Some New Cordial Digraphs

Sarang Sadawarte [a], Sweta Srivastav [a] and Rajiv Kumar [b]

[a] *Department of Mathematics, Sharda University, Greater Noida, India*
[b] *G L Bajaj Institute of Technology and Management, Greater Noida, India*

Abstract

A directed graph, also known as a digraph, is a graph in which each edge has direction. A linear directed cycle, Cm is a cycle whose all edges have the same direction. A linear directed graph is called directed cordial if it preserves binary or 0 – 1 labeling under certain condition. This paper dealt with directed cordial labeling of directed cycle Cm and square of directed Cm. We investigate directed path joining two copies of directed cycle Cm and directed square cycle Cm2 is directed cordial. Further we show that the directed path union of of r-copies of directed cycle and directed square cycle graph Cm2 is cordial under certain condition.

Keywords

Directed cycle, directed square cycle, cordial graph, directed cordial graph

1. Introduction

Graph theory is an important domain of discrete mathematics with voluminous applications in various streams. Graph labeling is an allotment of labels to edges or vertices or both. Numerous graph labeling schemes are studied and researched by many researchers. Gallian [3] reviewed and surveyed various labeling schemes invented by many researchers. In graph theory cordial labeling plays a vital role. It has ample of applications in many streams such as computer science, networking and communication network. Cahit [1] introduced cordial labeling in 1958. The directed cordial labeling of directed path was investigated and studied by Al-Shamiri [5] in 2019. He proved many results on linear directed path in the context of some graph operations are directed cordial. We study directed cordial labeling of directed cycle and their square subject to certain conditions. An overview provided on basic terminology and notations is needed for the presentation of our results. In the present work, we consider the *W* is directed graph.

2. Terminology and Notation

2.1. Definition

A Graph *W* is said to be linearly directed if all the edges have same direction. (clockwise or anticlockwise).

2.2. Definition

Consider *W* as digraph. Define a function ρ: $V(W) \rightarrow \{0, 1\}$
We define the edge labeling as follows $\rho^* : E(W) \rightarrow \{0, 1\}$

$$\rho^*(s_i\, s_{i+1}) = 2^{s_i} \pmod 2$$

WCNC-2022: Workshop on Computer Networks and Communications, April 22 – 24, 2022, Chennai, India.
EMAIL: 2020489011.sadawarte@dr.sharda.ac.in (Sarang Sadawarte)
ORCID: 0000-0002-4223-6949 (Sarang Sadawarte)

$s_\rho(0)$ and $s_\rho(1)$ represent number of vertices are assigned with label 0 and label 1 respectively. Similarly, $e_\rho(0)$ and $e_\rho(1)$ represent number of edges are assigned with label 0 and label 1 respectively. This binary labeling is known as cordial if both criteria preserves

a) $|s_\rho(0) - s_\rho(1)| \le 1$

b) $|e_\rho(0) - e_\rho(1)| \le 1$

The graph W which preserves cordiality is called as a cordial graph [1].

If directed graph is cordial, we call it as directed cordial graph.

2.3. Definition

A path $P_r = b_0, b_1 \ldots . b_{r-1}$ is an alternating sequence of different vertices with r – 1 length. The linear directed path P_r have the same orientation. (clockwise or anticlockwise).

2.4. Definition

A cycle C_m is a closed path. A cycle is said to be linear directed cycle C_m if all its edges have the same direction. (clockwise or anticlockwise).

2.5. Definition

A linear directed cycle graph C_m on m vertices is called directed square cycle graph if each pair of vertex has distance less than or equal to two. We denote directed square cycle graph as C_m^2 .

2.6. Definition

The graph W = {2-C_m : P_r } is constructed with merging two same copies of directed cycle graph with indefinite path length . We denote W = {r-C_m : P_r } as path union of *r* same copies of directed cycle graph.

3. Results

Theorem 3.1. The linear directed cycle C_m is directed cordial.

Proof: Let $t_1, t_2, \ldots t_m$ be successive vertices of directed cycle C_m.

