

Computational Methods of Integration of Deterministic Systems, which are Described by Systems of Ordinary Differential Equations

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Abstract

The result of the work is obtaining difference schemes for dynamic systems, which are described by systems of ordinary differential equations of the second order based on the classical methods of Bossak, Newmark and the generalized α -method. For the developed modifications of the methods, the corresponding software was developed in the MATLAB system of applied mathematics for conducting numerical experiments and testing methods.

Keywords 1

Deterministic system, Newmark's method, Bossack's method, generalized α -method, computational method.

1. Introduction

The development of computational methods for modeling dynamic systems occupies a significant place in solving scientific and technical tasks in today's conditions. A number of requirements are put forward to such methods, in particular, the methods that are developed should be stable and those that obtain a numerical solution with minimal errors [1; 2]. The rapid development of computer technology makes it possible to apply these methods in practice. Therefore, the work will demonstrate a tool for obtaining basic mathematical dependencies based on classical methods for the analysis of nonlinear systems, for example, systems described by Cauchy problems for second-order ordinary differential equations.

2. The main part of the article

To analyze dynamic systems, let's take a classical nonlinear system in the form of a nonlinear oscillator [3]. The nonlinear oscillator should be viewed as a generalized model suitable for describing oscillatory phenomena in systems of different physical nature.

A nonlinear oscillator is a system whose dynamics is described by a second-order differential equation of the general form [3; 4]:

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$$x'' + \delta x' + f(x) = 0, \quad (1)$$

where x – dynamic variable (or generalized coordinate), $f(x)$ – some non-linear function, δ – dissipation parameter. Depending on the different values of this parameter, different transformations of the phase portrait of the nonlinear oscillator occur. In order to form a complete view of the nonlinear oscillator near the catastrophe of the assembly, it is necessary to draw a function $f(x)$ species equality (1). In equation (1), the function $f(x)$ has the following form:

$$f(x) = \bar{a}x + b + x^3. \quad (2)$$

Equality (2) corresponds to an assembly type disaster. When varying parameter values \bar{a} and b the behavior of the system will also change. Therefore, the second type of disaster is called an "assembly" disaster. In the behavior of the phase portrait curves, it is possible to observe both trajectories without a jump, with smooth development, and with a jump in development.

The catastrophe theory explains the dependence of unstable forms of the phase portrait on the number of control parameters. Yes, if we have only one control parameter, we can observe a fold disaster. If we have two control parameters, as in our case \bar{a} i b , a build crash is observed.

Based on equalities (1) and (2), we can form a complete view of the nonlinear oscillator near the assembly catastrophe:

$$x'' + \delta x' + \bar{a}x + b + x^3 = 0. \quad (3)$$

In equation (3) δ is dissipation parameter, \bar{a} and b are control parameters [5; 6].

2.1. Newmark's method

The Newmark method is a numerical integration method for solving differential equations. It is widely used in the calculations of many engineering, technical, chemical problems for the numerical evaluation of dynamic characteristics. The method is named after a former professor of civil engineering at Illinois State University who developed it in 1959 for use in structural dynamics.

To derive the formulas and analyze them, consider the Cauchy problem for an ordinary differential equation of the second order. Its mathematical representation has the form:

$$\begin{aligned} x'' + gx' + ax + b + x^3 &= 0; \\ x(t_0) &= x_0, \quad x'(t_0) = x'_0. \end{aligned}$$

$$\begin{aligned} U_{n+1} &= U_n + hV_n + h^2 \left(\frac{1}{2} - \beta \right) A_n + h^2 \beta A_{n+1}, \\ V_{n+1} &= V_n + h(1 - \gamma)A_n + h\gamma A_{n+1}, \\ A_{n+1} + gV_{n+1} + aU_{n+1} + b + U_{n+1}^3 &= 0. \end{aligned} \quad (4)$$

Since the system (4) is nonlinear, then the classical Seidel method can be used for its numerical solution. For this system, Seidel's method will look like this:

$$\begin{aligned} U_{n+1}^{(k+1)} &= U_n + hV_n + h^2 \left(\frac{1}{2} - \beta \right) A_n + h^2 \beta A_{n+1}^{(k)}, \\ V_{n+1}^{(k+1)} &= V_n + h(1 - \gamma)A_n + h\gamma A_{n+1}^{(k)}, \\ A_{n+1}^{(k+1)} &= - \left(gV_{n+1}^{(k+1)} + aU_{n+1}^{(k+1)} + b + \left(U_{n+1}^{(k+1)} \right)^3 \right), \\ U_0 &= x_0, V_0 = x'_0, A_0 = -(gV_0 + aU_0 + b + U_0^3), \end{aligned} \quad (5)$$

where k is iteration number.

