FORT: a minimal Foundational Ontological Relations Theory for Conceptual Modeling Tasks

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Abstract

Foundational relations play an important role in the ontological foundations of conceptual modeling. Their investigation has been theoretically addressed in philosophical/ontological theories, and empirically offered in foundational ontologies (FOs). FOs are comprehensive theories that model the world as top-level entities and relations. Empirically, for modelers aiming to use foundational relations without an urge for entity types, FOs seem to be complex to comprehend, comply with, and integrate in practice. And since the practice of these relations is critical for conceptual modeling tasks, we present an approach that builds a well-founded entity-type free relations theory within a first-order-logic formalization, besides large complex FOs. The theory contributes to a minimal set of foundational ontological relations (FORT) by importing extant theories (mereotopology and location) and (re-)formalizing other relations (dependence, membership, constitution, and entity-location), while no FO has compromised this set.

Keywords

Foundational Relations, Foundational Ontologies, Conceptual Modeling, First-order Logic

1. Introduction

Ontology-driven conceptual modeling is the representation of a particular knowledge domain according to some ontological analysis concerning the modeling decisions made in the representation of this domain i.e. according to a particular world-view. Adding this level of ontological analysis is crucial to conceptual modeling in the sense that it guarantees the validation of the resulting model according to the intended semantics, and supports integration and interoperability services when the model is linked to other models having different world-views. Having a language that offers primitive relations and rule constraints offers an ontological tool, in which an ontological analysis is provided, and facilitates the task of ontology-driven conceptual modeling. Nowadays, such a language is offered by means of foundational ontologies (FOs); comprehensive theories comprising both foundational concepts and relations.
Foundational relations are basic primitive relations that play a fundamental role in the ontological foundations of conceptual modeling. They have been formally probed in applied philosophical and ontological theories e.g. mereology and mereotopology.

The use of foundational relations has proven itself crucial across domains; the parthood relation in aligning and correlating ontologies growing in the bio-informatics domain [1], the locative

ER'2022 Forum @ the 41st International Conference on Conceptual Modeling, October 17–20, 2022.
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(and topological) relations in disambiguating the spatial information embodied in biomedical ontologies and enhancing their automatic reasoning capabilities [2], and other relations as typologies of part-whole relations depending on the types of the participating entities in contextualizing parthood in cognitive sciences [3], linguistics [4], ontology [5], and conceptual modeling [6]. The application that motivates our usage of foundational relations is the ontological representation-of and reasoning-over the structural and spatial constraints of a physical entity, across any domain. This implies modeling the link between; an entity as a whole and its different inseparable parts (i.e. parthood and dependence); an entity as collective whole and the entities that it groups under certain semantics (i.e. membership); an entity and its constituents (i.e. constitution); an entity and the spatial region that locates it (i.e. location); and the spatial constraints among entities (i.e. entity-location). Hence, we are concerned with parthood (and connection), location, membership, and constitution relations, with the formal properties that characterize them.

Moreover, the use of foundational relations is limited with the use of FOs in which they are offered. However, (1) FOs model the world as entities (categories) and relations, thus constraining modelers in application domains to comply to these categories and map them to their domain categories, if the relations are to be used. Also, (2) FOs require a deep understanding of the ontological and philosophical commitments made in each in order to justify a choice of a FO where major differences rely at a high meta-physical level, thus quite complicating its comprehension and integration within an applicative conceptual modeling task. In addition to the fact that (3) no FO incorporates all the mentioned relations above or their interlinks, as will be detailed in Section 2. These three points highlight the difficulties that modelers face in the process of selecting, customizing and using foundational relations in their conceptual modeling tasks.

Thus, on the one hand these relations are critical in ontological-driven conceptual modeling tasks. And on the other hand, the theories in which they are offered are complex and require huge inclusive commitments. Therefore, we pose our research problem of why not have a theory of a minimal set of foundational ontological relations, besides large complex FOs? To overcome the preceding difficulties, our approach aims at answering the posed research question by building a well-founded entity-type free theory of a minimal set of foundational ontological relations (FORT). This theory serves as the first step towards a goal of providing a FORT-ontology, for modelers, to explicate the semantics and distinct properties of foundational relations. Hence, this paper contributes towards a theory of a minimal set of foundational ontological relations; dependence, parthood (components and elements), connection, location, membership, and constitution. The remainder of this paper is organized as follows. Section 2 outlines the followed methodology. Section 3 demonstrates preliminary research in the literature. Section 4 analyzes, picks up, and adapts, for each selected relation, a set a axioms that characterizes its intended semantics. Section 5 discusses the work presented and presents some final considerations.

2. Methodology

To fulfill the goal of a FORT-ontology as a tool for modelers to explicate the semantics of terminology systems [7], we follow a methodology inspired by the work in [8]:

1. formalize FORT in an expressive logical language that is adequate for the specification of

formal theories; a first-order-logic (FOL) specification.

