Models and Characteristics of Identification of Noise Stochastic Signals of Research Objects

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Abstract
The task of identifying stochastic noise signals as an information resource for research objects is an urgent task, confirmed by a significant number of publications. A formalized hierarchy of mathematical models of such signals from a vector space-time random field to a random variable is proposed, probabilistic and physical measures are indicated for measuring values and statistical evaluation of signal characteristics. The identification characteristics of noise signals are obtained based on the following constructive models of random functions - linear, conditionally linear, linear periodic, random linear fields.

Keywords 1
Noise signals, information resource, mathematical models, identification characteristics, linear random processes, white noise, color noise, characteristic functions, correlation functions

1. Introduction

Different studies of noise signals are carried out in different fields [1-3]. Noise signals have a stochastic nature of formation, the dynamics of changes in intensity and characteristics in space and time.

Traditionally, the analysis of such signals is carried out in two directions [4-6]:

- tasks of detecting signals in the presence of noise, using a significant number of methods to reduce the effect of noise in order to detect signals;
- tasks in which noise signals are the subject of research, mathematical models of signals are substantiated, their spatio-temporal and spectral characteristics are determined and statistically evaluated.

This paper is devoted to the second direction of analysis.

In most cases, the identification of various systems, including the "black box" system, linear, nonlinear, inertial, non-inertial, open-loop structures, feedback systems, and others, is performed as follows: based on the results of the analysis of the input and output signals of the research objects, a formalized model of the research object is justified and determined. The results of solving such identification problem are given in many publications in which the use of different methods is proposed [7-9]. Among these methods, we single out methods that use the following main idea – the input signal is a stochastic process of white noise [10, 11]. These methods are forming linear filters, white noise,

ITTAP'2022: 2nd International Workshop on Information Technologies: Theoretical and Applied Problems, November 22–24, 2022, Ternopil, Ukraine
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generating process, innovation process, and stochastic integral representations. The results of applying the method of stochastic integral representations are used in this paper to solve identification problems.

The identification of various phenomena, processes and objects is based on the research of an information resource – stochastic noise signals generated by the objects of research during their operation. Mathematical models and characteristics of the studied noise signals are information signs of their identification [10]. The input signal of the object of study is a stochastic process of white noise propagating in the space of the object and forming in each case its own space-time field of color noise. The measurement and registration of the values of such a field is carried out by various technical means in a limited amount of space and time [10, 12].

The theoretical foundations for the study of stochastic signals were laid at the end of the 17th century. In the works of the Swiss scientist Jacob Bernoulli, the statement of the problem of the convergence of the laws of the probability distribution of the sum of independent random variables with an unlimited growth of terms (the limit theorem) was formulated. In the 30s of the XX century, the central limit theorem was proved, according to which the laws of the probability distribution of the sum of independent random variables with the number of terms $n \to \infty$ are infinitely divisible, their special cases are the Gaussian (normal) and Poisson distribution laws. The characteristic functions of infinitely divisible probability distribution laws in the canonical forms of Levy, Levy-Khinchin and Kolmogorov are obtained [13-15].

The rapid development of electrical engineering, electronics, and radio engineering, starting from the 20s of the XX century, determined the relevance of the use of theoretical and applied studies of noise signals. This is due to the solution of a wide range of scientific and technical problems of transmission, detection, and identification of information signals under the influence of noise interference in various technical systems: radio communications and television, automatic control, radar, hydroacoustic, physiological signals and others [16-18].

Thus, at present time, the identification of information signals from white noise generated by moving elements is actively used for monitoring, diagnostics, and control of power equipment. The authors of [19] share this method implemented based on 2 approaches: 1) monosensory approach; 2) multisensory approach.

The monosensory approach is based on the study of the selected signal of the research object by choosing the appropriate statistical method and data processing method. In paper [20] it was proposed to carry out a preliminary normalization of diagnostic parameters using the Johnson distribution, which with three basic distribution groups, covers a wide class of empirical distributions. To assess the accuracy of the obtained normalized data, they were compared with the data obtained by replacing the resulting law with a Gaussian one. In paper [21] it was shown and studied some informative features for diagnostics of composite materials by impedance method using Hilbert transform. Among them there are instantaneous frequency of a signal, the integral of a phase characteristic on the selected interval and the integral of a difference signal phase characteristics. The features of using the average amplitude and harmonic wave of the vibration signal as the health indicator to represent the bearing condition status are shown in [22]. Authors of the paper [23] extract the mean squared error and the peak value of the vibration signal to construct the health indicator of bearing status using continuous wavelet transform. Another example of using wavelet transform for determining diagnostic signs is shown in [24].

In order to improve the reliability of fault identification, it is proposed to use a multi-sensor approach based on the search for a complex information signal (diagnostic sign) based on the measurement of several parameters [25-27]. As an example of multisensory approach, in paper [28] it was shown the using of vibrations, forces, temperature and acoustic signals for faulty diagnostics. In paper [29] it was shown that the multisignal-based faulty identification approach can make the identification result more reliable compared to the single signal-based approach. Authors of the paper [30] proposed a multisensor-based approach for the degradation identification of the mechanical component by evaluating the composite index which is combined with multiple sensor signals collected under multiple operational conditions. Features of a motor faulty identification model based on sensor data fusion are shown in [31].