The second version of Newmark's method can be presented in the following form:

$$\begin{aligned}U_{n+1} &= U_n + hV_n + h^2 \left(\frac{1}{2} - \beta \right) A_n + h^2 \beta A_{n+1}, \\V_{n+1} &= V_n + h(1 - \gamma)A_n + h\gamma A_{n+1}, \\A_{n+1} &= -(gV_{n+1} + aU_{n+1} + b + U_{n+1}^3).\end{aligned}$$

Then we will have

$$\begin{aligned}U_{n+1} &= U_n + hV_n - h^2 \left(\frac{1}{2} - \beta \right) (gV_n + aU_n + b + U_n^3) - h^2 \beta (gV_{n+1} + aU_{n+1} + b + U_{n+1}^3), \\V_{n+1} &= V_n - h(1 - \gamma)(gV_n + aU_n + b + U_n^3) - h\gamma(gV_{n+1} + aU_{n+1} + b + U_{n+1}^3),\end{aligned}$$

So,

$$\begin{aligned}U_{n+1}(1 + h^2 a \beta) &= U_n + hV_n - h^2 \left(\frac{1}{2} - \beta \right) (gV_n + aU_n + b + U_n^3) - h^2 \beta (gV_{n+1} + b + U_{n+1}^3), \\V_{n+1}(1 + g h \gamma) &= V_n - h(1 - \gamma)(gV_n + aU_n + b + U_n^3) - h\gamma(aU_{n+1} + b + U_{n+1}^3).\end{aligned}$$

Finally, the formulas for mathematical calculation take the form:

$$\begin{aligned}U_{n+1} &= \frac{U_n + hV_n - h^2 \left(\frac{1}{2} - \beta \right) (gV_n + aU_n + b + U_n^3) - h^2 \beta (gV_{n+1} + b + U_{n+1}^3)}{1 + h^2 a \beta}, \\V_{n+1} &= \frac{V_n - h(1 - \gamma)(gV_n + aU_n + b + U_n^3) - h\gamma(aU_{n+1} + b + U_{n+1}^3)}{1 + g h \gamma}.\end{aligned}$$

Values U_{n+1} and V_{n+1} are calculated using computational methods. For example, Seidel's method for this system of equations will have the form:

$$\begin{aligned}U_{n+1}^{(k+1)} &= \frac{U_n + hV_n - h^2 \left(\frac{1}{2} - \beta \right) (gV_n + aU_n + b + U_n^3) - h^2 \beta \left(gV_{n+1}^{(k)} + b + \left(U_{n+1}^{(k)} \right)^3 \right)}{1 + h^2 a \beta}, \\V_{n+1}^{(k+1)} &= \frac{V_n - h(1 - \gamma)(gV_n + aU_n + b + U_n^3) - h\gamma \left(aU_{n+1}^{(k+1)} + b + \left(U_{n+1}^{(k+1)} \right)^3 \right)}{1 + g h \gamma}.\end{aligned}\tag{6}$$

The third version of the Newmark method looks like this:

$$\begin{aligned}V_{n+1} &= \frac{V_n - h(1 - \gamma)(gV_n + aU_n + b + U_n^3) - h\gamma(aU_{n+1} + b + U_{n+1}^3)}{1 + g h \gamma}; \\U_{n+1} &= \frac{U_n + hV_n - h^2 \left(\frac{1}{2} - \beta \right) (gV_n + aU_n + b + U_n^3) - h^2 \beta (gV_{n+1} + b + U_{n+1}^3)}{1 + h^2 a \beta}.\end{aligned}\tag{7}$$

It should be noted that in (7) the second equation is substituted into the first equation. Next is the value U_{n+1} , which is then substituted into the equation:

$$V_{n+1} = \frac{V_n - h(1 - \gamma)(gV_n + aU_n + b + U_n^3) - h\gamma(aU_{n+1} + b + U_{n+1}^3)}{1 + g h \gamma}.$$

In this case, Seidel's method solves only one equation [7; 8].