2. associate the theory with another logical language that validates the existence of models for FORT using consistency checks, which is the case in FOs; e.g. Common Logic (CLIF) [9], Alloy [10], Prover9 [11] or TPTP-syntax [12] serializations. This work has been done along with a comparison and alignment between the relation-based content of extant FOs and FORT's relations in [13].
3. translate the FOL-specification into a decidable knowledge representation language that supports reasoning and inference services; a SROIQ-DL specification.
4. implement the T-boxes of the SROIQ formalization in an ontological language supporting its practices in conceptual modeling tasks; an OWL2-DL implementation.

The work is in progress, and this paper only presents the first step of formally building FORT [1], in which two tasks arise:

Selecting the minimal set of relations in FORT: within the logical, ontological, and philosophical literature on foundational ontologies, there is a common set of formal (domain-independent) relations that acquire a foundational role such as parthood (mereology), connection (topology), dependence, location, membership, and constitution. While no extant FO (formulates and) imports all the preceding relation theories[2] e.g. DOLCE [14] does not incorporate entity-to-entity location and membership, BFO [15] does not (and cannot) consider constitution, UFO [16] does not explicitly encompass location and so on, FORT tends to encompass all the mentioned relations. The "minimality of this set" claim is based on: (a) these relations have been commonly and exhaustively addressed in FOs as fundamental ones which assures the importance of including each in a set that is ought to be a one of foundational relations, and (b) the applicative goal that the approach is to serve (representing the structural and spatial constraints of a physical entity) necessitates that each mentioned relations is required and without which, the full representation of an entity cannot be supported. Moreover, these relations are generic in the sense that they are independent of a domain of application.

At this point, one might raise the question "why not simply extend an extant FO with the set of relations targeted in this paper?" Such a task might exceed some empirical and theoretical abilities of a theory to comprise a specific relation. Earlier in this work, we adopted an approach of extending BFO with a relations theory, including the constitution relation, in which the debate initiates. However, this approach results in violating theoretically the ontological commitments made by BFO, and empirically, the user-intended representation and inferences where the constituent and the constituted entity are regarded as identical i.e. the same individual. It is possible now to claim this is a particular case with only BFO being problematic. But what about other FOs? Why does DOLCE not account for a membership theory? Why does UFO not consider a comprehensive theory of location? The question here goes beyond "why not extend an extant FO?" to "can extant FOs accommodate for additional relations needed in practice?" If yes, why is there not any consideration yet?

[1] The authors are aware of the obligations that such a proposal issues such as the necessity of showing its applicability using real-world examples as modeling dilemmas, however this is to be presented in future and not within the scope of this paper's presentation.

[2] For a detailed relation-based analysis comparing and aligning FORT, DOLCE, BFO, and UFO with respect to one another, the reader is referred to [13]

Characterizing and axiomatizing relations in FORT: the relations are recognized as basic primitive relations i.e. they are not defined in terms of other relations and span multiple application domains [14]. The relations are characterized in terms of ground axioms i.e. some algebraic properties, and additional non-ground axioms (properties and constraints) that establish distinctions among them, and semantically possible interlinks across them.

A major limitation of the work is that, up to now, the context is assumed in an atemporal framework i.e. it captures reality as it exists at a single moment of time. Thus, we do not consider (yet) the behavior of these relations at different times. Though, we use the time predicate to axiomatize some notions that need the intrusion of time to be defined. However, we do not study the evolution of (the relata of these) relations with time neither their conservation of identity while undergoing changes, which is widely discussed subject [17].

3. Background and related work

3.1. Mereo(topo)logy

The part-whole relation is a fundamental relation that has been intensively addressed and actively occupying wide research areas in the domain of knowledge representation and reasoning. Logically, the formal accounts of the relation, grouped under the term "mereology" [18], started by Lesniewski [19] and were universally approved. Later, Lyons [20] and Cruse [21] pointed out that mereological relations cannot fully represent part-of relations where there are some cases of intransitivity in natural language, and initiated the hypothesis of a multiplicity of part-whole relations for cognitive tasks. Whereas, Simons [22] and Varzi [23] investigated different mereologies corresponding to different parthood relations and in link to topological theories.
Ground mereology **M** is the common core of any mereological theory presenting *parthood* as a reflexive, antisymmetric, and transitive relation, i.e. a partial order. $P(x, y)$ denotes "x is part of y". Based on P, mereological predicates are built for a wider semantic range; *proper-part*, *equal*, *overlap*, *underlap*, *overcross*, *undercross*, *proper-overlap*, and *proper-underlap*. Different mereological theories (Minimal, Extensional, Closure, and General mereologies) can be then generated by adding axioms, i.e. assumptions of composition and/or decomposition principles, to **M** to allow for finer grained types of part-whole relations or/and permit intransitivity in some cases.