Consider a function $\rho: V(W) \to \{0, 1\}$ resulted as following

Case I For m ≡ 0, 1 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\; p = 0, 3 \pmod 4 \\ 1\,;\; p = 1, 2 \pmod 4 \end{cases}$$

Case II For m ≡ 2, 3 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\; p = 0, 2 \pmod 4 \\ 1\,;\; p = 1, 3 \pmod 4 \end{cases}$$

Table 1: Cordial pattern for vertices and edges

Conditions on m	Vertices Pattern	Edges Pattern
m≡ 0(mod4)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1)$
m ≡ 1(mod4)	$v_\rho(0)+1 = v_\rho(1)$	$e_\rho(0) = e_\rho(1) +1$
m ≡2(mod4)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1)$

m ≡3(mod4)	$v_\rho(0)+1 = v_\rho(1)$	$e_\rho(0)+1 = e_\rho(1)$

The corresponding observed cases of m with labeling pattern of edges and vertices resulted in above table. Therefore, the conditions $|e_\rho(0) - e_\rho(1)| \leq 1$ and $|v_\rho(0) - v_\rho(1)| \leq 1$ are preserved and verified.

Example 1. The directed cordiality of C_6 is elaborated as shown in Figure 1.

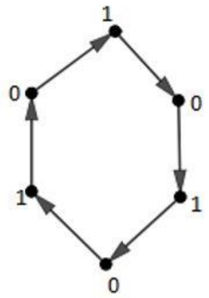

Figure 1: C_6 is directed cordial

Theorem 3.2. The linear directed square cycle C_m^2, $m \geq 6$ is directed cordial.
Proof: Let $t_1, t_2, \ldots t_m$ successive vertices of directed square cycle C_m.
Consider a function $\rho : V(W) \rightarrow \{0, 1\}$ resulted as following

For m ≡ 0, 2, 4 (mod6)

$$\rho(t_p) = \begin{cases} 1\ ;\ p = 1, 3, 5 \pmod 6 \\ 0\ ;\ p = 0, 2, 4 \pmod 6 \end{cases}$$

Table 2: Cordial pattern for vertices and edges

Conditions on m	Vertices Pattern	Edges Pattern
m≡ 0,2, 4(mod6)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1)$

The corresponding observed cases of m with labeling pattern of edges and vertices resulted in above table. Therefore, the conditions $|e_\rho(0) - e_\rho(1)| \leq 1$ and $|v_\rho(0) - v_\rho(1)| \leq 1$ are preserved and verified.

Example 2. The directed cordiality C_6^2 is elaborated as shown in Figure 2.

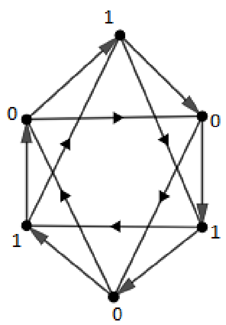

Figure 2: C_6^2 is directed cordial

Theorem 3.3. The graph W = {2-C_m : P_r }is directed cordial.

Proof: Let us denote 1st and 2nd copies of m-pan as $t_1, t_2, \ldots t_m$ and $s_1, s_2 \ldots s_m$ respectively. Let the vertices 1st, 2nd and r th of path P_r be represented by $x_1, x_2, \ldots, x_r$ which carries condition as first vertex $x_1 = t_1$ and r th vertex, $x_r = s_1$.

Consider a mapping : V (W) → {0, 1} as given below
Some cases observed

Case I For r ≡ 0(mod4), m ≡ 0, 1 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\ p = 0, 3 (\text{mod}4) \\ 1\,;\ p = 1, 2 (\text{mod}4) \end{cases}$$

$$\rho(s_p) = \begin{cases} 0\,;\ p = 1, 2 (\text{mod}4) \\ 1\,;\ p = 0, 3\ (\text{mod}4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p = 0, 3 (\text{mod}4) \\ 1\,;\ p = 1, 2 (\text{mod}4) \end{cases}$$