Thus, these works reflect the relevance and importance of using identification of noise stochastic signals for monitoring, diagnosing, and controlling tasks.

An urgent task is to study the stochastic signals of various objects of the natural and man-made environment (research objects) in order to create models and determine the information characteristics of their identification and study.

Formulation of the problem. It is necessary to study stochastic noise signals and further develop the theoretical foundations for creating models and determining the identification characteristics of stochastic noise signals of various objects of study as an information resource for their functioning.

2. General approach

An analysis of the results of a significant number of publications makes it possible to determine the features and specifics of noise signals.

Noise signals are characterized by:

- disturbance of the intensity and characteristics of signals stochastically appeared in space and time;
- actions of different energy sources form a significant range of signals in a wide frequency range;
- space-time and spectral characteristics of signals allow to identify the objects of research;
- a set of sources of various noise signals creates combinations of options for the functioning of research objects in a limited space.

Thus, the following statements can be formed.

Statement 1. The noise signal generated by the object of research is the result of the action of a large number of elementary stochastic impulse disturbances of various physical nature, including thermal, mechanical, electromagnetic, atmospheric, hydrodynamic and others, and is an information resource of the object.

Statement 2. Identification and, if possible, determination of the state of the research object is based on the use of a mathematical model of the set of space-time and spectral characteristics of the generated noise signal.

Statement 3. In the case of the functioning of two or more objects of study in a limited area of space, the noise signals generated by them create various combinations of additive and multiplicative mixtures, which are the subject of theoretical and applied research, including the problem of detecting a signal under interference conditions, i.e., signal under the action of other noise interference.

Methods for the formation and study of noise signals.

First method. The object of study is a source of noise signal (Fig. 1).

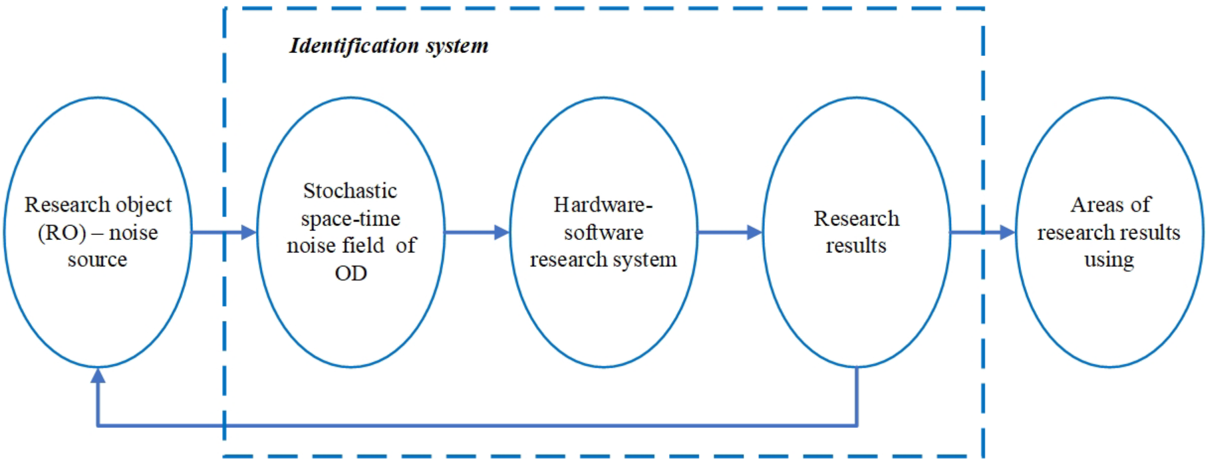

Figure 1: Schematic illustration of the implementation of the first method for the formation and study of noise signals

Second method. This method, in which the object of study is the space-time operator Z[·] of the test noise transformation, in most cases, white noise is used to identify both linear and non-linear systems, including physiological systems (Fig. 2).

