On Applying the Structured Model “State-Probability of Action” to Multi-Criteria Decision Making and Contradictory Reasoning

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Abstract
An approach to applying the structured model “state-probability of action” for decision making and reasoning under contradictory information got its further development. It was formulated in terms of nodes, each of which should be associated with a rectangular stochastic matrix “state-probability of action”, and decision making, reasoning etc. is carried out mainly by operating with matrices and vectors. A way to constructing matrices “state-probability of action” on the base of fuzzy sets and their membership functions has been suggested. A problem of equilibrium between two alternatives was explored for such a network. Some examples are provided, one of them represents the prospect of how the structured model “state-probability of action” can be combined with some elements of the Analytic Hierarchy Process for tackling the problem of multi-criteria decision making.

Keywords 1
Multi-criteria decision making, model “state-probability of action”, fuzzy sets, equilibrium of alternatives, contradictory reasoning, Analytic Hierarchy Process

1. Introduction. Related works

Currently there is a growing interest to models of individual and collective decision making in a multi-agent environment, especially to those models that take into account behavioral aspects of agents’ actions and various factors affecting their decisions. We consider also agents of influence whose goals are to make other agents accept decisions desirable for the influencers. Provided that an influencer is aware of a supposed parameterized model describing behavior of agents, they can try to affect decisions of other agents by manipulating parameters of that model – probably in an indirect way by sharing a certain information with other agents.

In [1] one possible approach to constructing such models has been suggested. This approach is based on considering a system of states corresponding to possible distributions of probabilities that an agent shall make available decisions if they are being in the certain state. Random walk across those states has been considered as well. A model based on this approach was called “state-probability of action” (or sometimes “state-probability of choice”) model. In [2] some parameters for this sort of models have been introduced and explored.

For describing complex decision making influenced by a set of different factors it appears promising to join together separate nodes, each of which corresponds to a particular judgement and is described by a separate model “state-probability of action”, to form a network reflecting relations between those nodes. A basic approach to describing such relations, that is the structured model “state-probability of action”, has been suggested in [3], but this approach needs a further development. A specific issue is how to use fragments of the network formed by “state-probability of action” nodes for describing

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uncertain logical reasoning, especially if this reasoning is carried out on the base of contradictory evidence. This is the problem this paper is devoted to.

2. Methodology, model, and techniques: "state-probability of choice" nodes and connections between them

Formally, for each node Q we have to specify a system of states $S^Q = \{s_1^{(Q)}, s_2^{(Q)}, \ldots\}$. Let $\xi^{(Q)}$ be a random variable that means which state from S^Q an agent is located in at the moment. We have to consider a vector of probabilities

$$\bar{p}^{(Q)} = (\bar{p}_1^{(Q)}, \bar{p}_2^{(Q)}, \ldots)$$

where $\bar{p}_i^{(Q)}$ is a probability that an agent is being in the state $s_i^{(Q)}$:

$$\bar{p}_i^{(Q)} = P(\xi^{(Q)} = s_i^{(Q)})$$

Instead of specifying $\bar{p}_i^{(Q)}$ explicitly, we might consider a random walk across Q and transitional probabilities in the corresponding Markov chain as it has been done in [1].

Let A and B be connected nodes, and B is a successor of A. We consider the relation $A \to B$ as an uncertain one, which means that the state S^B the agent is located in depends on their state from S^A. We specify the following probabilities:

$$y_{ij}^{(A\to B)} = P\left(\xi^{(B)} = s_j^{(B)} \middle| \xi^{(A)} = s_i^{(A)}\right)$$

We can introduce the $(m^A \times m^B)$-matrix $Y^{(A\to B)} = (y_{ij}^{(A\to B)})$, where $m^A = \left|S^{(A)}\right|$, $m^B = \left|S^{(B)}\right|$, $y_{ij}^{(A\to B)}$ are described above.

