The Metaphysics of Internal Controls

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Abstract

A quality internal control system has been seen as a remedy for various corporate governance issues. Two pieces of legislation, the Foreign Corrupt Practices Act (FCPA) and the Sarbanes-Oxley Act (SOX) deal with very different corporate governance issues, but each argue for a similar remedy. Both the FCPA and the SOX legislation argue that improved (or proper) internal controls are necessary to root out bribery of foreign officials, in the case of the FCPA, and (in the case of SOX) to support the accurate preparation of financial statements. An issue that has yet to be resolved is that the quality of internal control systems is subject to subjective assessments of the internal control deficiencies and their impact. This paper presents a mathematical model of internals controls based on Gödel number of axioms. This results in the representation of quality internal controls in terms of an integer. This approach also allows for inferences about financial statements and various auditing judgements.

Keywords

Internal Controls, Axioms, Gödel Numbering, Inferences

1. Introduction

Two pieces of legislation, the Foreign Corrupt Practices Act (FCPA) (US Congress, 1977) and the Sarbanes Oxley Act (SOX) [1] deal with very different corporate governance failures, but each argued for a similar remedy. Both the FCPA and the SOX legislation argue that improved (or proper) internal controls are necessary to root out bribery of foreign officials, in the case of the FCPA, and (in the case of SOX) to support the accurate preparation of financial statements. Previsous research suggest that in a well-controlled (or perhaps perfectly-controlled) company, all business events are specified and that all individuals adhere to these specifications. Therefore, an organization with a perfect system of internal control will exhibit two features. First, all potentially legitimate (acceptable) business events will be defined.1 Second, the organization's information system sufficiently captures information about those business allowing a person to make a judgement concerning whether actual business events have unfolded according to that definition. Thus, for a quality internal control system these two features are necessary: defining state changing business events and capturing necessary information about those events [3]The purpose of this paper is to present a mathematical model of internal controls which defines both the state changes and the information about these state changes.

Each step (business event) can have various controls applied to it, which should ensure that the event provides correct information to the system, and that the system provides reliable information. Srinidhi and Vasarhelyi [4] furthered the notion of internal controls as improving the reliability of data in an accounting information system, by integrating them with inferences made by auditors about the quality of the data.

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[1] Herein, defining a business event implies specifying the finer grained steps which are combined into business processes [2]

Srivastava [5] also presented an argument for a model of audits which considers this interdependency and additionally the auditor's inferences about the quality of information about account balances. Therefore, a company with all controls functioning perfectly, would have zero-material difference between what the firm's data should be versus the actual data. Using this as a basis for the perfect company, the difference between a target company (one being audited for example) and this prototype can be considered the "semantic distance" or the perceiver's (auditors for example) conceptualization of the difference between this company and the perfect company.

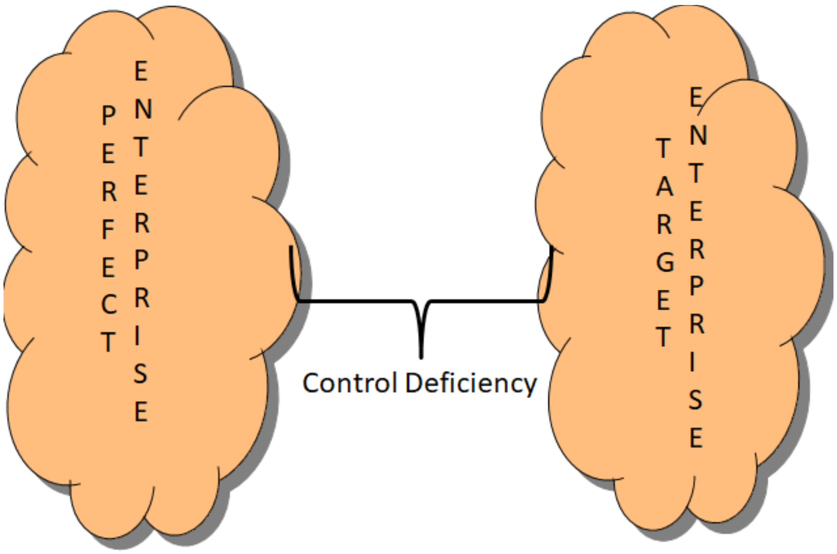

Figure 1: Semantic Distance (from [4])

2. Internal Control Axioms

This conceptualization of the company can be represented with axioms which describe the events and the parties that execute these events. An employee type is defined in terms of attributes a_1 to a_n.[2] These values for attributes are inserted via a business event which is defined in Equations 2 & 3.