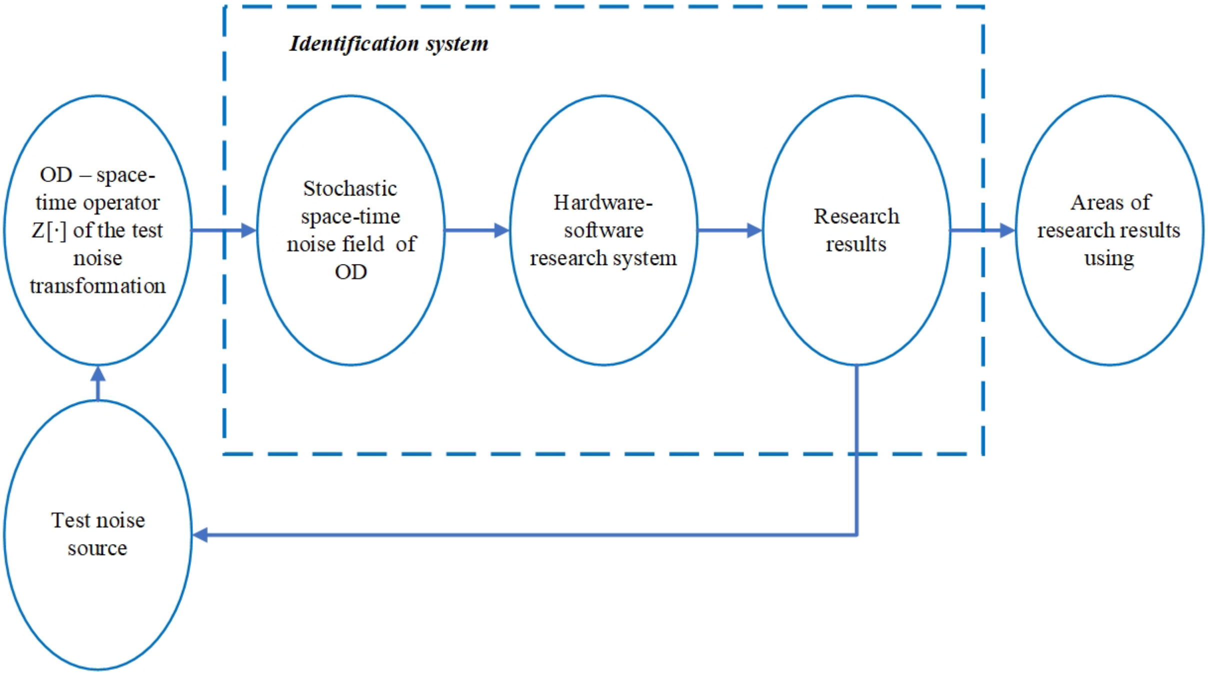

Figure 2: Schematic illustration of the implementation of the second method for the formation and study of noise signals

Mathematical models. The solution of various problems of studying noise signals is based on the use of mathematical models. Even though they do not reflect all the properties and characteristics of real objects, models play a fundamental role in theoretical, modeling and experimental studies.

Based on the obtained results of the study of noise signals, we can present the following.

Definition 1. A mathematical model of a noise field is a multidimensional random function with corresponding ranges of its arguments

$$\xi(\omega,\boldsymbol{r},t),$$

$$\omega\in\Omega,\ \boldsymbol{r}=(x,y,z)\in R^3,\ t\in T,\ D(\xi)=\Omega\times R^3\times T,\ E(\xi)=R$$

with different distribution laws.

Realizations of such a model form an ensemble of multidimensional deterministic functions

$$\left\{u_i(\boldsymbol{r},t),\ i=\overline{1,n}\right\}, D(u)=R^3\times T,\ E(u)=R.$$

Based on the processing of the ensemble of realizations, statistical estimates of the space-time characteristics of the field are determined.

Fig. 3 shows a formalized hierarchical structure of mathematical models, which is a sequence of one-dimensional and multidimensional random functions, where analytical expressions of mathematical models are presented in the framework, and their names are presented on top.

In the tasks of studying noise signals and fields, both continuous and discrete models are used. During conducting modeling and experimental studies, discrete models are mainly used, which are a special case of continuous models.

Depending on the research conditions, various combinations of continuous and discrete domains are used to define multidimensional models. In the complex of such models, there are:

- continuous models with continuous domains;
- discrete models with discrete domains, that is, given on the corresponding discrete lattices of variables;
- discrete-continuous or continuous-discrete models, if the general domains of the model is a combination of continuous and discrete domains of variables; thus, for a random space-time field $\xi(\omega,\boldsymbol{r},t)$, the number of combinations of continuous and discrete domains of variables is equal to $2^5-2=30$.

Fig. 4 shows a schematic illustration of the use of probabilistic and physical measures for one-dimensional and multidimensional random functions and their realizations for the problems of measuring values and statistical estimation of the characteristics of noise signals.