The matrix $Y^{(A\to B)}$ belongs to the class of so-called rectangular stochastic matrices [1, 3, 4], which is a generalization of well-known square stochastic matrices. A rectangular matrix $W = \left(w_{ij},\ i = \overline{1,m},\ j = \overline{1,n}\right)$ is said to be rectangular stochastic if it satisfices the following requirements:

$$\forall i \sum_{j=1}^{n} w_{ij} = 1,$$
$$\forall i,j \quad 0 \le w_{ij} \le 1$$

which means that the sum of elements in each row equals 1, but the matrix may be not a square one.

As it was stated in [3], within such a notation in terms of vectors and matrices the following relation takes place:

$$\bar{p}^{(B)} = \bar{p}^{(A)} \cdot Y^{(A\to B)}$$

Terminal nodes, which have no successors, have another meaning. They are related just to the action of decision making. Let there be n alternatives. We can introduce the "state-probability of action" model as follows: let its states numbering m represent possible distribution of probabilities of choice, and each element $h_{ij}, i = \overline{1,m}, j = \overline{1,n}$ of the matrix $H = (h_{ij})$ is the probability that an agent will choose the j-th alternative if they are being in the i-th state.

Let's introduce the vector $p_j, j = \overline{1,n}$, where p_j is the probability that an agent will choose the j-th alternative. Then

$$p = \bar{p}^{(H)} \cdot H$$

3. Equilibrium of alternatives

We will consider the most important particular case when there are two alternatives (*n=2*). For this case it is especially important to consider the situation of equilibrium of alternatives. This means that no alternative has advantage over the other, and

$$p = (0.5, \quad 0.5)$$

Equilibrium of alternatives is of great importance for collective decision making by majority of votes [1, 2]. It can be shown that if a number of agents is large enough, a situation of equilibrium is the only situation when the alternatives are on a par with each other and are chosen in turn. Otherwise, one alternative holds the steady advantage over the other, and it is permanently winning. But there can be different situations of equilibrium. So, if an agent of influence wants to boost up an alternative which is currently losing, they can try to reach one of equilibrium situations and then walk away from it in the desired direction. In [1] some sufficient conditions of equilibrium situations were found, which significantly rely on the principles of symmetry. This will be illustrated below.

To make the further considerations clearer, let's regard a basic illustrative example.

<u>Example 1</u>

Let the terminal matrix *H* be as follows [1]:

$$H = \begin{pmatrix} 1 & 0 \\ 0.9 & 0.1 \\ 0.75 & 0.25 \\ 0.6 & 0.4 \\ 0.5 & 0.5 \\ 0.4 & 0.6 \\ 0.25 & 0.75 \\ 0.1 & 0.9 \\ 0 & 1 \end{pmatrix}$$

As it was shown in [1], for securing the situation of equilibrium between the alternatives the vector $\bar{p}^{(H)}$ should be symmetric. For example, let's take the following one:

$$\bar{p}^{(H)} = (0.3, 0.1, 0., 0., 0.2, 0., 0., 0.1, 0.3)$$

Then

$$p = \bar{p}^{(H)} \cdot H = (0.5, 0.5)$$

This means that the alternatives should be chosen with the equal probabilities, and equilibrium of alternatives holds.

Let's assume that an agent of influence succeeds in changing the vector $\bar{p}^{(H)}$ – for example, it changes as follows:

$$\bar{p}^{(H)} = (0.3, \quad 0.1, \quad 0., \quad 0.1, \quad 0.1, \quad 0., \quad 0., \quad 0.1, \quad 0.3)$$

Then $p = (0.51, 0.49)$. Equilibrium of alternatives has been broken, and the first alternative will now win permanently.

The problem is that the matrix *H* is postulated in a very arbitrary way. There is no sound reason for its elements to be chosen as they are, and they could have very different values. In addition to this, the states represented by H hardly can be clearly interpreted. Now we are going to discuss how this problem might be tackled.