Case II For r ≡ 1(mod4), m ≡ 0 (mod4)

$$\rho(t_p) = \rho(s_p) = \begin{cases} 0\,;\ p = 0,\ 3 (\text{mod}4) \\ 1\,;\ p = 1,\ 2 (mod4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p = 0, 3 (\text{mod}4) \\ 1\,;\ p = 1, 2 (mod4) \end{cases}$$

Case III For r ≡ 2 (mod4), m ≡ 0, 1 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\ p = 0, 3 (\text{mod}4) \\ 1\,;\ p = 1, 2 (\text{mod}4) \end{cases}$$

$$\rho(s_p) = \begin{cases} 0\,;\ p = 1, 2 (\text{mod}4) \\ 1\,;\ p = 0, 3\ (\text{mod}4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p = 0, 2 (\text{mod}4) \\ 1\,;\ p = 1, 3 (\text{mod}4) \end{cases}$$

Case IV For r ≡ 3 (mod4), m ≡ 0, 1 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\ p = 0, 3 (\text{mod}4) \\ 1\,;\ p = 1, 2 (\text{mod}4) \end{cases}$$

$$\rho(s_p) = \begin{cases} 0\,;\ p = 1, 2 (\text{mod}4) \\ 1\,;\ p = 0, 3\ (\text{mod}4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p = 2, 3 (\text{mod}4) \\ 1\,;\ p = 0, 1 (\text{mod}4) \end{cases}$$

Case V For r ≡ 0(mod4), m ≡ 2, 3 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\ p = 0, 2 (\text{mod}4) \\ 1\,;\ p = 1, 3 (\text{mod}4) \end{cases}$$

$$\rho(s_p) = \begin{cases} 0\,;\ p = 1, 3 (\text{mod}4) \\ 1\,;\ p = 0, 2\ (\text{mod}4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p\ =\ 0,3(\mathrm{mod}4) \\ 1\,;\ p\ =\ 1,2(\mathrm{mod}4) \end{cases}$$

Case VI For r ≡ 1(mod4), m ≡ 2 (mod4)

$$\rho(t_p) = \ \rho(s_p) = \ \begin{cases} 0\,;\ p\ =\ 0,\ 3(\mathrm{mod}4) \\ \ 1\,;\ p\ = 1,\ 2(mod4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p\ =\ 0,3(\mathrm{mod}4) \\ 1\,;\ p =\ 1,2(mod4) \end{cases}$$

Case VII For r ≡ 2 (mod4), m ≡ 2, 3 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\ p\ =\ 0,2(\mathrm{mod}4) \\ 1\,;\ p\ =\ 1,3(\mathrm{mod}4) \end{cases}$$

$$\rho(s_p) = \begin{cases} 0\,;\ p\ =\ 1,3(\mathrm{mod}4) \\ 1\,;\ p\ =\ 0,2\ (\mathrm{mod}4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p\ =\ 0,2(\mathrm{mod}4) \\ 1\,;\ p\ =\ 1,3(\mathrm{mod}4) \end{cases}$$

Case VIII For r ≡ 3 (mod4), m ≡ 2, 3 (mod4)

$$\rho(t_p) = \begin{cases} 0\,;\ p\ =\ 0,2(\mathrm{mod}4) \\ 1\,;\ p\ =\ 1,3(\mathrm{mod}4) \end{cases}$$

$$\rho(s_p) = \begin{cases} 0\,;\ p\ =\ 1,3(\mathrm{mod}4) \\ 1\,;\ p\ = 0,2\ (\mathrm{mod}4) \end{cases}$$

$$\rho(x_p) = \begin{cases} 0\,;\ p\ =\ 2,3(\mathrm{mod}4) \\ 1\,;\ p\ =\ 0,1(\mathrm{mod}4) \end{cases}$$