Equation 1:

$$(\forall x EmployeeType(x) \supset (\exists a_1 Attribute(a_1) \wedge Slot_1(x, a_1)) \wedge \left((\exists a_2 At(a_2) \wedge Slot_2(x, a_2)\right) \ldots \left((\exists a_n At(a_n) \wedge Slot_n(x, a_n))\right) \; for\ EmployeeType\ with\ n\ slots$$

Where critical attribute a_n is not Null, i.e. to be a "Cashier" there is a critical event that transforms a non-cashier to a cashier and this event creates a valid value for a_n. A similar equation can define other objects, i.e. EventTypes and ResourceTypes, such as sales, cash receipts, raw materials, inventory receiving, etc. This would allow for a definition of not just the types of agents but also the economic events and the resources in the organization. In a well-controlled organization one of the numerous EmployeeTypes would be assigned responsibility for executing that particular business event. Therefore, for each EmployeeType a definition of the set of business events that they will be ResponsibleFor is also required.

Equation 2:

$$\forall x BusinessEvent(x) \supset (\exists y EmployeeType(y) \wedge ResponsibleFor(y, x))$$

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[2] This formulation is based on frame representations from Brachman [6] and Hayes [7].

For each business event at MOF level 1, defined as those events for which an organization's state changes, an instance of Equation 2 would be required. There would be a separate axiom for all business events defined in the organization. Not all business events insert a specific attribute. Some business events update attributes, i.e. change an employee's name. Other business events simply view an attribute, i.e. a store manager that is responsible for approving credit may view a credit limit. This means that an axiom is required to represent the possible changes to an attribute. Equation 3 depicts the Alters axiom which defines the relationship between a business event and a particular attribute. Where V = View, C = Change or update, I = Insert, & D = Delete) come from the database operations enumerated by Tsichritzis & Lochovsky (1982, p. 63). The entire set of attributes define each object from the numerous attributes included in Equation 1, so each attribute will have at least one Alters axiom (an Insert) associated with it. In addition, there will also be other instances of Equation 3, describing BusinessEvents which View, Change, and Delete an attribute. These instances of Equation 3 combine with Equation 2 will define not only which EmployeeType can Insert a value for an attribute, but also which EmployeeType can view the attribute.

Equation 3:

$$\forall x BusinessEvent(x) \supset (\exists y(Attribute(y) \exists z Operation(z) \wedge Alters(x,y,z) \wedge (z = V \vee z = C \vee z = I \vee z = D))$$

A fourth axiom, depicted in Equation 4, formulates the accountability hierarchy of the firm's superior-subordinate structure.

Equation 4:

$$\forall x \exists y(EmployeeType(x) EmployeeType(y) \supset Accountable(x,y))$$

Equations 2, 3, and 4 deal specifically with a critical component of effective internal controls, which is the development of an effective organizational structure that includes establishing appropriate roles for people in the organization [8]. Specifically, a perfectly controlled company has a definition of all the employee types, resource types, and event types (Eq 1). Each employee type is responsible for a specific set of business events (Eq 2). Each business event alters a specific attribute (Eq 3). Finally, each employee type is accountable to another employee type (Eq 4).

These equations now define (not name) what are the state changes (Eq 3) who is responsible for this state change (Eq 2), who is accountable for the individual making the state change, and finally the definition (in terms of attributes) the Resources, Events, and Agents (Eq 1). Using these equations, it is possible to define a perfectly-controlled company.