4. Getting matrices for terminal nodes

One approach to making the model "state-probability of choice" more structured and intelligible was suggested in [3]. Firstly, it comprises distinguishing groups of states having more or less clear interpretation. For instance, such groups can be as follows:

- proponents of a decision
- those who hesitate
- opponents of a decision

A number of groups can be larger.

In terms of concepts introduced in Section 2, this is a separate node associated with a model "state-probability of action" with its own system of states L connected to the terminal one. So, we should somehow determine a transition matrix $Y^{(L\to H)}$ from L to H. Then we can get the more ingenious aggregated matrix $H^* = Y^{(L\to H)} \cdot H$. Given a vector $\bar{p}^{(L)}$, which specifies probabilities that an agent is being in each state of L, the vector of choice probabilities p takes a view

$$p = \bar{p}^{(L)} \cdot Y^{(L\to H)} \cdot H = \bar{p}^{(L)} \cdot H^*$$

Again, a question of equilibrium arises, and now it is connected with so-called centrosymmetric matrices [5, 6]. A $(m \times n)$ – matrix A is said to be centrosymmetric, if

$$a_{ij} = a_{m-i+1,n-j+1} \ \forall i = \overline{1,m}; j = \overline{1,n}$$

It is known that if both matrices $Y^{(L\to H)}$ and H are centrosymmetric, then their product H^* is centrosymmetric as well. Then, if $\bar{p}^{(L)}$ is a symmetric vector and H^* is a centrosymmetric matrix, the vector p shall be symmetric, that is $p = (0.5,\ 0.5)$, and therefore equilibrium of alternatives holds.

In [3] the matrix $Y^{(L\to H)}$ was just specified explicitly. But it seems reasonable to elaborate more flexible approaches to getting this matrix.

5. A fuzzy approach to getting transitional matrix

For specifying the transitional matrix $Y^{(L\to H)}$, we suggest an approach based on fuzzy sets. To formulate the idea more or less formally, let's consider a family of fuzzy sets $U(l, H)$ with the membership functions $\mu_{U(l,H)}(x)$, which indicates the grade of relation of the state x represented by H to the certain state l from L. For instance, the state corresponding to the distribution $(0.6, 0.4)$ relates in large measure to the state corresponding to hesitating agents but not to proponents or opponents of the decision.

So, we can get a matrix $U = (u_{ij})$, where $u_{ij} = \mu_{U\left(s_i^{(L)},H\right)}(s_j^{(H)})$. The matrix $Y^{(L\to H)}$ can be obtained from this matrix by means of the well-known exponential transformation

$$y_{ij}^{(L\to H)} = \frac{e^{\beta u_{ij}}}{\sum_j e^{\beta u_{ij}}}$$

where $\beta > 0$ is a certain parameter.

This looks similar to what we did in [2], but in that paper we did not take into consideration neither related systems of states nor fuzzy sets.

Let's illustrate this by the following example.

<u>Example 2</u>

Let's take the terminal matrix H corresponding to a certain decision the same as in (1), and there are three groups of states on the level L: proponents of the decision, hesitating agents and opponents of the decision. We can take U as follows:

$$U = \begin{pmatrix} 1 & 0.8 & 0.3 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.4 & 0.9 & 1 & 0.9 & 0.4 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0.3 & 0.8 & 1 \end{pmatrix}$$

This matrix is centrosymmetric, so Y has to be centrosymmetric as well. If we take $\beta = 1$, it approximately equals

$$Y^{(L \to H)} = \begin{pmatrix} 0.2192 & 0.1795 & 0.1089 & 0.0891 & 0.0807 & 0.0807 & 0.0807 & 0.0807 & 0.0807 \\ 0.0674 & 0.0745 & 0.1006 & 0.1658 & 0.1833 & 0.1658 & 0.1006 & 0.0745 & 0.0674 \\ 0.0807 & 0.0807 & 0.0807 & 0.0807 & 0.0807 & 0.0891 & 0.1089 & 0.1795 & 0.2192 \end{pmatrix}$$

Then the aggregated matrix

$$H^{*(\beta=1)} = Y^{(L \to H)} \cdot H \approx \begin{pmatrix} 0.6167 & 0.3833 \\ 0.5 & 0.5 \\ 0.3833 & 0.6177 \end{pmatrix}$$

is centrosymmetric. Equilibrium of alternatives will hold for any symmetric vector $\bar{p}^{(L)}$.