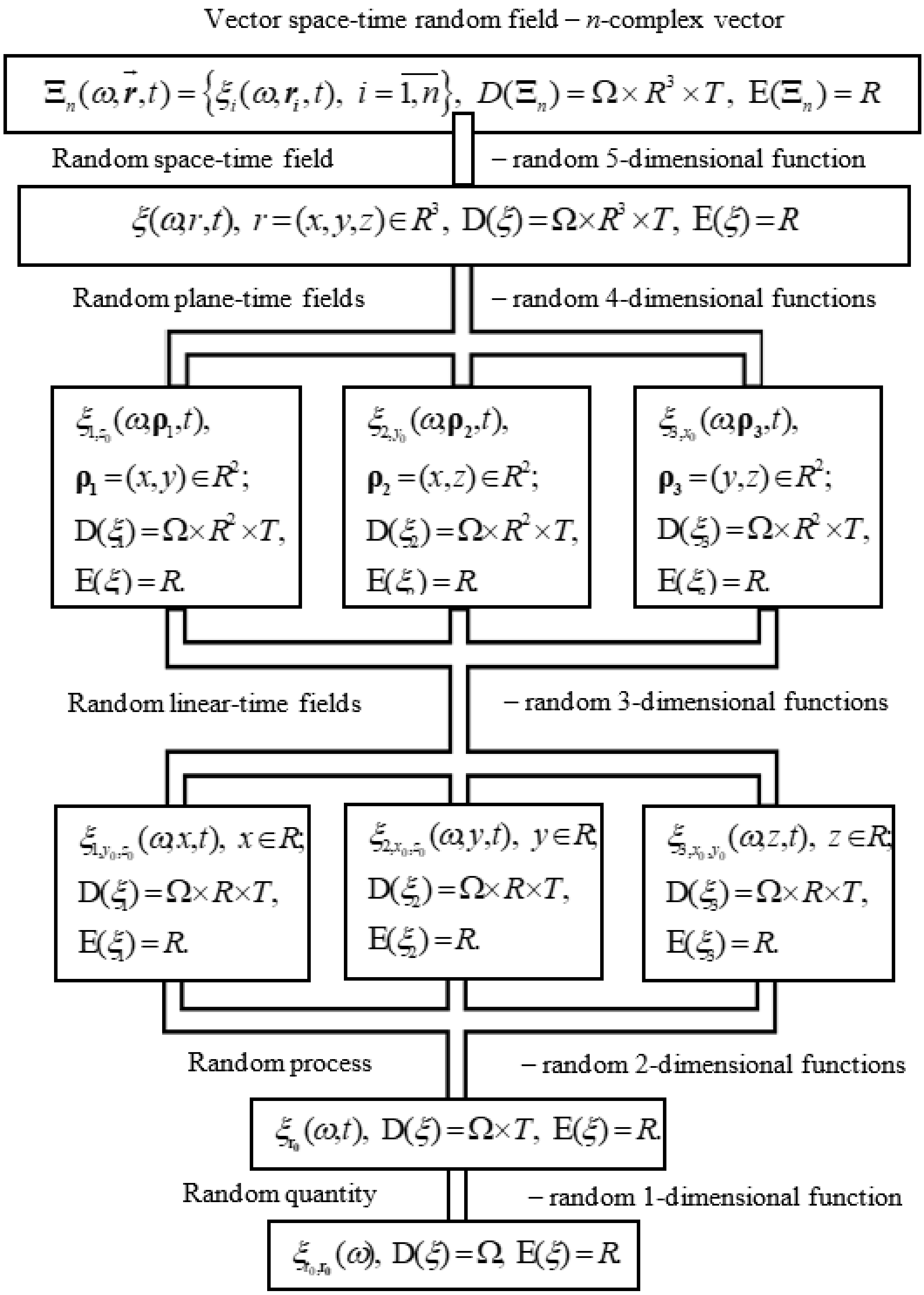

Figure 3: Schematic illustration of a formalized hierarchical structure of mathematical models of stochastic noise fields and signals

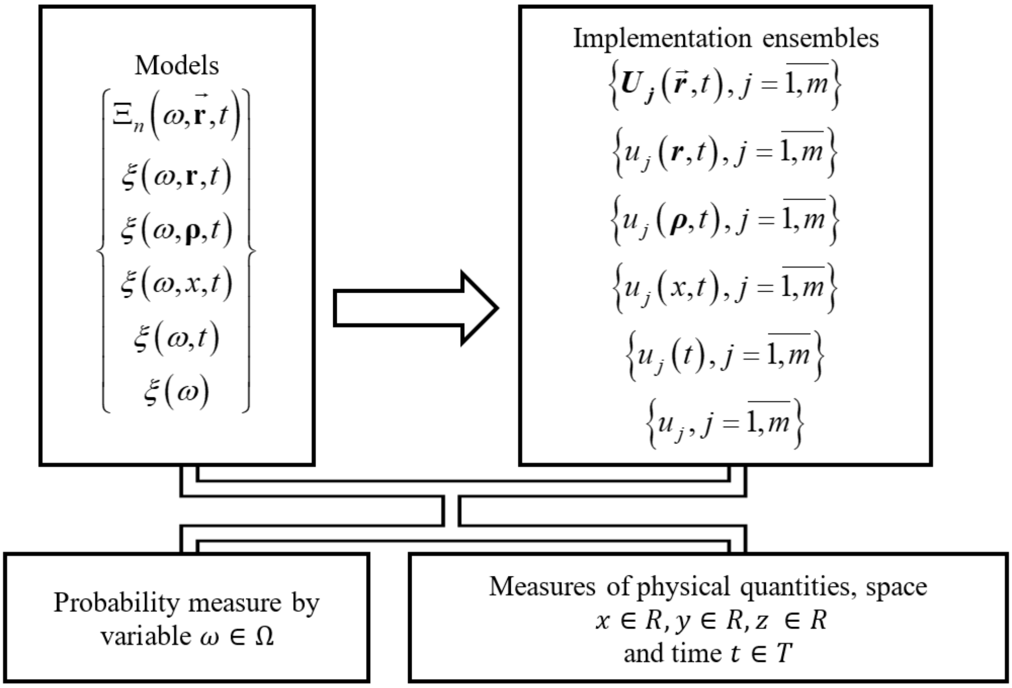

Figure 4: Schematic illustration of the use of a probability measure and measures of physical quantities of space and time for one-dimensional and multidimensional random functions and the corresponding ensembles of their realizations

The presented sets of continuous and discrete mathematical models of stochastic noise signals are written in a general form. Therefore, it is important to use the following constructive mathematical models that reflect the specifics and basic properties of stochastic noise signals for identifying research objects.