Table 3: Cordial pattern for vertices and edges

Conditions on r, m	Vertices Pattern	Edges Pattern
r≡ 0(mod4), m ≡ 0,1,2,3(mod4)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1) + 1$
r ≡ 0(mod4), m ≡ 0,2(mod4)	$v_\rho(0) = v_\rho(1) + 1$	$e_\rho(0) = e_\rho(1)$
r ≡0(mod4), m ≡ 0,1,2,3(mod4)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1) + 1$
r ≡0(mod4), m ≡ 0,1,2,3(mod4)	$v_\rho(0) = v_\rho(1) + 1$	$e_\rho(0) = e_\rho(1)$

The corresponding observed cases of m with labeling pattern of edges and vertices resulted in above table. Therefore, the conditions $|e_\rho(0) - e_\rho(1)| \leq 1$ and $|v_\rho(0) - v_\rho(1)| \leq 1$ are preserved and verified.

Example 3 The graph W = {2-C_5 : P_6 } elaborated in Figure 3 is directed cordial.

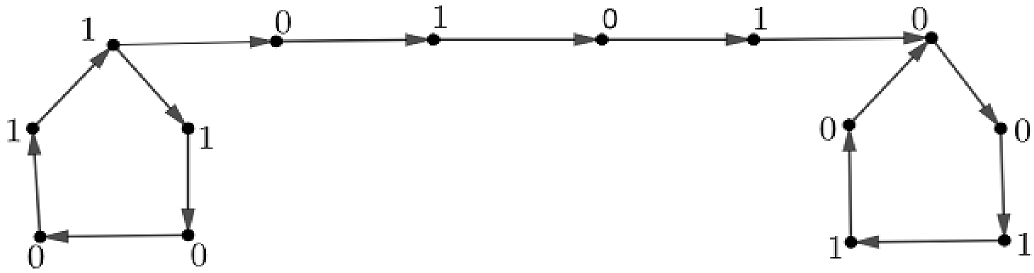

Figure 3: The graph W = {2-C_5 : P_6 } is directed cordial

Theorem 3.4. The graph W = {r-C_m : P_r }is directed cordial.

Proof. Consider the vertices of 1st, 2nd , . . . , r th copy graph as $t_{p1}, t_{p2}, \dots, t_{pr}$. Let $x_1, x_2, \dots, x_r$ be the vertices of path P_r with condition $x_1 = t_{p1}, x_2 = t_{p2}, \dots, x_r = t_{pr}$.

Consider $: V(W) \rightarrow \{0, 1\}$ with following
We observe following cases

Case I: For r ≡ 0(mod4), m ≡ 0, 1 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0 ; \ p = 1,2(\text{mod}4) \\ 1 ; \ p = 0,3(mod4) \end{cases}$$

$$\rho(t_{pq-2}) = \rho(t_{pq-3}) = \begin{cases} 0 ; \ p = 0,3(\text{mod}4) \\ 1 ; \ p = 1,2(\text{mod}4) \end{cases}$$

Case II: For r ≡ 1(mod4), m ≡ 0, 1 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0 ; \ p = 0,3(\text{mod}4) \\ 1 ; \ p = 1,2(mod4) \end{cases}$$

$$\rho(t_{pq-1}) = \rho(t_{pq-2}) = \begin{cases} 0 ; \ p = 1,2(\text{mod}4) \\ 1 ; \ p = 0,3(\text{mod}4) \end{cases}$$

Case III: For r ≡ 2(mod4), m ≡ 0, 1 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0 ; \ p = 0,3(\text{mod}4) \\ 1 ; \ p = 1,2(mod4) \end{cases}$$

$$\rho\ (t_{pq-2}) = \rho\ (t_{pq-4}) = \begin{cases} 0 ; \ p = 1,2(\text{mod}4) \\ 1 ; \ p = 0,3(mod4) \end{cases}$$

Case IV: For r ≡ 3(mod4), m ≡ 0, 1 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0 ; \ p = 0,3(mod4) \\ 1 ; \ p = 1,2(mod4) \end{cases}$$

$$\rho(t_{pq-1}) = \rho(t_{pq-4}) = \begin{cases} 0 ; \ p = 1,2(\text{mod}4) \\ 1 ; \ p = 0,3(mod4) \end{cases}$$