2.2. Generalized α -method

Formulas for solving a nonlinear oscillator near the assembly catastrophe using the generalized α -method are derived similarly to Newmark's method. The difference is that the values of all variables at $n + 1$ time steps are taken as averages, taking into account the average theorem.

The equality that we need to solve by the method of generalized α has the following form:

$$x'' + \delta x' + \bar{a}x + b + x^3 = 0. \quad (8)$$

In equation (8), δ is dissipation parameter, \bar{a} and b control parameters.

As before, let's mark $x'' = a, x' = v$, and let's complete the equality (8). Now let's write down the system of differential equations at $n + 1$ instant of time according to the method of generalized α .

$$\begin{cases} v_{n+1} = v_n + (1 - \gamma)\Delta t a_n + \gamma\Delta t a_{n+1}, \\ x_{n+1} = x_n + \Delta t v_n + \left(\frac{1}{2} - \beta\right)\Delta t^2 a_n + \beta\Delta t^2 a_{n+1}, \\ a_{n+1-\alpha_m} + \delta v_{n+1-\alpha_f} + \bar{a}x_{n+1-\alpha_f} + b + x_{n+1-\alpha_f}^3 = 0, \end{cases} \quad (9)$$

where $a_{n+1-\alpha_m}, v_{n+1-\alpha_f}, x_{n+1-\alpha_f}$ are equal to:

$$\begin{aligned} a_{n+1-\alpha_m} &= (1 - \alpha_m)a_{n+1} + \alpha_m a_n, & v_{n+1-\alpha_f} &= (1 - \alpha_f)v_{n+1} + \alpha_f v_n, \\ x_{n+1-\alpha_f} &= (1 - \alpha_f)x_{n+1} + \alpha_f x_n. \end{aligned} \quad (10)$$

Moreover, the values of all constants are equal to:

$$\gamma = \frac{1}{2} - \alpha_m + \alpha_f, \beta = \frac{1}{4}(1 - \alpha_m + \alpha_f)^2, \alpha_m = \frac{2\rho_\infty - 1}{\rho_\infty + 1}, \alpha_f = \frac{\rho_\infty}{\rho_\infty + 1}.$$

With the condition that $0 \leq \rho_\infty \leq 1$.

Let's rewrite (9) considering (10):

$$\begin{cases} v_{n+1} = v_n + (1 - \gamma)\Delta t a_n + \gamma\Delta t a_{n+1} \\ x_{n+1} = x_n + \Delta t v_n + \left(\frac{1}{2} - \beta\right)\Delta t^2 a_n + \beta\Delta t^2 a_{n+1} \\ (1 - \alpha_m)a_{n+1} + \alpha_m a_n + \delta(1 - \alpha_f)v_{n+1} + \delta\alpha_f v_n + \\ + \bar{a}(1 - \alpha_f)x_{n+1} + \alpha_f x_n + b + (1 - \alpha_f)x_{n+1}^3 + \alpha_f x_n^3 = 0 \end{cases} \quad (11)$$

From equality (11) we see that x is a vector, and therefore velocity and acceleration are also vectors. We obtained a closed system of $3k$ algebraic equations, with $3k$ unknowns.

Everything that is on the $n + 1$ step is unknown and it is necessary to look for their value. To do this, we move all variables $n + 1$ steps to the left, and n steps to the right:

$$\begin{cases} \gamma\Delta t a_{n+1} - v_{n+1} = -v_n - (1 - \gamma)\Delta t a_n \\ \beta\Delta t^2 a_{n+1} - x_{n+1} = -x_n - \Delta t v_n - \left(\frac{1}{2} - \beta\right)\Delta t^2 a_n \\ (1 - \alpha_m)a_{n+1} + \delta(1 - \alpha_f)v_{n+1} + \bar{a}(1 - \alpha_f)x_{n+1} + (1 - \alpha_f)x_{n+1}^3 = \\ = -\alpha_m a_n - \delta\alpha_f v_n - \alpha_f x_n - b - \alpha_f x_n^3 \end{cases} \quad (12)$$