3. Models and characteristics of identification

The conducted studies of stochastic noise signals using publications [32-34] made it possible to obtain the following results on the creation of models and characteristics of identifications.

3.1. Linear random process (LRP)

Definition 2. A linear random process $\xi(\omega,t), t\in(-\infty,\infty)$, defined on some probability space $\{\Omega,\Im,\mathbf{P}\}$ is denoted as follows:

$$\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\tau,t)d\eta(\omega,\tau),\ \omega\in\Omega,\ t\in(-\infty,\infty),$$

where $\eta(\omega,\tau), \tau\in(-\infty,\infty)$, $\mathbf{P}(\eta(\omega,0)=0)=1$ is a Hilbert stochastically continuous random process with independent increments (generating process); $\varphi(\tau,t), \tau,t\in(-\infty,\infty)$ is non-random function (kernel) such that $\int_{-\infty}^{\infty}|\varphi(\tau,t)|^p d\kappa_p(\tau)<\infty, \forall t, p=1,2$, where $\kappa_p(\tau)$ is cumulant function of the p-th order of the generating process with independent increments $\eta(\omega,\tau)$.

Identification characteristics of a continuous time LRP:

- one-dimensional characteristic function

$$f_\xi(u;t)=\exp\left[iu\int_{-\infty}^{\infty}\varphi(\tau,t)da(\tau)+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi(\tau,t)}-1-iux\varphi(\tau,t)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right],$$

- two-dimensional characteristic function

$$f_\xi(u_1,u_2;t_1,t_2)=\exp\left[i\sum_{k=1}^{2}u_k\int_{-\infty}^{\infty}\varphi(\tau,t_k)da(\tau)+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{2}u_k\varphi(\tau,t_k)}-1-ix\sum_{k=1}^{2}u_k\varphi(\tau,t_k)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right],$$

- cumulant function of the n-th order, $m>1$

$$\kappa_m\left[\xi(\omega,t_1),\xi(\omega,t_2),...,\xi(\omega,t_m)\right]=\int_{-\infty}^{\infty}\varphi(\tau,t_1)\varphi(\tau,t_2)\cdot...\cdot\varphi(\tau,t_m)dc_m(\tau);$$

$$c_m(\tau)=\int_{-\infty}^{\infty}x^{m-2}d_xK(x;\tau),$$

- expected value

$$\mathbf{M}\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\tau,t)da(\tau),$$

- correlation function

$$R_\xi(t_1,t_2)=\int_{-\infty}^{\infty}\varphi(\tau,t_1)\varphi(\tau,t_2)db(\tau),$$

- expected value of a stationary LRP

$$\mathbf{M}\xi(\omega,t)=a\int_{-\infty}^{\infty}\varphi(s)ds=const,$$

- correlation function of a stationary LRP

$$R_\xi(\tau)=b\int_{-\infty}^{\infty}\varphi(s)\varphi(s+\tau)ds,$$

- cumulants of the n-th order of a stationary LRP

$$\kappa_m\left[\xi(\omega,t)\right]=\kappa_m\int_{-\infty}^{\infty}\varphi^m(s)ds=const.$$

Identification characteristics of a discrete time LRP:

- one-dimensional characteristic function

$$f_\xi(u;t)=\exp\left[iu\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t}a_\tau+\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi_{\tau,t}}-1-iux\varphi_{\tau,t}\right)\frac{d_xK(x;\tau)}{x^2}\right],$$

- two-dimensional characteristic function

$$f_\xi(u_1,u_2;t_1,t_2)=\exp\left[i\sum_{k=1}^{2}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_k}a_\tau+\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{2}u_k\varphi_{\tau,t_k}}-1-ix\sum_{k=1}^{2}u_k\varphi_{\tau,t_k}\right)\frac{d_xK(x;\tau)}{x^2}\right],$$

- expected value

$$\mathbf{M}\xi_t(\omega)=\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t}a_\tau,$$

- correlation function

$$R_{t_1,t_2}=\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_1}\varphi_{\tau,t_2}\sigma_\tau^2,$$

- expected value of a stationary LRP

$$\mathbf{M}\xi_t(\omega)=a\sum_{\tau=-\infty}^{\infty}\varphi_\tau=const,$$

- correlation function of a stationary LRP

$$R_s=\sigma^2\sum_{\tau=-\infty}^{\infty}\varphi_\tau\varphi_{\tau+s}.$$

3.2. *n*-dimensional linear random process

Identification characteristics of *n*-dimensional linear random process:

- expected value

$$\mathbf{M}\xi_k(\omega,t)=\sum_{m=1}^{n}\int_{-\infty}^{\infty}\varphi_{km}(\tau,t)da_m(\tau),\ k=\overline{1,n},$$

- correlation function

$$R_{kl}(t_1,t_2)=\sum_{m=1}^{n}\int_{-\infty}^{\infty}\varphi_{km}(\tau,t_1)\varphi_{lm}(\tau,t_2)db_m(\tau),\ \ k,l=\overline{1,n}.$$

3.3. Linear space-time random field (LSTRF)

Definition 3. A LSTRF defined on some probability space $\{\Omega,\mathfrak{F},\mathbf{P}\}$ is a field $\xi(\omega,\mathbf{r},t)$, $\omega\in\Omega$, $\mathbf{r}\in\mathbf{R}^3$, $t\in(-\infty,\infty)$ that allows an integral representation of the form:

$$\xi(\omega,\mathbf{r},t)=\int_{\mathbf{R}^3}\int_{-\infty}^{\infty}\varphi(\boldsymbol{\rho},\tau;\mathbf{r},t)d\eta(\omega,\boldsymbol{\rho},\tau),$$

where $\eta(\omega,\boldsymbol{\rho},\tau),\boldsymbol{\rho}\in\mathbf{R}^3,\tau\in\mathbf{R}$ is a Hilbert space-time stochastically continuous random field with independent increments (generating field); $\varphi(\boldsymbol{\rho},\tau;\mathbf{r},t)$ is non-random function (kernel), such that $\int_{\mathbf{R}^3}\int_{-\infty}^{\infty}|\varphi(\boldsymbol{\rho},\tau;\mathbf{r},t)|^p\,d\kappa_p(\boldsymbol{\rho},\tau)<\infty,\ \forall\mathbf{r},t,\ p=1,2,$ where $\kappa_p(\boldsymbol{\rho},\tau)$ is a cumulant function of the *p*-th order of generating field with independent increments $\eta(\omega,\mathbf{r},t)$.

Identification characteristics of a LSTRF:

- characteristic function

$$\ln f_\xi(u;\mathbf{r},t)=iua\int_{\mathbf{R}^3}\int_{-\infty}^{\infty}\varphi(\boldsymbol{\rho},\tau;\mathbf{r},t)d\boldsymbol{\rho}d\tau+\int_{-\infty}^{\infty}\int_{\mathbf{R}^3}\int_{-\infty}^{\infty}\left(e^{iux\varphi(\boldsymbol{\rho},\tau;\mathbf{r},t)}-1-iux\varphi(\boldsymbol{\rho},\tau;\mathbf{r},t)\right)\frac{dK(x)}{x^2}dxd\boldsymbol{\rho}d\tau,$$

- expected value

$$\mathbf{M}\xi(\omega,\mathbf{r},t)=a\int_{\mathbf{R}^3}\int_{-\infty}^{\infty}\varphi(\boldsymbol{\rho},\tau;\mathbf{r},t)d\boldsymbol{\rho}d\tau,$$

- correlation function

$$R_\xi(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)=b\int_{\mathbf{R}^3}\int_{-\infty}^{\infty}\varphi(\boldsymbol{\rho},\tau;\mathbf{r}_1,t_1)\varphi(\boldsymbol{\rho},\tau;\mathbf{r}_2,t_2)d\boldsymbol{\rho}d\tau.$$

3.4. Conditional linear random process (CLRP)

Definition 4. A CLRP $\xi(\omega,t),\omega\in\Omega,t\in(-\infty,\infty)$ defined on some probability space $\{\Omega,\mathfrak{F},\mathbf{P}\}$ is a stochastic integral of the form:

$$\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)d\eta(\omega,\tau),$$

where $\varphi(\omega,\tau,t),\tau,t\in(-\infty,\infty)$ is a real random function (kernel); $\eta(\omega,\tau),\tau\in(-\infty,\infty)$, $\mathbf{P}(\eta(\omega,0)=0)=1$ is a real Hilbert stochastically continuous random process with independent increments (generating process); $\mathbf{M}\eta(\omega,\tau)=a(\tau)<\infty$ and $\mathbf{D}\eta(\omega,\tau)=b(\tau)<\infty\ \ \forall\tau$, random functions

$\varphi(\omega,\tau,t)$ and $\eta(\omega,\tau)$ are stochastically independent, $\int_{-\infty}^{\infty} |\varphi(\omega,\tau,t)||da(\tau)| < \infty$, $\int_{-\infty}^{\infty} |\varphi(\omega,\tau,t)|^2 \, db(\tau) < \infty$, $\forall t$ with probability 1.

Let $\phi(\tau,t)$ be expected value and $R_\varphi(\tau_1,\tau_2;t_1,t_2)$ is correlation function of CLRP kernel [32].