Case V: For r ≡ 0(mod4), m ≡ 2, 3 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0\,;\ p\ =\ 1,3(\mathrm{mod}4) \\ 1\,;\ p\ =\ 0,2(mod4) \end{cases}$$

$$\rho(t_{pq-2})\ =\ \rho(t_{pq-3})\ = \begin{cases} 0\,;\ p\ =\ 0,2(\mathrm{mod}4) \\ 1\,;\ p\ =\ 1,3(\mathrm{mod}4) \end{cases}$$

Case VI: For r ≡ 1(mod4), m ≡ 2, 3 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0\,;\ p\ =\ 0,2(\mathrm{mod}4) \\ 1\,;\ p\ =\ 1,3(mod4) \end{cases}$$

$$\rho(t_{pq-1}) = \rho(t_{pq-2}) = \begin{cases} 0\,;\ p\ =\ 1,3(\mathrm{mod}4) \\ 1\,;\ p\ =\ 0,2(\mathrm{mod}4) \end{cases}$$

Case VII: For r ≡ 2(mod4), m ≡ 2, 3 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0\,;\ p\ =\ 0,2(mod4) \\ 1\,;\ p\ =\ 1,3(mod4) \end{cases}$$

$$\rho(t_{pq-2}) = \rho(t_{pq-4}) = \begin{cases} 0\,;\ p\ =\ 1,3(\mathrm{mod}4) \\ 1\,;\ p\ =\ 0,2(mod4) \end{cases}$$

Case VIII: For r ≡ 3(mod4), m ≡ 2, 3 (mod4)

$$\rho(t_{pq}) = \begin{cases} 0\,;\ p\ =\ 0,2(mod4) \\ 1\,;\ p\ =\ 1,3(mod4) \end{cases}$$

$$\rho(t_{pq-1}) = \rho(t_{pq-4}) = \begin{cases} 0\,;\ p\ =\ 1,3(\mathrm{mod}4) \\ 1\,;\ p\ =\ 0,2(mod4) \end{cases}$$

Table 4: Cordial pattern for vertices and edges

Conditions on r, m	Vertices Pattern	Edges Pattern
r ≡ 0(mod4),m ≡ 0, 2 (mod4)	$v_\rho(0)$ =$v_\rho(1)$	$e_\rho(0)$ =$e_\rho(1)$ +1
r ≡ 1(mod4), m ≡ 0, 2 (mod4)	$v_\rho(0)$ =$v_\rho(1)$	$e_\rho(0)$ =$e_\rho(1)$
r ≡ 2(mod4), m ≡0, 2 (mod4)	$v_\rho(0)$ =$v_\rho(1)$	$e_\rho(0)$ =$e_\rho(1)$ +1
r ≡ 3(mod4), m ≡ 0, 2 (mod4)	$v_\rho(0)$ =$v_\rho(1)$	$e_\rho(0)$ =$e_\rho(1)$
r ≡ 0(mod4),m ≡ 1, 3 (mod4)	$v_\rho(0)$ =$v_\rho(1)$	$e_\rho(0)$ =$e_\rho(1)$+1
r ≡ 1(mod4),m ≡ 1, 3 (mod4)	$v_\rho(0)$+1 =$v_\rho(1)$	$e_\rho(0)$ =$e_\rho(1)$ +1
r ≡ 2(mod4),m ≡ 1, 3 (mod4)	$v_\rho(0)$ =$v_\rho(1)$	$e_\rho(0)$ =$e_\rho(1)$ +1
r ≡ 3(mod4),m ≡ 1, 3 (mod4)	$v_\rho(0)$ =$v_\rho(1)$ +1	$e_\rho(0)$+1 =$e_\rho(1)$

The corresponding observed cases of m with labeling pattern of edges and vertices resulted in above table. Therefore, the conditions $|e_\rho(0) - e_\rho(1)| \leq 1$ and $|v_\rho(0) - v_\rho(1)| \leq 1$ are preserved and verified.