We see that equality (12) is written in vector form. Therefore, to simplify the notations, we introduce the vectors:

$$\begin{aligned} A_{n+1} &= -v_n - (1 - \gamma)\Delta t a_n, \quad B_{n+1} = -x_n - \Delta t v_n - \left(\frac{1}{2} - \beta\right)\Delta t^2 a_n, \\ C_{n+1} &= -\alpha_m a_n - \delta\alpha_f v_n - \alpha_f x_n - b - \alpha_f x_n^3. \end{aligned}$$

$$\begin{cases} \gamma\Delta t a_{n+1} - v_{n+1} = A_{n+1} \\ \beta\Delta t^2 a_{n+1} - x_{n+1} = B_{n+1} \\ (1 - \alpha_m)a_{n+1} + \delta(1 - \alpha_f)v_{n+1} + \bar{a}(1 - \alpha_f)x_{n+1} + (1 - \alpha_f)x_{n+1}^3 = C_{n+1} \end{cases} \quad (13)$$

Next, we divide the first equality (13) by $\gamma\Delta t$, and the second equality – by $\beta\Delta t^2$

$$\begin{cases} a_{n+1} - \frac{1}{\gamma\Delta t}v_{n+1} = \frac{1}{\gamma\Delta t}A_{n+1}, \\ a_{n+1} - \frac{1}{\beta\Delta t^2}x_{n+1} = \frac{1}{\beta\Delta t^2}B_{n+1}, \\ (1 - \alpha_m)a_{n+1} + \delta(1 - \alpha_f)v_{n+1} + \bar{a}(1 - \alpha_f)x_{n+1} + (1 - \alpha_f)x_{n+1}^3 = C_{n+1}. \end{cases} \quad (14)$$

Now subtract the first from the second equality (14), and from the third the first multiplied by $(1 - \alpha_m)$.

$$\begin{cases} \frac{1}{\gamma\Delta t}v_{n+1} - \frac{1}{\beta\Delta t^2}x_{n+1} = \frac{1}{\beta\Delta t^2}B_{n+1} - \frac{1}{\gamma\Delta t}A_{n+1}, \\ \left(\delta(1 - \alpha_f) + \frac{1 - \alpha_m}{\gamma\Delta t}\right)v_{n+1} + \bar{a}(1 - \alpha_f)x_{n+1} + (1 - \alpha_f)x_{n+1}^3 = C_{n+1} - \frac{1 - \alpha_m}{\gamma\Delta t}A_{n+1}. \end{cases} \quad (15)$$

For simplification, we introduce $D = (\delta(1 - \alpha_f) + (1 - \alpha_m)/(\gamma\Delta t))$. Rewrite (15):

$$\begin{cases} v_{n+1} - \frac{\gamma}{\beta\Delta t}x_{n+1} = \frac{\gamma}{\beta\Delta t}B_{n+1} - A_{n+1}, \\ Dv_{n+1} + \bar{a}(1 - \alpha_f)x_{n+1} + (1 - \alpha_f)x_{n+1}^3 = C_{n+1} - \frac{1 - \alpha_m}{\gamma\Delta t}A_{n+1}. \end{cases} \quad (16)$$

Next, we subtract the first multiplied by D from the second equality (16).

$$\begin{aligned} \bar{a}(1 - \alpha_f)x_{n+1} + \frac{\gamma}{\beta\Delta t}Dx_{n+1} + (1 - \alpha_f)x_{n+1}^3 &= \\ = C_{n+1} - \frac{1 - \alpha_m}{\gamma\Delta t}A_{n+1} + DA_{n+1} - \frac{\gamma}{\beta\Delta t}DB_{n+1}. \end{aligned} \quad (17)$$

In order to avoid division by zero, let's multiply equality (17) by $\beta\Delta t$:

$$\begin{aligned} (\bar{a}\beta\Delta t(1 - \alpha_f) + \gamma D)x_{n+1} + (1 - \alpha_f)\beta\Delta tx_{n+1}^3 &= \\ = \beta\Delta tC_{n+1} - \gamma DB_{n+1} + \left(\beta\Delta tD - \frac{1 - \alpha_m}{\gamma}\beta\right)A_{n+1}. \end{aligned} \quad (18)$$