Identification characteristics of a CLRP:

- one-dimensional characteristic function

$$f_\xi(u;t) = \mathbf{M}\exp\left[iu\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)da(\tau) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi(\omega,\tau,t)} - 1 - iux\varphi(\omega,\tau,t)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right],$$

- two-dimensional characteristic function

$$f_\xi(u_1,u_2;t_1,t_2) = \mathbf{M}\exp\left[i\sum_{k=1}^{2}u_k\int_{-\infty}^{\infty}\varphi(\omega,\tau,t_k)da(\tau) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{2}u_k\varphi(\omega,\tau,t_k)} - 1 - ix\sum_{k=1}^{2}u_k\varphi(\omega,\tau,t_k)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right],$$

- expected value

$$\mathbf{M}\xi(\omega,t) = \int_{-\infty}^{\infty}\phi(\tau,t)da(\tau),$$

- correlation function

$$R_\xi(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_\varphi(\tau_1,\tau_2;t_1,t_2)da(\tau_1)da(\tau_2) + \int_{-\infty}^{\infty}\mathbf{M}\left(\varphi(\omega,\tau,t_1)\varphi(\omega,\tau,t_2)\right)db(\tau),$$

- expected value of a stationary CLRP

$$\mathbf{M}\xi(\omega,t) = a\int_{-\infty}^{\infty}\phi(s)ds = const,$$

- correlation function of a stationary CLRP

$$R_\xi(t_2-t_1) = a^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_\varphi(\tau_1,\tau_2;t_2-t_1)d\tau_1 d\tau_2 + b\left[\int_{-\infty}^{\infty}R_\varphi(\tau,\tau;t_2-t_1)d\tau + \int_{-\infty}^{\infty}\phi(s)\phi(t_2-t_1+s)ds\right].$$

3.5. Linear periodic random process (LPRP)

Definition 5. Let $\xi(\omega,t) = \int_{-\infty}^{\infty}\varphi(\tau,t)d\eta(\omega,\tau)$ is a LRP. And let there be a real number $T_0 > 0$ such that for any τ and t the following holds:

$$\varphi(\tau,t) = \varphi(\tau+T_0, t+T_0),$$

and the following relations hold

$$d\kappa_1(\tau) = d\kappa_1(\tau+T_0),\ d\kappa_2(\tau) = d\kappa_2(\tau+T_0);$$
$$d_x d_\tau L(x,\tau) = d_x d_\tau L(x,\tau+T_0),$$

where $\kappa_1(\tau)$ and $\kappa_2(\tau)$ are cumulant functions of the first and second order of the generating process with independent increments $\eta(\omega,t)$, and $L(x,\tau)$ is its Poisson jump spectrum in Levy form.

Then LPRP is a linear random process whose characteristic function satisfies the cyclostationarity condition

$$f_\xi(u_1,u_2,...,u_n;t_1,t_2,...,t_n) = f_\xi(u_1,u_2,...,u_n;t_1+T_0,t_2+T_0,...,t_n+T_0).$$

Identification characteristics of a continuous time LPRP:

- period: real number $T_0 > 0$,
- one-dimensional characteristic function

$$f_\xi(u;t) = \exp\left[iu\int_{-\infty}^{\infty}\varphi(\tau,t)da(\tau) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi(\tau,t)} - 1 - iux\varphi(\tau,t)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right],\ t\in[0,T_0],$$

- two-dimensional characteristic function

$$f_{\xi}(u_1,u_2;t_1,t_2)=\exp\left[i\sum_{k=1}^{2}u_k\int_{-\infty}^{\infty}\varphi(\tau,t_k)da(\tau)+\right.$$

$$\left.+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{2}u_k\varphi(\tau,t_k)}-1-ix\sum_{k=1}^{2}u_k\varphi(\tau,t_k)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right], t_1\in(-\infty,\infty), t_2\in[0,T_0],$$

- expected value

$$\mathbf{M}\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\tau,t)da(\tau), t\in[0,T_0],$$

- correlation function

$$R_{\xi}(t_1,t_2)=\int_{-\infty}^{\infty}\varphi(\tau,t_1)\varphi(\tau,t_2)db(\tau), t_1\in(-\infty,\infty), t_2\in[0,T_0].$$

Identification characteristics of the discrete time linear random process:

- period: real number $T_0>1$,
- one-dimensional characteristic function

$$f_{\xi}(u;t)=\exp\left[iu\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t}a_\tau+\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi_{\tau,t}}-1-iux\varphi_{\tau,t}\right)\frac{d_x K(x;\tau)}{x^2}\right], t\in\overline{1,T_0},$$

- two-dimensional characteristic function

$$f_{\xi}(u_1,u_2;t_1,t_2)=\exp\left[i\sum_{k=1}^{2}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_k}a_\tau+\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{2}u_k\varphi_{\tau,t_k}}-1-ix\sum_{k=1}^{2}u_k\varphi_{\tau,t_k}\right)\frac{d_x K(x;\tau)}{x^2}\right];$$

$$t_1\in\mathbf{Z}, t_2\in\overline{1,T_0},$$

- expected value

$$\mathbf{M}\xi_t(\omega)=\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t}a_\tau, t\in\overline{1,T_0},$$

- correlation function

$$R_{t_1,t_2}=\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_1}\varphi_{\tau,t_2}\sigma_\tau^2, t_1\in\mathbf{Z}, t_2\in\overline{1,T_0}.$$

3.6. Conditional linear cyclostationary random process (CLCRP)

Conditional linear cyclostationary random process is defined as CLRP [32] but it's kernel and generating process have cyclostationary properties.