Kurt Gödel is recognized as being responsible for two incompleteness theorems [9]–[11]. While the substance of his theorems is beyond this paper, one step in his proofs becomes invaluable for representing and combining internal control axioms into a representation of a perfectly controlled firm. Gödel recognized that each component of an axiom can be represented as numbers and sequences of numbers. The EmployeeType axiom expressed in Equation 1 formulated for Cashiers can be converted to a Gödel number as shown in Figure 2.

$$\underset{2^{x1}}{(\forall} EmployeeType(\underset{3^{x2}}{Cashier)} \; \underset{5^{x2}}{\supset} \; \underset{7^{x4}}{(\exists} Attribute(\underset{11^{x5}}{Name)} \underset{13^{x7}}{\wedge} \underset{17^{x11}}{Slot_1}(\underset{3^{x2}}{Cashier}, \underset{11^{x4}}{Name))} \; \underset{13^{x8}}{\wedge} \; \ldots$$

Figure 2: Gödel Numbering of the Axiom Defining the Cashier Employee Type [4]

In the same manner all the other MOF level 1 internal control axioms can also be represented as an integer. This results in a definition of a complete set (or perfectly controlled firm) internal controls as an integer as follows:

Complete Set of Internal Controls (CSIC) = ET*BE*RF*AC*……

Figure 3: Complete Set of Internal Controls (see [4])

Using a similar approach, a Gödel number for all distinct company types can be derived. For example, to determine the definition of a perfectly controlled retail company, the integer representing the axioms related to raw materials (hiring of raw material buyers, conversion processes, responsibility for vetting raw material vendors, and so on) would divided into the CSIC with the result being the $CISC_{retail}$.

$CISC_{retail}$= CISC/ Gödel number for raw material controls

Additionally, the control state for each company can also be determined as follows. The internal controls present in a company can be represented as a Gödel number. The following calculation would indicate whether the company being evaluated has the requisite controls.

Control difference = $CISC_a$ / Gödel number for target company

It is also important to examine the conclusions or inferences from these axioms to verify and formulate an understanding the ontology. The results of these inferences (or formulae) are obtained from initial formulae through a series of symbolic manipulations [12]. The examination of the inferences can also make it clear what aspects of the ontology's domain are represented in the axioms.

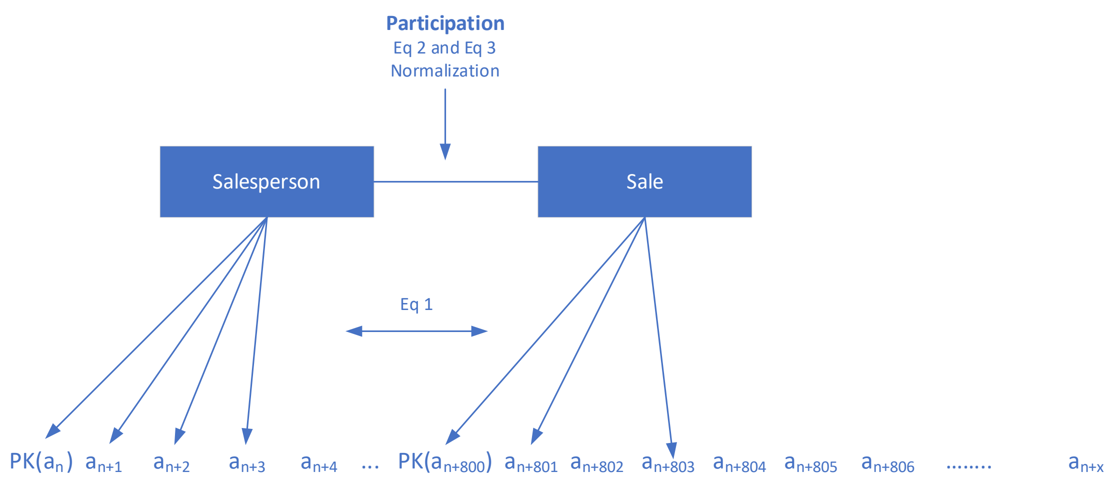

Figure 4: Inferences Related to REA Ontological Classes[4]

Figure 4 defines the insideParticipate, the Economic Event (Sale) and the Economic Agent (Salesperson) from the Accountability Layer at the M1 level for the Revenue Cycle. The same process described in Figure 4 can be used to derive the other classes and associations of the REA ontology. The axioms can also be used to make conclusions about financial statement objects. Figure 5 shows the formulation of the financial statement balance for Cash. Similar inferences can be used to create balances for other balance sheet amounts. The income statement items can be inferred from the value for the events over a specific date range.