From here we find x . From the first equality (16), we find v :

$$v_{n+1} - \frac{\gamma}{\beta\Delta t}x_{n+1} = \frac{\gamma}{\beta\Delta t}B_{n+1} - A_{n+1}, \quad v_{n+1} = \frac{\gamma(x_{n+1} + B_{n+1})}{\beta\Delta t} - A_{n+1}.$$

From the first equality (14), we find a :

$$a_{n+1} - \frac{1}{\gamma\Delta t}v_{n+1} = \frac{1}{\gamma\Delta t}A_{n+1}, \quad a_{n+1} = \frac{v_{n+1} + A_{n+1}}{\gamma\Delta t}.$$

Let's write down the obtained formulas of the generalized α method for solving a nonlinear oscillator near the assembly catastrophe:

$$\begin{aligned} & (\bar{\alpha}\beta\Delta t(1 - \alpha_f) + \gamma D)x_{n+1} + (1 - \alpha_f)\beta\Delta tx_{n+1}^3 - \\ & -\beta\Delta tC_{n+1} + \gamma DB_{n+1} - \left(\beta\Delta tD - \frac{1 - \alpha_m}{\gamma}\beta\right)A_{n+1} = 0, \\ & v_{n+1} = \frac{\gamma}{\beta\Delta t}(x_{n+1} + B_{n+1}) - A_{n+1}, \quad a_{n+1} = \frac{1}{\gamma\Delta t}(v_{n+1} + A_{n+1}). \end{aligned} \quad (19)$$

The solution (19) will be a curve, any changes of which depend on the parameter values $\delta, \bar{\alpha}$ and b . Formulas (19) demonstrate formulas for solving a nonlinear oscillator near the assembly catastrophe by the generalized α -method.

2.3. Bossak's method

Bossak's method is a continuation of Newmark's method. The derivation of the formulas for solving the nonlinear oscillator near the assembly catastrophe is carried out similarly to the Newmark's method. The difference is that the values of the second derivatives at $n + 1$ time steps are taken as averages (taking into account the average theorem).

The equality that we need to solve by Bossak's method has the form (3). Everything that is on the $n + 1$ step is unknown and it is necessary to find their value. To do this, we will transfer all the variables $n + 1$ steps to the left, and n steps to the right:

$$\begin{cases} \gamma\Delta t a_{n+1} - v_{n+1} = -v_n - (1 - \gamma)\Delta ta_n \\ \beta\Delta t^2 a_{n+1} - x_{n+1} = -x_n - \Delta tv_n - \left(\frac{1}{2} - \beta\right)\Delta t^2 a_n \\ (1 - \alpha)a_{n+1} + \delta v_{n+1} + \bar{\alpha}x_{n+1} + b + x_{n+1}^3 = -\alpha a_n \end{cases} \quad (20)$$

We see that the equalities in (20) are written in vector form. Therefore, to simplify the notations, we introduce the vectors

$$A_{n+1} = -v_n - (1 - \gamma)\Delta t a_n, \quad B_{n+1} = -x_n - \Delta tv_n - \left(\frac{1}{2} - \beta\right)\Delta t^2 a_n, \quad C_{n+1} = -\alpha_m a_n.$$

$$\begin{cases} \gamma\Delta t a_{n+1} - v_{n+1} = A_{n+1} \\ \beta\Delta t^2 a_{n+1} - x_{n+1} = B_{n+1} \\ (1 - \alpha)a_{n+1} + \delta v_{n+1} + \bar{\alpha}x_{n+1} + b + x_{n+1}^3 = C_{n+1} \end{cases} \quad (21)$$

Next, we divide the first equality (21) by $\gamma\Delta t$, and the second equality – by $\beta\Delta t^2$.