Mereology provides a sound and formal common core for the analysis and representation of part-whole relations that is highly beneficial from a mathematical and philosophical aspect. However, as discussed in [24] and following several authors (e.g. [25], [26], and [3]), its application as a theory of parthood for conceptual and ontological modeling tasks is problematic. This is due to either the theory considered too strong to hold for part-whole relations at the conceptual level, or too weak to characterize the distinctions between different typologies of part-whole relations.

Moreover, a purely mereological theory is too tight to serve as a true theory both; parts and wholes i.e. it does not handle the global property of wholeness (the properties of whole entities) [27]. For example, the relationship between an entity and its surface, or the relation of an entity being around some other entity, which reveal the limits of pure mereology to capture very basic spatial relations. This is where the need of a complementary topological analysis is required to characterize entries and the spatial relations existing between them. The integration of the topological theory of connection, characterized as reflexive and symmertic, with the mereological theory of parthood is made using a bridging principle $((\forall x, y)P(x, y) \rightarrow \forall z(C(z, x) \rightarrow C(z, y))c)$. This is called the enclosure axiom; stating that C is monotonic with respect to P; if an entity x is part of another y, then whatever is connected to x is connected to y i.e. whatever is connected to a part is also connected to the whole. Ground mereotopology **MT** is the theory defined by the P-axioms and the C-axioms. In **MT**, the variety of distinct relations [27] [23] [28] that can be asserted allows for connection without sharing parts (external connection), and richer spatial subrelations of P (tangential and interior parts), O (tangential and interior overlap), and U (tangential and interior underlap).

3.2. Location

The location relation has been studied in several aspects. First, the entity-to-region location relations; these are location relations that paradigmatically hold between entities (physical objects, social groups, events, holes, tropes, cavities...) and regions [29] under the substantivalism assumption. The study of this relation poses questions concerning the link between a located entity and the region at which it is located i.e. whether they share common parts or not. With the aim of formulating the axioms that govern the location relation and its interaction with parthood and other mereo(topo)logical relations, some philosophers have formulated the logics of location. These logics aim to capture and represent the systematic link between (a) the mereological properties (and relations) between located entities and (b) the mereological properties of (and relations between) the locations of these entities. In other words, capturing the ways in which (a) must mirror (b). Moreover, debates concerning the framing of the location predicate arise; "is exactly located" (which several authors adopt e.g. [23] and [30]) or "is weakly located at" (discussed and presented in [31]). In [23], the primitive location relation L captures the intuition "being (exactly) in a place", focusing on space only. $L(x, y)$ takes place between an entity x that is not a spatial region and y that is the spatial region at which x is located. L is formalized as conditionally reflexive (and not reflexive) i.e. reflexivity hold only on the region entities, and functional since it represents exact location and not a any notion of minimal address location. Second, the region-to-region location relations; these are relations holding in a domain comprising only spatial regions i.e. the primitive L and its successive definitions collapse onto plain mereotopological relations by interpreting L as identity i.e. a reflexive, transitive and symmetric [23]. In approaches as in [2] for anatomical reasoning, both arguments of L range over a function R_x that is the region of an entity x, and L is interpreted in two fashions; (a) with link to mereology i.e. a locative relation between two entities holds if an entity is part-of or overlaps the other; and (b) without a link to mereology i.e. locative relations between partially/wholly coinciding entities without sharing parts.

4. Foundational relations in FORT

In this Section, we demonstrate the relations in FORT as shown in figure 1 below.

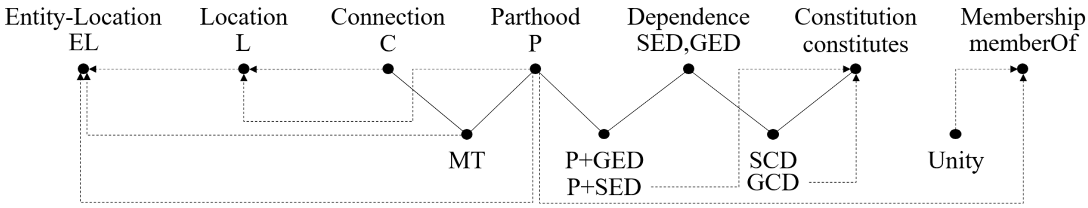

Figure 1: FORT relations, their interlinks (plain lines) and participation, as axioms, in the formalization of other relations (dotted lines).