Example 4: The graph W = {5-C_m : P_5 }is directed cordial as shown in Figure 4.

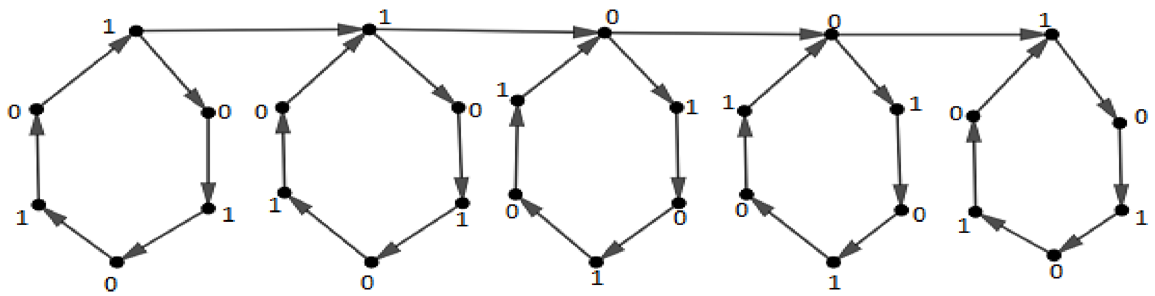

Figure 4: The graph W = {5-C_m : P_5 }is directed cordial

Theorem 3.5. The graph W = {2-C_m^2 : Pr } merged by two copies of directed cycle graph with path of indefinite length is cordial.(For m ≥ 6)

Proof. Consider vertices of 1st and 2nd copies of C_m^2 as $t_1, t_2, \ldots .t_m$ and $s_1, s_2 \ldots s_m$ respectively. Let vertices 1st, 2nd and r th of path P_r be represented by $x_1, x_2, \ldots, x_r$ which carries condition as first vertex $x_1 = t_1$ and r th vertex, $x_r = s_1$.

Consider : V (W) → {0, 1} as stated,
When m ≡ 0, 2, 4(mod6), we observe same cases given below

Case I For r ≡ 0 (mod4)

$$\rho(t_p) = \begin{cases} 1 ; \ p = 1, 3, 5(\text{mod}6) \\ 0 ; \ p = 0, 2, 4(\text{mod}6) \end{cases}$$

$$\rho(s_p) = \begin{cases} 1 ; \ p = 0, 2, 4(\text{mod}6) \\ 0 ; \ p = 1, 3, 5(\text{mod}6) \end{cases}$$

$$\rho(x_r) = \begin{cases} 1 ; \ r = 1, 2(\text{mod}4) \\ 0 ; \ r = 0, 3(\text{mod}4) \end{cases}$$

Case II For r ≡ 1(mod4)

$$\rho(t_p) = \rho(s_p) = \begin{cases} 1 ; \ p = 1, 3, 5(\text{mod}6) \\ 0 ; \ p = 0, 2, 4(\text{mod}6) \end{cases}$$

$$\rho(x_r) = \begin{cases} 1 ; \ r = 1, 2(\text{mod}4) \\ 0 ; \ r = 0, 3(\text{mod}4) \end{cases}$$

Case III For r ≡ 2, 3 (mod4)

$$\rho(t_p) = \begin{cases} 1 ; \ p = 1, 3, 5(\text{mod}6) \\ 0 ; \ p = 0, 2, 4(\text{mod}6) \end{cases}$$

$$\rho(s_p) = \begin{cases} 1 ; \ p = 0, 2, 4(\text{mod}6) \\ 0 ; \ p = 1, 3, 5(\text{mod}6) \end{cases}$$

$$\rho(x_r) = \begin{cases} 1 ; \ r = 0, 1(\text{mod}4) \\ 0 ; \ r = 2, 3(\text{mod}4) \end{cases}$$