$$\begin{cases} a_{n+1} - \frac{1}{\gamma\Delta t}v_{n+1} = \frac{1}{\gamma\Delta t}A_{n+1} \\ a_{n+1} - \frac{1}{\beta\Delta t^2}x_{n+1} = \frac{1}{\beta\Delta t^2}B_{n+1} \\ (1 - \alpha)a_{n+1} + \delta v_{n+1} + \bar{\alpha}x_{n+1} + b + x_{n+1}^3 = C_{n+1} \end{cases} \quad (22)$$

Now subtract the first from the second equality (22), and subtract the first multiplied by $(1 - \alpha)$ from the third.

$$\begin{cases} \frac{1}{\gamma\Delta t}v_{n+1} - \frac{1}{\beta\Delta t^2}x_{n+1} = \frac{1}{\beta\Delta t^2}B_{n+1} - \frac{1}{\gamma\Delta t}A_{n+1}, \\ \left(\delta + \frac{1-\alpha}{\gamma\Delta t}\right)v_{n+1} + \bar{a}x_{n+1} + b + x_{n+1}^3 = C_{n+1} - \frac{1-\alpha}{\gamma\Delta t}A_{n+1}. \end{cases} \quad (23)$$

For simplification, we introduce $D = (\delta + (1 - \alpha)/(\gamma\Delta t))$. Rewrite (23):

$$\begin{cases} v_{n+1} - \frac{\gamma}{\beta\Delta t}x_{n+1} = \frac{\gamma}{\beta\Delta t}B_{n+1} - A_{n+1} \\ Dv_{n+1} + \bar{a}x_{n+1} + b + x_{n+1}^3 = C_{n+1} - \frac{1}{\gamma\Delta t}A_{n+1} \end{cases} \quad (24)$$

Next, we subtract from the second equality (24) the first multiplied by D .

$$\bar{a}x_{n+1} + b + x_{n+1}^3 + \frac{\gamma}{\beta\Delta t}Dx_{n+1} = C_{n+1} - \frac{1-\alpha}{\gamma\Delta t}A_{n+1} - \frac{\gamma}{\beta\Delta t}DB_{n+1} + DA_{n+1}. \quad (25)$$

In order to avoid division by zero, let's multiply equality (25) by $\beta\Delta t$:

$$\beta\Delta t(\bar{a}x_{n+1} + b + x_{n+1}^3) + \gamma Dx_{n+1} = \beta\Delta tC_{n+1} - \frac{(1-\alpha)\beta}{\gamma}A_{n+1} - \gamma DB_{n+1} + DA_{n+1}. \quad (26)$$

From here we find x . From the first equality (24), we find v :

$$v_{n+1} - \frac{\gamma}{\beta\Delta t}x_{n+1} = \frac{\gamma}{\beta\Delta t}B_{n+1} - A_{n+1}, \quad v_{n+1} = \frac{\gamma(x_{n+1} + B_{n+1})}{\beta\Delta t} - A_{n+1}.$$

We find a from the first equality:

$$a_{n+1} - \frac{1}{\gamma\Delta t}v_{n+1} = \frac{1}{\gamma\Delta t}A_{n+1}, \quad a_{n+1} = \frac{1}{\gamma\Delta t}(v_{n+1} + A_{n+1}).$$

Let's write down the obtained formulas of Bossack's method for solving a nonlinear oscillator near the assembly catastrophe:

$$\begin{aligned} &\beta\Delta t(\bar{a}x_{n+1} + b + x_{n+1}^3) + \gamma Dx_{n+1} - \beta\Delta tC_{n+1} + \\ &\quad + \frac{(1-\alpha)\beta}{\gamma}A_{n+1} + \gamma DB_{n+1} - DA_{n+1} = 0, \\ &v_{n+1} = \frac{\gamma}{\beta\Delta t}(x_{n+1} + B_{n+1}) - A_{n+1}, \quad a_{n+1} = \frac{1}{\gamma\Delta t}(v_{n+1} + A_{n+1}). \end{aligned} \quad (27)$$

Solution (27) is represented as a curve.

2.4. Programs and Calculation experiments

The general structure of the developed software consists of modules, the names of which are shown in Figure 1. The software was developed in the computer mathematics environment MATLAB.