4.1. Ontological dependence

The original analysis on the dependence notion started with Husserl [32] within a classical philosophical context, upon which later formalizations were proposed. In [33], Fine and Smith considered dependence as a quasi-mereological primitive relation introduced in terms of 4 axioms; reflexive, transitive, and links to parthood. Later in [22], Simons criticizes the preceding axioms as resembling a sort of a topological relation, and presents dependence within a modal-logic approach, using the existence relation as the primitive. Several authors also addressed the notion of dependence such as; Thomasson by introducing different kinds of temporal dependence; constant and historical in [34]; and Vieu and Aurangaue by specializing kinds of generic dependence (functional dependence) both within modal logic formalizations in [35]. Moreover, in foundational ontologies such as in BFO [15]; it is studied between qualities, realizable entities (e.g. roles and functions), or processes (also process boundaries), and (in)dependent continuants or processes, and in DOLCE [14]; it is deeply axiomatized (ontological and spatial dependence notions) in a modal approach using the presence primitive relation.

Our consideration of ontological dependence is a property characterizing the persistence semantics between two entities (individuals) or entity types (categories). It plays an important role in the representation and reasoning over relations e.g. parthood or connection in particular. We investigate ontological dependence as based on a primitive existence relation (in contrast to Smith and Fine's primitivity) and at both; instances and universals levels, as inspected in [36] and following to DOLCE's distinction between specific and generic constant dependence, within a non-modal formalization. For the primitive existence relation we use the binary predicate E, with the notation $E(x,t)$ standing for "entity x exists at time t". The introduction of t as an instance of time in E does not put the approach in a temporal framework, but simplifies the representation of the framework at instants of time, when necessary.

An entity x is specifically existentially dependent entity y, denoted $SED(x,y)$, iff; at any time t, x cannot exist at t unless y exists at t; & x and y are different entities; & x exists at some t (Dd1). For example, a person is be specifically existentially dependent on its brain.

$$\forall(x,y)SED(x,y) =_{df} \forall t(E(x,t) \rightarrow E(y,t)) \wedge \neg(x=y) \wedge \exists t E(x,t) \quad \text{(Dd1)}$$

By (Dd1), it follows that $SED(x,y)$ is irreflexive, and implied to be transitive.

$$(\forall x,y,z)SED(x,y) \wedge SED(y,z) \rightarrow SED(x,z) \quad \text{(Dt1)}$$

An entity type ϕ is generically existentially dependent on entity type φ, denoted $GED(\phi,\varphi)$, iff; at any time t, for every x instance of ϕ, x cannot exist at t unless there exists some instance y of φ att and x and y are different entities; & there exists time t such that there exists instance x of ϕ; & ϕ and φ are disjoint (Dd2). For example, a person might be generically constantly dependent on having a heart.

$$\forall(\phi,\varphi)GED(\phi,\varphi) =_{df} \forall x,t((\phi(x)\wedge E(x,t)) \rightarrow \exists y(\varphi(y)\wedge E(y,t)))\wedge \exists x,t(\phi(x)\wedge E(x,t))\wedge\neg\exists z(\phi(z)\wedge\varphi(z)) \quad \text{(Dd2)}$$

Given that ϕ and α are disjoint, generic existential dependence is transitive.

$$GED(\phi,\varphi)\wedge GED(\varphi,\alpha)\wedge\neg\exists x(\phi(x)\wedge\alpha(x)) \rightarrow GED(\phi,\alpha) \quad \text{(Dt2)}$$

4.2. Parthood and dependence:

Using parthood P, satisfying the axioms of **CEM**, we extend P with dependence to allow for semantic inferences on the persistence of wholes according to some parts inline with [14]. Relations grouped under this section provoke for part entities that are inseparable from their wholes i.e. essential and mandatory parts. The notion of separability that we seek for is not that of physical connection/disconnection of a part/whole from its whole/parts by any means of physical (e.g. by hand, clippers, scissors) or chemical (e.g. chemical filtration process) separation which depends on some granularity level of separation. Instead, separability is elucidated by means of ontological dependence (specific and generic existential dependencies) which serves an important role in the reasoning over the persistence conditions of the parts and whole entities. Thus, adding semantically specialized parthood relations and benefiting the usage and representation of dependencies in conceptual modeling tasks. Using the two preceding dependency definitions of section 4.1, we introduce the notions of components and elements.

4.2.1. Componenthood:

x is a ComponentOf y iff; x is a part of y & y is generically existentially dependent on x (Pd1). For example; the engine is a component of the car, the heart is a component of the body of a living being.

$$(\forall x,y)ComponentOf(x,y) =_{df} P(x,y)\wedge GED(\phi(y),\varphi(x)) \quad \text{(Pd1)}$$

From the axioms of P and Pd1, ComponentOf is a strict partial order relation satisfying weak supplementation axiom.