Like [2], the parameter β can be interpreted as a parameter which indicates the degree of agents' decisiveness, and an influencer can try to manipulate this parameter. The resulting matrix $H^{*(\beta=1)}$ appears not to be very good due to the low value of β. If we take the increased value, for instance $\beta = 5$, which means that the agents become more decisive, we will get

$$H^{*(\beta=5)} \approx \begin{pmatrix} 0.9487 & 0.0513 \\ 0.5 & 0.5 \\ 0.0513 & 0.9487 \end{pmatrix}$$

Another promising approach is related to introducing fuzzy variables such as triangle, trapezoid etc. ones.

6. The model "state-probability of action" and reasoning

Now we a going to show how the structured model "state-probability of action" can be applied for uncertain and contradictory reasoning. Reasoning is typically carried out on the base of the knowledge by applying certain rules of inference to the known facts. But if we regard an agent's knowledge as a logical system, it may be contradictory (especially if the agent is a human being). Therefore, for a statement A an agent may infer both A and its negation $\bar{A}$. In such a situation, we consider a probabilistic approach, which means that an agent can accept A with a certain probability, and this probability can be calculated with the help of the model "state-probability of action". In this paper we are going to develop an approach preliminary outlined in [7], re-formulate it in terms of systems of states and to provide some more ingenious examples.

Let an agent consider a decision L and therefore two related alternatives – to accept L or to reject L. Within the model "state-probability of action" we have to introduce systems of states corresponding to the rules of inference. For a single inference rule $A \Longrightarrow B$ we consider two systems of states: $S^{(A)}$ and $S^{(B)}$. So, we have to specify the vector $\bar{p}^{(A)}$ and the matrix $Y^{(A \Rightarrow B)}$, then $\bar{p}^{(B)}$ can be obtained by the formula (). If the rule is a terminal one, that is the rule is $A \Longrightarrow L$, then we should specify the vector $\bar{p}^{(A)}$, the matrix $Y^{(A \Longrightarrow L)}$ and in addition to this the matrix H^* described above.

Finally, for the rule $A \Longrightarrow L$ the vector of probabilities equals

$$p = \bar{p}^{(A)} \cdot Y^{(A \Longrightarrow L)} \cdot H^* \quad (1)$$

Multipliers in the equation (1) can be grouped in different ways. It can be rewritten in the following form:

$$p = \bar{p}^{(A)} \cdot R$$

where $R = Y^{(A \Longrightarrow L)} \cdot H^*$.

Surely, a chain of logical inference may be longer.

If both $Y^{(A \Longrightarrow L)}$ and H^* are centrosymmetric rectangular stochastic matrices, then their product R shall be a centrosymmetric rectangular stochastic matrix as well. Provided that $\bar{p}^{(A)}$ is a symmetric vector and R is a centrosymmetric matrix, equilibrium of alternatives holds.

Another equivalent form of (1) can be constructed as follows:

$$p = \bar{p}^{(L)} \cdot H^*, \quad (2)$$

$$\bar{p}^{(L)} = \bar{p}^{(A)} \cdot Y^{(A \Longrightarrow L)}$$

We will use this form below.