Table 5: Cordial pattern for vertices and edges

Conditions on r, m	Vertices Pattern	Edges Pattern
r ≡ 0(mod4),m ≡ 0, 2, 4 (mod6)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1) + 1$
r ≡ 1(mod4), m ≡ 0, 2, 4(mod6)	$v_\rho(0) = v_\rho(1) + 1$	$e_\rho(0) = e_\rho(1)$
r ≡ 2(mod4), m ≡ 0, 2, 4(mod6)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1) + 1$
r ≡ 3(mod4), m ≡ 0, 2, 4(mod6)	$v_\rho(0) = v_\rho(1) + 1$	$e_\rho(0) = e_\rho(1)$

The corresponding observed cases of r, m with labeling pattern of edges and vertices resulted in above table. Therefore, the conditions $|e_\rho(0) - e_\rho(1)| \le 1$ and $|v_\rho(0) - v_\rho(1)| \le 1$ are preserved and verified.

Example 5 The graph W = {$2\text{-}C_6^2 : P_5$ } is directed cordial as shown in Figure 5.

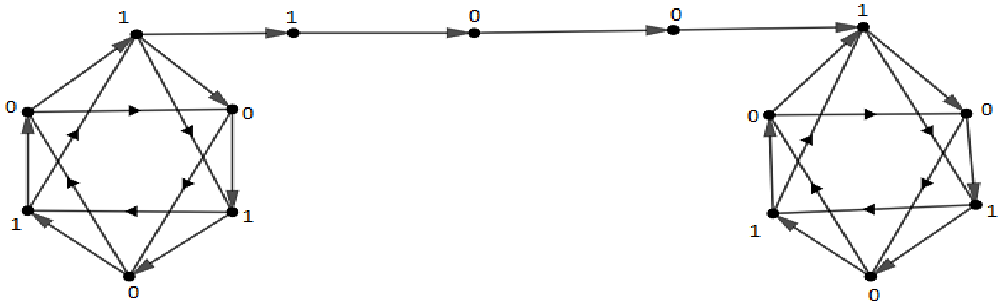

Figure 5: The graph W = {$2\text{-}C_6^2 : P_5$ } is directed cordial

Theorem 3.6. The graph W = {$r\text{-}C_m^2 : P_r$ }, m ≥ 6 preserves directed cordial labeling.
Proof. Let us represent the vertices of 1st, 2nd , . . . , r th copy as $t_{p1}, t_{p2}, \ldots, t_{pr}$. Let $x_1, x_2, \ldots, x_r$ be the vertices of path P_r with condition $x_1 = t_{p1}, x_2 = t_{p2}, \ldots, x_r = t_{pr}$.
Consider $\rho : V(W) \to \{0, 1\}$ When m ≡ 0, 2, 4(mod6).
We examine same cases.

Case I For r ≡ 0(mod4)

$$\rho(t_{pq}) = \begin{cases} 1\,;\ p = 0,2,4(\text{mod}6) \\ 0\,;\ p = 1,3,5(\text{mod}6) \end{cases}$$

$$\rho(t_{pq-2}) = \rho(t_{pq-3}) = \begin{cases} 1\,;\ p = 1,3,5(\text{mod}6) \\ 0\,;\ p = 0,2,4(\text{mod}6) \end{cases}$$

Case II For r ≡ 1(mod4)

$$\rho(t_{pq}) = \begin{cases} 1\,;\ p = 1,3,5(\text{mod}6) \\ 0\,;\ h = 0,2,4(\text{mod}6) \end{cases}$$

$$\rho(t_{pq-1}) = \rho(t_{pq-2}) = \begin{cases} 1\,;\ \text{p} = 0,2,4(\text{mod}6) \\ 0\,;\ \text{p} = 1,3,5(\text{mod}6) \end{cases}$$

Case III For r ≡ 2 (mod4)

$$\rho(t_{pq}) = \begin{cases} 1\,;\ p = 1,3,5(\text{mod}6) \\ 0\,;\ p = 0,2,4(\text{mod}6) \end{cases}$$

$$\rho(t_{pq-2}) = \rho(t_{pq-4}) = \begin{cases} 1\,;\ p = 0,2,4(\text{mod}6) \\ 0\,;\ p = 1,3,5(\text{mod}6) \end{cases}$$