4.2.2. Elementhood:

x is an ElementOf y iff; x is an part of y & y is specifically existentially dependent on x (Pd2). For instance; the tin layer is an element of the brocade, the brain is an element of the human's body. Elements parts of a whole are those whose existence is elementary. For instance, the spatial existence of the tin layer in the brocade is (along the other layers) what makes up the identity of a brocade as a whole.

$$(\forall x, y)\text{ElementOf}(x, y) =_{\text{df}} P(x, y) \wedge \text{SED}(y, x) \quad \text{(Pd2)}$$

Similar to componenthood, ElementOf is a strict partial order relation i.e. proper parthood satisfying weak supplementation. Following [24], components represent mandatory parts while elements present essential ones. At this point, it is possible to assert additional predicates e.g. ElementOverlap to infer about entities sharing a common element as being identical.

4.3. Location

In FORT, we treat the location relation in a three-fold manner.

4.3.1. Region-to-region locative relations:

we use P to show the mereo(topo)logical relations without the commitment to formalizing a region entity type.

4.3.2. Entity-to-region locative relations:

we import the location theory in [23] using the L primitive ("exactly located at region") and borrowing the axioms, definitions and theorems (7.1 to 7.9, L1 to L7, 7.14 to 7.24), excluding that of identity. The borrowed formalization of L establishes; links to parthood and connection; reasoning on the location of the mereotopological properties of an entity with respect to its location; reasoning on the location of an entity with respect to the mereotopological properties of its location; and reasoning about the location of the mereotopological properties of an entity with respect to the mereotopological properties of its location.

4.3.3. Entity-to-non-region locative relations:

motivated by the work in [2], we introduce an entity-location relation using a primitive EL indicating "located at/on/in", with links to parthood and connection. We believe that this aspect of locative relations is essential to represent the spatial links (non-region) between entities that are temporarily or permanently located in spaces, but never share parts with each other or with these spaces. For example; the painting is located on the wall, the basket is located at the top of the fridge. These examples hold even though the wall and the top of the fridge are not regions (i.e. L solely cannot hold) and neither the painting is part of the wall nor the basket is parts of the fridge (i.e. P cannot hold). In such cases, neither pure mereological relations nor solely region-location relations are useful to accomplish the task.
Turning now to specifying the axioms of EL. We assume that every entity is located in/at itself i.e. reflexive (ELa1), and that transitivity is guaranteed i.e. is x is located in y and y is located in z then x is located in z (ELa2). As links to mereology, we assert the axiom (ELa3); if x is part of y then x is located in y, which directly implies (from ELa2) the theorems (ELt1); if x is part of y and y is located in z then x is located in z, and (ELt2); if x is located in y and y is part of then x is located in y. The preceding two theorems serve in the reasoning about the entity-location of entity x, with respect to the entity-location of its whole y (e.g. the red dear is part of the painting, and the painting is located at the wall, then the red dear is located at the wall), and the location of an entity x with respect to the whole of its location entity z (e.g. the painting is located at the rock

wall, and the wall is part a of the Rocher du Château, then the painting is located in the Roche du Chateaux site). In both cases, the entity x is (not necessarily) part of the entity y, rather than a pure locative relation. As links with the locative relation L, using (7.16) and (L.3), theorems (ELt3); if x is entity-located in y, and y is exactly located in z, then x is wholly located in z, and (ELt4); if x is entity-located in y, then x's spatial region if part of y's spatial region, are provable respectively.

$$(\forall x)\mathrm{EL}(x,x) \tag{ELa1}$$

$$(\forall x,y,z)(\mathrm{EL}(x,y)\wedge \mathrm{EL}(x,z)\rightarrow \mathrm{EL}(x,z)) \tag{ELa2}$$

$$(\forall x,y)(\mathrm{P}(x,y)\rightarrow \mathrm{EL}(x,y)) \tag{ELa3}$$

$$(\forall x,y,z)(\mathrm{P}(x,y)\wedge \mathrm{EL}(y,z)\rightarrow \mathrm{EL}(x,y)) \tag{ELt1}$$

$$(\forall x,y,z)(\mathrm{EL}(x,y)\wedge \mathrm{P}(y,z)\rightarrow \mathrm{EL}(x,y)) \tag{ELt2}$$

$$(\forall x,y,z)(\mathrm{EL}(x,y)\wedge \mathrm{L}(y,z)\rightarrow \mathrm{WL}(x,z)) \tag{ELt3}$$

$$(\forall x,y,z,w)(\mathrm{EL}(x,y)\wedge \mathrm{L}(x,z)\wedge \mathrm{L}(y,w)\rightarrow \mathrm{P}(z,w)) \tag{ELt4}$$

Furthermore, one can define other locative predicates, such as tangential and interior entity locative relations, using mereotopological definitions of tangential and interior parts, as well as partial and whole entity locative relations using locative definitions of partial and whole locations.

