^{R-k} Q_j\,,$$

To eliminate, on the one hand, the trivial decision about detection, and on the other hand, the trivial decision about non-detection of AO. If all Qi and Qj are different and the sum of any group Qi and Qj does not coincide with the sum of any other group of them, then for different combinations of the values of xi и xj (2m – 1) different values of L> 0 are possible. Choosing the threshold z0 in the intervals between the value Qi, Qj, and their different sums, it is possible to formulate (2m – 1) different rules. If the following condition is fulfilled:

$$Fi = F0,\ Di = D0,\ i = \overline{1,k} \text{ и}$$

$$Q1 = \ldots = Qk = Q \text{ и } Fj = F0,\ Dj = D0,\ j = \overline{1,m-k} \text{ и}$$

$$Q1 = \ldots = Qm\text{-}k = Q,$$

then in (12) Q can be taken out of the sum sign and both parts can be divided by this constant value. Then expression (12) can be written as follows:

$$L = \sum_{i=1}^{k} x_i + \sum_{j=1}^{R-k} x_j \gtrless z_{01} \qquad (13)$$

As it follows from (13), for all possible xi и xj the value L> 0 can take only m different values. In this case, we obtain the well-known rule for detecting "k out of m", according to which the signal is assumed to be detected if the previous detection occurred at least in k out of m processing channels. It is easy to notice that the decision rules, with insignificant differences in the weight coefficients, will include all m decision rules of the "k out of m" type obtained from (13).

Given the fixed probabilities of previous decisions in the processing channels *Fi and Di,* different decision rules result in different values of the probabilities F and D. To select the optimal rule, i.e. the threshold in (12), we obtain an expression for the probabilities of erroneous alarm and AO's detection with optimal processing. To simplify the calculations, we shall consider only two processing channels formed by the primary and secondary SS. Since the xj obey the Bernoulli distribution with the density:

$$W(x_i) = P_i\delta(x_i - 1) + (1 - P_i)\delta(x_i)\,,$$

then for the random variable zi = Qixi we obtain the probability density and the characteristic function formulated as follows:

$$W(z_i) = P_i\delta(z_i - Q_i) + (1 - P_i)\delta(z_i)\,,$$

$$\Theta(u) = P_i e^{iuQ_i} + (i - P_i)$$

In this case, the characteristic function of the L-sum of independent values (13) is as follows:

$$\Theta_L(u) = \prod_{i=1}^{m} \Theta_i(u) = \prod_{i=1}^{m}\left(P_i e^{iuQ_i} + (1 - P_i)\right) \qquad (14)$$

The inverse Fourier transform gives the probability density L*:*

$$\begin{aligned} W_L(u) = {} & \prod_{i=1}^{m}(1 - P_i)\delta(z) + \\ & + \sum_{k=1}^{m}\sum_{i_1=1}^{m-k-1}\cdots\sum_{i_k=i_{k-1}+i}^{m} P_{i_1}P_{i_2}\ldots P_{i_k}\cdot \\ & \cdot\delta\left(z - \sum_{r=1}^{k} Q_{ir}\right)\prod_{j=1}^{m}\left(1 - P_j\right). \end{aligned} \qquad (15)$$

When k changes from 1 to m, the multiplicity of the sum in (15) normally also changes from 1 to m. Since we are interested in the case when m = 2, then:

$$W_L(u)=\prod_{i=1}^{2}(1-P_i)\delta(z)+\sum_{i_1=1}^{2}P_{i_1}\delta\left(z-Q_{i_1}\right)\cdot$$
$$\cdot\prod_{j=1}^{2}\left(1-P_j\right)+P_{i_1}P_{i_2}\delta\left[z-\left(Q_{i_1}+Q_{i_2}\right)\right] \qquad (16)$$

At the same time, note that the availability factor of this system is present in one of the processing channels (namely. in the second one). We obtain the probability of erroneous alarm or detection by substituting Fi или *Di*, in (15) and integrating it from z0 к ∞. Since z0 > 0, the first term in (15) does not contribute to the calculated integral. The same applies to all the terms wherein the argument of the δ-function $\sum_{r=1}^{k}Q_{ir}<z_0$. If the greater nearest z0 value of the sum of the weighting coefficients including n equals to $\sum_{r=1}^{k}Q_{ir}$, then the probability of exceeding the threshold z0 can be formulated as follows:

$$P=\sum_{k=n}^{m}\sum_{i_1=1}^{m-k+1}\dots\sum_{i_k=i_{k-1}+1}^{m}P_{i1}P_{i2}\dots P_{ik}\prod_{j=1}^{m}\left(1-P_j\right). \qquad (17)$$

For instance, for the case m = 2, when the condition "1 of 2" is fulfilled, expression (17) can be written as follows:

$$P = P2(1 – P1) + P1P2.$$

Consequently, if there is no signal, then *F* = F2(l – F1) + F1F2,

and under the signal's influence

$$D = D2(1 – D1) + D1D2.$$

If *F1* = … = *Fm* = *F0* и *D1* = … = *Dm* = *D0*, then

$$Q1 = ... = Qm$$

And for expression (13) it can be written as follows:

$$P=\sum_{n=k}^{m}C_m^n\tilde{P}^n\left(1-\tilde{P}\right)^{m-n} \qquad (18)$$

Using expressions (15) and (18) for given values of *Fi*, *Di*, the initial probabilities F and D can be calculated for any value of the threshold z0 and the corresponding decision function. The larger is z0 (i.e. the stricter the decision rule), the smaller is the F and D. According to the Neumann-Pearson criterion, if it is necessary that $F << \alpha 0$, then the optimal decision function will result in the pair of the largest values of F and D under the condition $F << \alpha 0$.

The proposed algorithm (13), which is optimal for *Q1* = … = *Qm*, is slightly simpler than (12) since it does not require an analysis of the probabilities Fi and *Di* in each channel and, consequently, the calculation of *Qi*. Therefore, in what follows we shall use expression (13) for different values of *Qi* hereinafter. It should be noted that we mainly consider the specifics of the general use of dissimilar SS signals under the condition that the weight coefficients differ insignificantly. This leads to a conclusion that the transition from the algorithm (12) to the algorithm (13) for different values of *Qi* reduces the number of decision functions, hence the choice among pairs of values F and D. If one of m rules of the algorithm (13) is revealed, then the transition from (12) to (13) will not lead to energy losses.

Thus, general detection improvement consists in choosing one of the decision functions satisfying algorithm (12) for the joint processing and setting the same relative thresholds in all channels providing such values of *Fi* that, given the chosen decision function, result in the required value of the resulting probability F.

3. Conclusion

It is noteworthy that the choice of the decision function in the general detection of joint SS signals, as a rule, should be determined not only by the requirements for the best detection of signals in such systems. Since AO's state affiliation must be determined during the identification process, this provides for the strengthening of the decision function in the proposed algorithm, although the resulting characteristics of detection may deteriorate.

The discussed method can be used to solve problems of improving the quality of data support using joint airspace surveillance systems in airport areas. For this purpose, in particular, the improvement of data processing in the network of the airspace surveillance system should be implemented within the framework of the implementation of a single data field for providing technological processes at airports proposed in [4].

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