Case IV For r ≡ 3 (mod4)

$$\rho(t_{pq}) = \begin{cases} 1\,;\ p = 1,3,5(\text{mod}6) \\ 0\,;\ p = 0,2,4(\text{mod}6) \end{cases}$$

$$\rho(t_{pq-1}) = \rho(t_{pq-4}) = \begin{cases} 1\,;\ p = 0,2,4(\text{mod}6) \\ 0\,;\ p = 1,3,5(\text{mod}6) \end{cases}$$

Table 6: Cordial pattern for vertices and edges

Conditions on r, m	Vertex Pattern	Edges Pattern
r ≡ 0(mod4),m≡ 0, 2, 4 (mod6)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1) + 1$
r ≡ 1(mod4), m≡ 0, 2, 4 (mod6)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1)$
r≡ 2(mod4),m≡ 0, 2, 4 (mod6)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1) + 1$
r ≡ 3(mod4), m≡ 0, 2, 4 (mod6)	$v_\rho(0) = v_\rho(1)$	$e_\rho(0) = e_\rho(1)$

The corresponding observed cases of r, m with labeling pattern of edges and vertices resulted in above table. Therefore, the conditions $|e_\rho(0) - e_\rho(1)| \le 1$ and $|v_\rho(0) - v_\rho(1)| \le 1$ are preserved and verified.

Example 6 The graph W = $\{6\text{-}C_6^2 : P_6\}$ is directed cordial as shown in Figure 6.

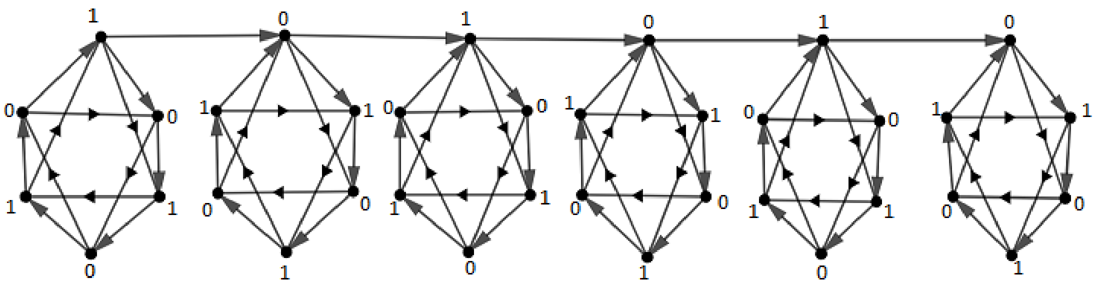

Figure 6: The graph W = $\{6\text{-}C_6^2 : P_6\}$ is cordial

4. Conclusion

In this paper we investigated linear directed cycle and their square is directed cordial. The directed path merged with two copies of directed cycle C_m and directed square cycle C_m^2 is directed cordial. We proved that directed path unions of r-copies of these graphs are directed cordial under certain condition. To investigate and elaborate various families of graph which preserves same results is an open problem for researchers. In the branch of graph theory, labeling is widely applicable in coding, circuit designing, communication networking and data base management.

5. Acknowledgements

Authors are highly grateful to anonymous referee for their valuable inputs and comments.

6. References

[1] Cahit, Cordial graphs: A Weaker version of graceful and harmonious graphs, Ars combinatorica, 23 (1987), 201–207.
[2] I.Cahit, On cordial and 3-equitable labelings of graphs, Utilitas Math, 370 (1990), 189– 198.
[3] J.A. Gallian 2016. A Dynamic survey of graph labeling, The Electronics Journal of Combinatorics, (2016), DS6.
[4] F.Harary, Graph Theory, Addison-Wesley, Reading, Massachusetts,(1972).
[5] Al-Shamiri, M.M.A., Nada, S.I., Elrokh, A.I. and Elmshtaye, Y. Some Results on Cordial Digraphs. Open Journal of Discrete Mathematics, (2020), 10, 4-12.