4.4. Membership

Upon the characterization of the membership relation; a debate arises two views on the relation. The first considers the relations as a part-whole relation typology i.e. formalized using parthood such as in formal ontological studies e.g. [37] following the analysis in [38], and in the meronymic literature on part-whole relations e.g. [4] and [39]. The second view analyzes membership as a primitive independent relation e.g. [22]. However in both, the members of a whole participating in a membership relation acquire a unifying relation (also referred to as a uniform structure in meronymic literature) that binds all the members together and a maximality constraint on the members with respect to this relation. Furthermore, an imperative point in the formalization of membership derives from ranging the relation over collective wholes, also called aggregates, and proceeding with characterizing collectives/aggregates as a mandatory for characterizing membership. Characterizing aggregates can be performed by; a relation holding among members [40]; a common role played by all members [41]; specifying a single entity type or a least common subsumer type that all members are instantiated to as in [42] and [43] respectively; the uniqueness of the collective's decomposition into members in [44].

In FORT, we regard membership as a primitive relation distinct from parthood inline with that in [22]. More precisely, we follow Simons's approach in attaching a notion of unity to the range of the membership relation i.e. the aggregate, and adopt the two preceding aspects as follows. First, we formalize membership, notating "is member of", using the binary predicate memberOf as irreflexive (Ma1) and asymmetric (Ma2).

$$(\forall x)\neg \mathrm{memberOf}(x,x) \tag{Ma1}$$

$$(\forall x, y)\text{memberOf}(x, y) \rightarrow \neg\text{memberOf}(y, x) \tag{Ma2}$$

Second, we proceed with characterizing the range by implanting a number of axioms. We employ axioms from BFO's axiomatization of an aggregate entity [15]; an aggregate has more than one member at least one time (Ma3); all proper parts of an aggregate overlap some member (Ma4); and all members of an aggregate are disjoint proper parts (Ma5). Then we add the axioms (Ma6) stating that an aggregate is the exact sum of its members.

$$(\forall x, y)\text{memberOf}(y, x) \rightarrow \exists t, m_1, m_2(m_1 \neq m_2 \wedge E(x, t) \wedge \text{memberOf}(m_1, x)$$
$$\wedge\text{memberOf}(m_2, x)) \tag{Ma3}$$
$$(\forall x, p, y)\text{memberOf}(y, x) \wedge PP(p, x) \rightarrow \exists o(\text{memberOf}(o, x) \wedge O(o, p)) \tag{Ma4}$$
$$(\forall x, y)\text{memberOf}(x, y) \rightarrow (PP(x, y) \wedge \forall m(\text{memberOf}(m, y) \rightarrow x = m \vee \neg O(m, x))) \tag{Ma5}$$
$$(\forall x, y)\text{memberOf}(y, x) \rightarrow (\forall w(O(w, x) \leftrightarrow \exists m(\text{memberOf}(m, x) \wedge O(w, m)))) \tag{Ma6}$$

Third, we advance with characterizing the members of an aggregate by sharing a characteristic property: a unity according to a unifying relation. The dispute resides on the ground axioms of the unifying relation; more precisely transitive in [38] and intransitive in [37]. In our paper, we endorse Gangemi's intransitivity and proceed with its formalization. Let $\mathfrak{R}_R$ denote a finite set of binary predicates that are unifying relations representing characteristic relations of entities, such that $\forall R_i \in \mathfrak{R}_R$, R_i is conditionally reflexive (Ra1) and symmetric (Ra2). Then we define the unification, denoted $\mathscr{U}_{R_i}(z)$ as; an entity z is unified under R_i iff z is the sum of entities in the domain of R_i, and all entities that possess R_i and are parts of z are linked by R_i (Rd1).

$$(\forall x, y)R_i(x, y) \rightarrow R_i(x, x) \wedge R_i(y, y) \tag{Ra1}$$
$$(\forall x, y)R_i(x, y) \rightarrow R_i(y, x) \tag{Ra2}$$
$$(\forall z)\mathscr{U}_{R_i}(z) =_{df} \forall r(R_i(r, r) \rightarrow P(r, z)) \wedge \forall m(O(m, z) \leftrightarrow \exists r(R_i(r, r) \wedge O(m, r))) \wedge \forall a, b$$
$$(R_i(a, a) \wedge R_i(b, b) \wedge P(a, z) \wedge P(b, z) \rightarrow R_i(a, b)) \tag{Rd1}$$

Some interpretation must be given about the choice of predicates in Rd1: The first statement, z is the sum of entities in the domain of R_i, indicate that each entity that is satisfied by R is a part of z. However, alone, it is not sufficient to assert that z is exactly the sum of the entities satisfying R_i and not more. For example; consider p1, p2, and p3 as the entities satisfying R (i.e. $R(p1, p1)$, $R(p2, p2)$, and $R(p3, p3)$), and consider z as the entity built by p1, p2, p3, and some other entity g. In this case, z satisfies the first statement, knowing that it does hold as the intended meaning. Thus, a second statement is needed to strengthen the declaration of the sum of R's entities; for every entity m that is overlapping z; there must exist an r entity belonging to the domain of R, such that m overlaps r. The third statement, for each pair of entities that are parts of z "and satisfying R_i", the entities must be linked with R_i too. The predicate "and satisfying R_i" is to ensure that parts of parts of z (that do not satisfy R) are not (necessarily) linked by R_i. For instance, the hand of a person of jury (who is part of a jury) should not be linked with the role of being a jury member, so that the jury is unified by the relation: being a jury member.
After defining $\mathscr{U}_{R_i}(z)$, we further axiomatize the range of memberOf as a unified entity (Ma7);

each aggregate has a unified relation R_i under which it is unified at all the times that it exists.