It is possible that the decision L can be affected by different factors, and this typically can lead to contradictions. To make a closer look, let's consider the following set of rules:

$$A_1 \Longrightarrow L,$$
$$\dots$$
$$A_q \Longrightarrow L$$

By applying the equation (2) we can get q different vectors:

$$\bar{p}^{(L)(k)} = \bar{p}^{(A_k)} \cdot Y^{(A_k \Longrightarrow L)}, k = \overline{1, q}$$

The vector $\bar{p}^{(L)}$ can be obtained by combining all $\bar{p}^{(L)(k)}$. In particular, we can get their convex combination:

$$\bar{p}^{(L)} = \sum_{k=1}^{q} w_k \, \bar{p}^{(L)(k)},$$

$$0 \leq w_k \leq 1, \quad (3)$$

$$\sum_{k=1}^{q} w_k = 1$$

We assume that the matrix H^* is centrosymmetric. It can be shown that if all vectors $\bar{p}^{(L)(k)}$ are symmetric, their convex combination is symmetric and therefore equilibrium of alternatives holds for any values of w_k. Otherwise, for ensuring equilibrium the proper values of w_k should be specially picked.

Let's regard some examples.

7. Examples of combining evidence

Example 3

Firstly, we are going to illustrate one popular rule of what may be called paranormal logic, namely the rule "If *A* implies *B* and *B* is desirable, then *A* is true". Surely, this rule is incorrect from the logical point of view, but people are often driven by it in their practice.

So, we have the rule $A \Longrightarrow B$, B is here the terminal node. As it was explained before,

$$p = \bar{p}^{(A)} \cdot Y^{(A \Rightarrow B)} \cdot H^*$$

As for A, one evidence is the uncertain information whether it is true or false. It appears to be more flexible if we pick out a separate rule $K \Longrightarrow A$, where K denotes the statement "There is the evidence that A is true". Based on this rule, we can get the vector

$$\bar{p}^{(A)(1)} = \bar{p}^{(K)} \cdot Y^{(K \Rightarrow A)}$$

Another rule is $W \Longrightarrow A$, where W denotes the statement "B is desirable". So,

$$\bar{p}^{(A)(2)} = \bar{p}^{(W)} \cdot Y^{(W \Rightarrow A)}$$

and the vector $\bar{p}^{(A)}$ should be obtained by combining $\bar{p}^{(A)(1)}$ and $\bar{p}^{(A)(2)}$.

For all rules we are taking systems of states like to that we used for getting aggregated matrix H^* (proponents, opponents and hesitating agents).

Let's postulate the specific values.

The matrix H^* will be taken from the example 2 with $\beta = 5$:

$$H^* = \begin{pmatrix} 0.9487 & 0.0513 \\ 0.5 & 0.5 \\ 0.0513 & 0.9487 \end{pmatrix}$$

For the rule $A \Longrightarrow B$ we are taking the matrix

$$Y^{(A \Rightarrow B)} = \begin{pmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0 & 0.2 & 0.8 \end{pmatrix}$$

For the reason of simplicity, we are taking the same matrices $Y^{(K \Rightarrow A)}$ and $Y^{(W \Rightarrow A)}$:

$$Y^{(K \Rightarrow A)} = Y^{(W \Rightarrow A)} = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \end{pmatrix}$$

But vectors $\bar{p}^{(K)}$ and $\bar{p}^{(W)}$ are very different. Assuming that there is no reliable information about A, we may take

$$\bar{p}^{(K)} = (0.1, \quad 0.2, \quad 0.7)$$

But if a majority of agents wants B to be accepted, we may take

$$\bar{p}^{(W)} = (0.7, \quad 0.2, \quad 0.1)$$

Then

$$\bar{p}^{(A)(1)} = \bar{p}^{(K)} \cdot Y^{(K \Rightarrow A)} = (0.11, \quad 0.24, \quad 0.65)$$

$$\bar{p}^{(A)(2)} = \bar{p}^{(W)} \cdot Y^{(W \Rightarrow A)} = (0.65, \quad 0.24, \quad 0.11)$$

If we performed reasoning on the base of available knowledge about *A* only, our further calculations would be as follows:

$$p = \bar{p}^{(A)(1)} \cdot Y^{(A \Rightarrow B)} \cdot H^* = (0.3062, \quad 0.6938),$$

which means that *B* should be rejected.