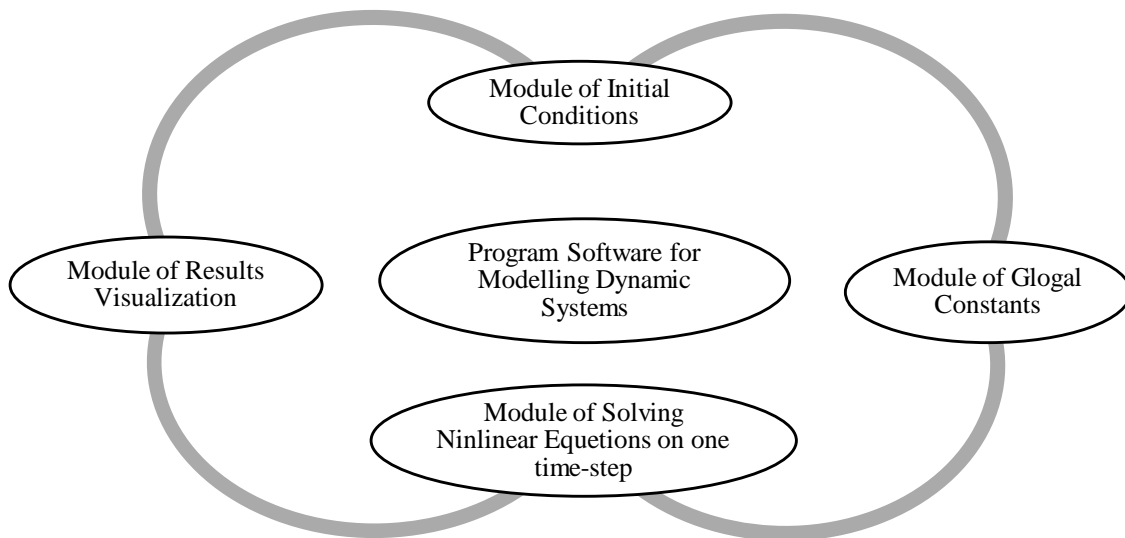


Figure 1. Structure of Software of Modelling Dynamic Systems

The developed fragment of the program in MATLAB, with the help of which the solution is found at each time step, has the following form:

```

t(i+1) = t(i) + h;
A_n = -v(i) - (1-gamma)*h*A(i);
B_n = -x(i) - h*v(i) - (0.5-beta)*h^2*A(i);
C_n = -a1_m*A(i) - g*k2*v(i) - a*k2*x(i) - (k2*x(i))^3 - b;
D = beta*h^2*C_n - B_n*k1 - g*k3*h*(gamma*B_n - beta*h*A_n);

A = k3*beta*h^2;
B = 3*k2*k3^2*beta*h^2*x(i);
C = (3*k2^2*k3*x(i)^2+a*k3)*beta*h^2 + k1 + k3*g*gamma*h;
x(i+1) = Newton(x(i));
v(i+1) = gamma/(beta*h) * (x(i+1) + B_n) - A_n;
A(i+1) = 1/(gamma*h) * (v(i+1) + A_n);
i = i+1;

```

The obtained dependences of the methods for the considered dynamic systems were tested on different values of the initial conditions and model constants. Some of the results are shown in Figure 2 and Figure 3. Initial Condition is $x(0) = 1; v(0) = 2$.