$$(\forall x, y)\text{memberOf}(y, x) \rightarrow \exists i(\mathscr{U}_{R_i}(x) \wedge (\forall t(E(x, t) \rightarrow \mathscr{U}_{R_i}(x))) \tag{Ma7}$$

In contrast to [44] in characterizing collectives, we consider that the characteristic property unifying the whole of a membership relation (or as it is formalized being the plurality constituting the collective), holds at all times of the existence of the collective, and applies on any plurality that constitutes. In other words, even if this sum (plurality) changes with time e.g. a member ceases, then the whole of the membership relation (collective) still maintains its unification under R_i. While in their approach, they consider that for the plurality to be x to be characterized by a property F at t notated $F_t x$, it has to be wholly present at t, notated $\varepsilon_t^w x$. Our interpretation for our disagreement is that we consider the aggregate as unified by R_i at all the times that it exists even if some members change or decreased in number. This only meas that these members do not satisfy R_i anymore, while the aggregate is still unified by R_i.

Fourth, we use what preceded to infer the conditions under which two aggregates are considered identical in (Mt1) if they are unified by the same unification relation.

$$(\forall x, y, w, z)\text{memberOf}(x, y) \wedge \text{memberOf}(w, z) \wedge \exists i(\mathscr{U}_{R_i}(y) \wedge \mathscr{U}_{R_i}(z)) \rightarrow y = z \tag{Mt1}$$

4.5. Constitution

In the ontological literature on constitution, its formalization varies according upon two philosophical views of the world. A multiplicative-based view approach allows for different entities to be co-localized in the same space-time. Different entities signify incompatible essential properties, such as persistence properties, yet related. Whereas, a reductionist-based view approach presupposes that each space-time location contains at most one entity, and the incompatible essential properties are only unintended different interpretations of different perspectives that one can assume about spatio-temporal entities. Thus, a main difference between the two views regards the mode of existence of entities populating the world at a metaphysical level [45]. FOs adopt different philosophical views which develop highly in some foundational relations such as constitution. We demonstrate the difference through a popular example; the vase and the clay which constitutes it.

For instance, DOLCE [14] is a descriptive ontology (multiplicative view) adopting a cognitive based representation of the world underlying natural language and human common-sense. With constitution, DOLCE recognizes a vase, as constituted by an amount of clay, and clay, as an amount of matter. A vase and amount of clay are taken as two different types that are co-localized in the same space-time location. DOLCE supports the claim of constitution is not identity based on three arguments following [46] and [47]. Firstly, the two entities have different histories; clay can be present before the vase. Secondly, the two entities have different persistence conditions; the clay can persists upon a change of change while the vase ceases to exist; and the vase can undergo a replacement of a certain amount of clay by another amount, while a piece of clay cannot i.e if replaced, the piece of clay is not the same piece anymore. Thirdly, the two entities differ in their essential metaphysical relational properties i.e. the clay can exist without any artificial intervention while a vase needs an intended intervention to exist. While BFO [15] is a realist ontology (reductionist view) capturing the world as (multiple) particular perspectives of reality i.e. a possibly multiple instantiations of the same particular individual. In contrast to DOLCE, BFO regards the entities participating in a constitution relation, e.g. the vase and the clay, as the same spatio-temporal individual that instantiates different universals at the same spacetime location.

In FORT, we adhere to the multiplicative view to formalize constitution. First, we build the primitive constitution relation using the binary predicate constitutes as a strict partial order relation (Ca1-Ca3), and link it to existence (Ca4) and parthood (Ca5) relations.

$$(\forall x)\neg constitutes(x,x) \tag{Ca1}$$

$$(\forall x,y)constitutes(x,y) \rightarrow \neg constitutes(y,x) \tag{Ca2}$$

$$(\forall x,y,z)constitutes(x,y) \wedge constitutes(y,z) \rightarrow constitutes(x,z) \tag{Ca3}$$

$$(\forall x,y)constitutes(x,y) \rightarrow \exists t(E(x,t) \wedge E(y,t)) \tag{Ca4}$$

$$(\forall x,y,z)constitutes(x,y) \wedge P(z,y) \rightarrow \exists x'(P(x',x) \wedge constitutes(x',z)) \tag{Ca5}$$