Similarly, if we proceeded the reasoning on the base of $\bar{p}^{(A)(2)}$, B should be accepted. And if we carry out the combining:

$$\bar{p}^{(A)} = 0.5 \cdot \bar{p}^{(A)(1)} + 0.5 \cdot \bar{p}^{(A)(2)} = (0.38, \quad 0.24, \quad 0.38)$$

we will get

$$p = \bar{p}^{(A)} \cdot Y^{(A \Rightarrow B)} \cdot H^* = (0.5, \quad 0.5)$$

and equilibrium of alternatives will hold.

Another example illustrates how the structured model "state-probability of action" can be combined with some elements of the Analytic Hierarchy Process (AHP) [8-12], which is a very famous method of hierarchical multi-factor decision making.

Example 4

Assume there are two alternatives and agents are to choose one of them. Similar to AHP, we consider decision making affected by multiple criteria. But instead of constructing pairwise comparison matrices for each criterion, we are trying to introduce states reflecting degrees of advantage of one alternative over the other. We stipulate the rule $A \Longrightarrow L$ with the following meaning: "if an alternative *L* is better with respect to any criterion, then *L* should be chosen". We introduce the states for *A* as follows:

- *L* is significantly better than a competing alternative;
- *L* is better in some measure;
- both alternatives are equivalent;
- *L* is worse in some measure;
- *L* is significantly worse.

Certainly, systems of states may be quite different. For instance, states may correspond to the standard grades of Saaty scale, or we may use any other scale of preferences. Some reviews of different scales for pairwise comparisons can be found in [13, 14].

For the rule $A \Longrightarrow L$ we will specify the following matrix:

$$Y^{(A \Rightarrow L)} = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.6 & 0.3 & 0.1 \\ 0 & 0.2 & 0.8 \\ 0.1 & 0.3 & 0.6 \\ 0 & 0.05 & 0.95 \end{pmatrix}$$

For each *k*-th criterion we will specify its particular vector $\bar{p}^{(A)(k)}$. Let there be *4* criteria, and

$$\bar{p}^{(A)(1)} = (0.9, 0.1, 0, 0, 0.)$$
$$\bar{p}^{(A)(2)} = (0.6, 0.2, 0.1, 0.1, 0)$$
$$\bar{p}^{(A)(3)} = (0, 0.1, 0.1, 0.2, 0.6)$$

$$\bar{p}^{(A)(4)} = (0, 0, 0, 0.1, 0.9)$$

Either *L* or the competing alternative shall gain an advantage with respect to particular criteria. For combining criteria by using the formula (), we have to specify the coefficients w_k. For instance, we can take the Perronian vector (that is the normalized main eigenvector) of a pairwise comparison matrix across criteria, which is absolutely typical for the AHP.

Let the comparison matrix be

$$C = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \frac{1}{2} & 1 & 2 & 3 \\ \frac{1}{3} & \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 \end{pmatrix}$$

Then its Perronian vector approximately equals

$$w_k = (0.4673, \quad 0.2772, \quad 0.1601, \quad 0.0954)$$

With respect to this combined vector, final probabilities of alternatives are

$$p = (0.6836, \quad 0.3164)$$

and the chosen alternative shall be *L*.

Equilibrium of alternatives will hold if the Perronian vector of *C* is symmetric. This can take place if *C* is centrosymmetric, for example if it is as follows:

$$C = \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} & 1 \\ 4 & 1 & 1 & 4 \\ 4 & 1 & 1 & 4 \\ 1 & \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}$$

In this paper we don't consider possible consistency or inconsistency of centrosymmetric pairwise comparisons, this issue needs to be specially studied.