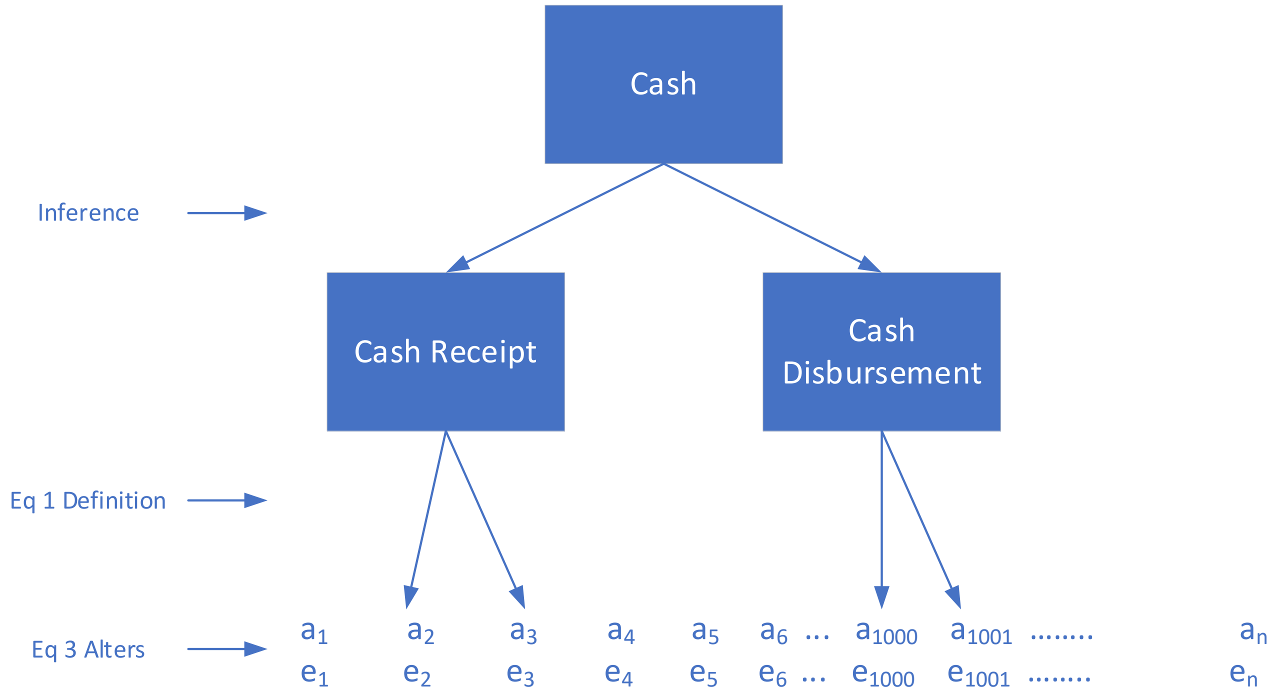

Figure 5: Inference of Cash Balance (see [4])

3. Conclusions

This paper demonstrated that axioms can also be used to create both the classes and association of the REA ontology. Additionally, using the axioms can also infer the organizational economic units. These organizational units and the axioms can also infer whether other controls such as segregation of duties are present. These inferences are critical as they demonstrate that the axioms are equivalent to the REA Ontology and therefore the internal controls are also equivalent the ontology. Previous research has shown that different accounting numbers can be derived from various implementations which used the REA ontology as the framework for the database schema. By showing that the axioms can are integral to the ontology it can be concluded that any implementation that uses these axioms will also include these internal controls. Therefore, the REA ontology is a complete business ontology as it not only includes the objects and associations of the domain, but the internal controls.

4. References

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