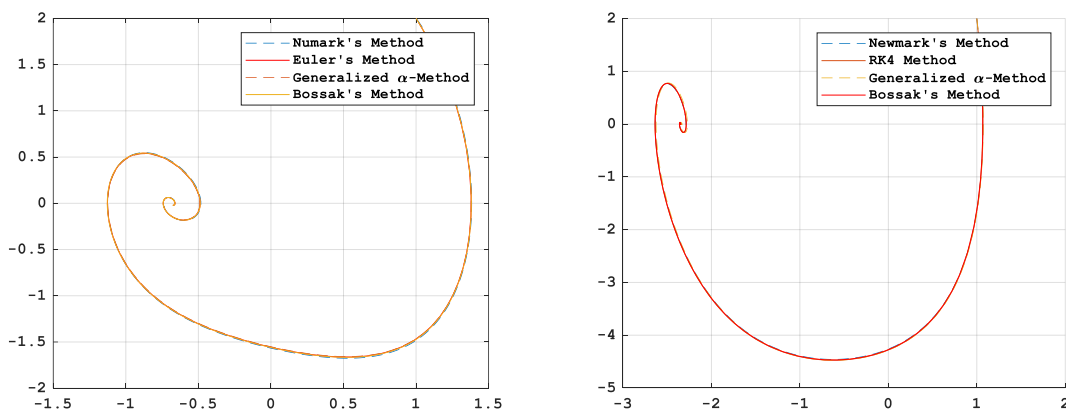


Figure 2. Result of numeric solution for equation (3) with parameters: left – $a = b = g = 1$; right – $a = 3; b = 20; g = 4$

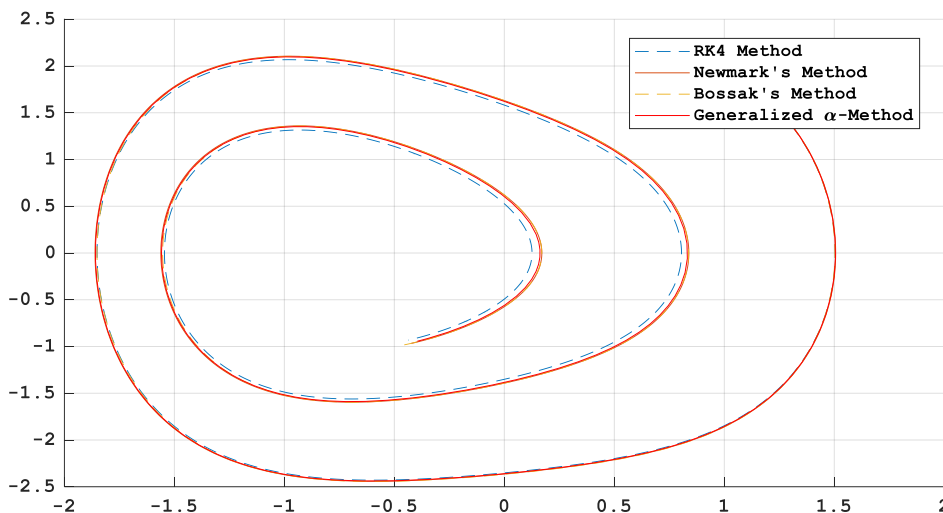


Figure 3. Result of numeric solution for equation (3) with parameters $a = 0,5$; $b = 1$; $g = 0,2$

Classical methods (Euler's method and Runge-Kutta method) gave less accurate results.

3. General conclusions

In the work, the main dependencies are obtained on the basis of the implicit methods of Newmark, Bossak and the generalized α -method for the example of a nonlinear oscillator near the assembly catastrophe. Curves of changes are plotted for different values of control parameters. The programs of implicit methods of Newmark, Bossack and the generalized α -method, Euler method and Runge-Kutta method with automatic step selection for the solution of a nonlinear oscillator near the assembly catastrophe are also implemented. Appropriate software for modeling deterministic nonlinear systems, which are described by Cauchy problems of the second order for ordinary differential equations, has been developed.

4. References

- [1] Franke, M.E., Lynch, P.M., Healy, A.J., "A 50-Year History of the Dynamic Systems and Control Division" ASME J. Dyn. Sys. Meas., and Control. Pp. 223–233.
- [2] Craig A. Kluever. Dynamic Systems: Modeling, Simulation, and Control. ISBN: 978-1-119-63540-6. Wiley; 1st edition. 2015, 496 p.
- [3] Kuznetsov A.P. Fluctuations. Catastrophes. Bifurcations. Chaos. Saratov. 2000. 99 p.
- [4] Kuznetsov A.P., Kuznetsov S.P., Ryskin N.M. Nonlinear oscillations. M.: Fizmatlit, 2002. 309 p.
- [5] Lambert J. Computational Methods in Ordinary Differential Equations, 1973.
- [6] Shampine L. Numerical solution of ordinary differential equations, Chapman & Hall, 1994.
- [7] Shampine L., Gordon M. Computer Solution of Ordinary Differential Equations, Freeman, 1975.
- [8] Shampine L., Gladwell I., Thompson S. Solving ODEs with MATLAB, Cambridge, 2003.