Identification characteristics of the continuous time CLCRP:

- period: real number $T_0>0$,
- one-dimensional characteristic function

$$f_{\xi}(u;t)=\mathbf{M}\exp\left[iu\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)da(\tau)+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi(\omega,\tau,t)}-1-iux\varphi(\omega,\tau,t)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right],\ t\in[0,T_0],$$

- two-dimensional characteristic function

$$f_{\xi}(u_1,u_2;t_1,t_2)=\mathbf{M}\exp\left[i\sum_{k=1}^{2}u_k\int_{-\infty}^{\infty}\varphi(\omega,\tau,t_k)da(\tau)+\right.$$

$$\left.+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{2}u_k\varphi(\omega,\tau,t_k)}-1-ix\sum_{k=1}^{2}u_k\varphi(\omega,\tau,t_k)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right], t_1\in(-\infty,\infty), t_2\in[0,T_0],$$

- expected value

$$\mathbf{M}\xi(\omega,t)=\int_{-\infty}^{\infty}\phi(\tau,t)da(\tau),\ t\in[0,T_0],$$

- correlation function

$$R_{\xi}(t_1,t_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_{\varphi}(\tau_1,\tau_2;t_1,t_2)da(\tau_1)da(\tau_2)+$$

$$+\int_{-\infty}^{\infty}\mathbf{M}\big(\varphi(\omega,\tau,t_1)\varphi(\omega,\tau,t_2)\big)db(\tau),\ t_1\in(-\infty,\infty),\ t_2\in[0,T_0].$$

The resulting set of identification characteristics of random linear models of stochastic noise signals is used in theoretical studies of various research objects. For practical use, such a combination is potentially possible and is implemented to a limited extent. The rapid development of modern information technologies and their implementation in the creation of complex hardware and software technical systems in various industries, including energy, determine the development of appropriate information support for such systems. The results obtained in this paper can be used to create such information support.

4. Discussion of research results

Today, the intensity of publications highlights the relevance of ensuring the reliable functioning of complex technical objects based on the use of modern hardware and software monitoring, identification, and diagnostic systems. These systems form and transmit actual information at the signal level, the state, and dynamics of its change, both components (means, mechanisms) and technical objects as a whole. Stochastic noise signals are the most integrated information resource for the functioning of such objects. The results of this paper, namely, the models and characteristics of the identification of noise stochastic signals of research objects, determine the potential for their use. To date, these opportunities are not fully realized, but the rapid development of information technology confirms the trend of increasing their use. More specifically, such models and characteristics of identification can be applied in the creation of information support for intelligent multi-level systems for monitoring, identification and diagnosis, the development and creation of which is carried out by the developed countries of the world. The results of the use of models and the determination of the characteristics of the identification of noise stochastic signals of the functioning of various technical objects are obtained in the paper.

The characteristics of the following models are given:

- linear random processes with continuous and discrete time;
- linear stationary random process;
- conditional linear random process;
- conditional stationary random process;
- n-dimensional linear random process;
- linear space-time random field;
- linear periodic random process with continuous and discrete time;
- conditional linear cyclostationary random process with continuous time.

The results of this paper are a contribution to the theoretical foundations of creating models and determining the characteristics of the identification of noise stochastic signals in the study of the functioning of a wide range of technical objects.

5. Conclusions

Based on the results of the conducted research, the following conclusions can be made.

1. The development and creation of modern hardware and software systems for monitoring, identification, and diagnosis of complex technical objects in different areas determines the relevance of identifying stochastic noise signals as an information resource for the functioning of such objects.

2. The general approach of formalization of mathematical models of stochastic noise signals is used. A formalized hierarchy of multidimensional and one-dimensional random functions from a vector space-time random to a random quantity is presented, covering the range of options for noise signals studying. Probabilistic and physical measures are indicated during measuring values and statistical processing of signal characteristics. Formalization of noise signal models allows to determine the direction of research and requires the use of constructive models of noise signals.
3. The conducted studies made it possible to obtain a set of characteristics for identifying constructive models of stochastic noise signals: a linear random process, a vector n-dimensional linear random process, a linear space-time random field, a linear periodic random process, a conditional linear cyclostationary random process in the form of characteristic functions and moment functions.
4. A new result for solving problems in the theory of linear stochastic dynamical systems is the substantiation of the model and identification characteristics of a conditional linear random process as the basis for applying the method of stochastic shaping linear filters driven by the white noise.
5. The use of linear models of random processes and fields, their identification characteristics make it possible to study complex hardware and software technical systems.
6. The results obtained in the complex are a further development of the theoretical foundations for creating models and determining the identification characteristics of noise stochastic signals.

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