Second, on the link with the dependence relations, following DOLCE's specific and generic constant constitution. We define two relations; specific constitutional dependence SCD (Cd1) and generic constitutional dependence GCD (Cd2). Using the definitions and axioms of dependence in 4.1, theorems (Ct1-Ct4) are implied.

$$(\forall x,y)SCD(x,y) =_{df} \exists tE(x,t) \wedge \forall t(E(x,t) \rightarrow constitutes(y,x)) \tag{Cd1}$$

$$(\forall \phi,\psi)GCD(\phi,\psi) =_{df} \neg\exists z(\phi(x) \wedge \psi(z)) \wedge \forall x(\phi(x) \rightarrow \exists tE(x,t)) \wedge \forall x,t(\phi(x) \wedge E(x,t) \rightarrow \exists y(\psi(y) \wedge constitutes(y,x)) \tag{Cd2}$$

$$(\forall x,y)SCD(x,y) \rightarrow SED(x,y) \tag{Ct1}$$

$$(\forall \phi,\psi)GCD(\phi,\psi) \rightarrow GED(\phi,\psi) \tag{Ct2}$$

$$(\forall x,y,z)SCD(x,y) \wedge SCD(y,z) \rightarrow SCD(x,z) \tag{Ct3}$$

$$(\forall \phi,\psi,\varphi)GCD(\phi,\psi) \wedge GCD(\psi,\varphi) \wedge \neg\exists z(\phi(z) \wedge \varphi(z)) \rightarrow GCD(\phi,\varphi) \tag{Ct4}$$

Third, we adjoin constitution with a dependence between the relata types. The dependence that we seek for is not specific i.e. the existence of vase (v1) does not depend specifically and constantly on that of the instance of clay (c1). This is to say that (c1) can be replaced with another piece (c2) without violating the persistence of v1, hence no specific existential dependence of v1 on c1 ensues. Nevertheless, any other instance v# of the same type "vase", could not have been artificially created without the presence of some clay, any instance c# of the type clay. Hence the dependence regarded in constitution is general constitutional dependence (GCD) between classes. To represent GCD, they types shall be disjoint to ensure that the causal existential connection between instances of the classes comes to an end. While DOLCE asserts generic constant constitution between categories (e.g. $GK(NAPO,M)$ a generic constant dependence between a non-agential physical object and amount of matter), we permit the relation itself to apply GCD between the relata of the relation without the obligation of instantiating the types to categories that ensure constitutional dependence. This is done via the axiom (Ca6) asserting GCD between the relata types and ensuring their disjointedness.

$$(\forall x,y,\phi,\psi)constitutes(x,y) \wedge \phi(x) \wedge \psi(y) \rightarrow GCD(\psi,\phi) \tag{Ca6}$$

Fourth, we link constitution to parthood. Since the matter of the constitution relation is taken to be mereologically invariant [14], i.e. it changes identity when some parts change, then parts of matter are considered to be essential ones (Ca7).

$$(\forall x, y)\text{constitutes}(x, y) \rightarrow \forall z(P(z, x) \rightarrow \text{ElementOf}(z, x)) \quad \text{(Ca7)}$$

5. Conclusion

In this paper, we have contributed to the ontological foundations of conceptual modeling by proposing and building a foundational ontological relations theory (FORT), within a first-order logic formalization. FORT addresses the research problem of providing a comprehensive theory of pure foundational relations besides large complex FOs. A basic assumption made is to distinguish parthood from other primitive relations such as membership and constitution, rather than part-whole relations typologies defined in terms of parthood. Such a demarcation is adopted to (a) refrain a philosophical debate on the consideration of these relations as part-whole typologies, (b) delimit the scopes upon which transitivity holds, and (c) advocate for the additional semantics that each relation acquires divergently from one another.

While a major limitation of the approach lies in being atemporal, the forte points of FORT are twofold. First, it is free of entity-types yet normalizes constraints on the relata of the relation which makes the theory straightforward to integrate within extant theories, without obliging the compliance to a hierarchy of entity types. Second, it is ample for representing the internal structure, spatial conditions, and interrelations between entities via the selected set of relations. The importing of concrete extant relation theories such as mereology and location derives in FORT being adequate at ontological level with the existing philosophical and formal literature. The selection and reformulation of other relation theories such as membership, constitution, entity-location, componenthood, and elementhood serves the goal of a minimal, yet comprehensive, set of foundational ontological relations.

For conceptual modeling tasks, this is a first step towards a contribution that would add value to existing approaches. As explained in the methodology part, upcoming steps include the development of an ontological language of primitive relations and rule constraints, corresponding to the FOL formalization, which is crucial to the community.

Acknowledgments

We are indebted to Claudio Masolo and Emilio Sanfilippo for their helpful comments and multiple feedbacks on earlier versions of this work.

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