If a pairwise comparison matrix is considered as a parameter of a behavioral model, an influencer can try to affect these comparisons – maybe by influencing experts to change their opinions and thereby to modify their comparisons.

8. Results, conclusions and discussion

In this paper, the structured model "state-probability of action" has got a further development, which opens a prospect of constructing a model of decision making under the uncertain and/or contradictory information on the base of a network of connected nodes, each of which implements the model "state-probability of action". At a closer look, each node should be associated with a rectangular stochastic matrix "state-probability of action", and decision making, reasoning etc. is carried out mainly by operating with matrices and vectors. For given or assumed statements, vectors of initial probabilities that an agent is being in the certain state related to these statements are to be provided. Such vectors might be specified explicitly, but they can be also obtained from a Markov chain with the given transitional probabilities, those are the probabilities of transitions across the states.

Such a model comprises operating with chains of reasoning and combining contradictory evidence. In some measure it is similar to those like probabilistic Bayesian networks, belief networks, knowledge graphs etc. [15-18]. But the suggested model places more articulate emphasis on behavioral aspects of decision making and on considering possible contradictions in available evidence.

An approach to constructing matrices "state-probability of action" on the base of fuzzy sets and their membership functions has been suggested. It appears that such a fuzzy approach might be considerably entrenched by using fuzzy numbers of different kinds (triangle, trapezoid etc.).

The model "state-probability of action" was designed to be parametrized. If such a model describes a real situation of decision making and an agent of influence is aware of this model, then they can try to manipulate the parameters with the aim of making other agents accept decisions desirable for influencers. They can do it by sharing information with other agents; models aimed at describing dissemination of information across communities are rapidly developing now [19-22].

Some parameters for nodes implementing the structured model "state-probability of action" have been suggested and discussed in [2] and, following that, in this paper. The main of them are as follows (including but not limited to):

- decisiveness of agents
- pairwise comparisons between different criteria the final decision depends on if the suggested approach is being combined with the Analytic Hierarchy Process
- weighting coefficients in formula (3), which are proposed to be used for combining evidence; meaningfully these coefficients may reflect how those pieces of evidence are important and/or reliable and how we do trust them
- fuzzy membership functions and types of fuzzy numbers which can be applied for forming matrices "state-probability of action", especially for terminal nodes
- transitional probabilities across states.

This list of parameters of the model surely can be extended.

Two examples, which illustrate possible ways of applying the structured model "state-probability of action" to contradictory decision making and reasoning, have been provided. One of them illustrates a very proliferated, despite its actual incorrectness, rule of reasoning "If *A* implies *B* and *B* is desirable, then *A* is true", which relates to what can be characterized as a paranormal logic and may be closely bound with a conflict between knowledge and wishes of an agent, especially of a human being. The other example illustrates the prospect of how the structured model "state-probability of action" can be combined with some elements of the Analytic Hierarchy Process for tackling the problem of multi-criteria decision making. This appears especially important if decisions are made algorithmically on the base of certain parametrized procedures.

For both examples, the problem of equilibrium between alternatives, which can be found within the model by combining contradictory pieces of evidence, has been explored. Those situations of equilibrium can be found on the base of symmetric vectors and centrosymmetric matrices, it appears interesting to search for non-symmetric ones. If the model is combined with the AHP, it appears important to investigate how consistent or inconsistent pairwise comparison matrices may be.

As an overall final remark, the suggested model "state-probability of action" insofar as it is a probabilistic model admitting clear fuzzy generalization, and it is a model placing special emphasis on behavioral aspects of decision making, can find various applications as for modeling individual and collective decisions in socio-economic systems (political activity, information wars, fluctuations of ratings gained by political parties, voting on elections etc.) so in multi-agent systems of algorithmic decision making, especially if decisive rules applied in those systems are weakly formalized, unclear and volatile.

9. References

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