Boundary Problems of Thermo-Electro Elasticity in the Generalized Cylindrical Coordinates of Telecommunication Systems

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Abstract

The condition of the constituent elements of telecommunications and radio-electronic complexes is largely determined by the trends of changes in parameters of electro-radio components, properties of mechanical structures, and parameters of antenna and guidance systems. And one of the indicators that must be taken into account is thermoelectric elasticity. Information on models of thermo-electro elasticity of multi-layered structures, which form various basic elements of telecommunication and radio-electronic complexes, is necessary for determining the dynamics of changes in the technical condition of equipment under certain operating conditions. These cases can be described with the help of boundary problems of thermo-electro elasticity. A class of static boundary value problems is effectively solved for bodies bounded by coordinate surfaces of generalized cylindrical coordinates ρ, α, z (ρ, α orthogonal curved coordinates on the plane, and z linear coordinates). The body is affected by a stationary temperature and electric field, surface disturbances (given voltages, displacements, or a combination of them) $z = 0$ and $z = z_1$ homogeneous conditions of a special type are set on the remaining part of the surface. An elastic body is assumed to be transtropic (transversally isotropic), with an isotropy plane $z = const$. The transtropic layers of a multilayer body contact along the plane $z = const$. In the work with the method of separation of variables, exact solutions to several boundary problems about the thermo-electro elastic equilibrium of single and multilayer bodies are constructed.

Keywords

Thermoelectroelasticity, radio-electronic equipment, transtopic medium, functions

1. Introduction

The condition of the constituent elements of telecommunications and radio-electronic complexes is largely determined by the trends of changes in parameters of electro-radio components, properties of mechanical structures, and parameters of antenna and guidance systems. Monitoring and control of the specified parameters are carried out by special measuring equipment, in particular using non-destructive, including radio wave control. Since the equipment is operated in certain external conditions, it is important to study the changes in the determining parameters of the basic components depending on the characteristics of the environment, in particular the temperature. At the same time, one of the indicators that must be taken into account is thermoelectric elasticity. Information on models of thermo-electro elasticity of multi-layered structures, which form various basic elements of telecommunication and radio-electronic complexes, is necessary for determining the dynamics of changes in the technical condition of equipment under certain operating conditions. These conditions can be described with the help of boundary problems of thermo-electro elasticity [1–4].

The boundary value problem of elastic equilibrium of a homogeneous layer (related to the problems considered in this article) was first considered by Lamet and Clapeyron. In

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subsequent studies, the solutions of these authors were simplified and generalized. A fairly complete list of works devoted to this issue is given in the bibliography [5–12].

In all these works, the solution was based on the formulas of the double integral transformation, in most cases for a homogeneous layer in the absence of temperature and electrical disturbances. In this article, using the method of separation of variables and double series, solutions of static boundary and boundary-contact problems of thermo-electro elastic are constructed [5, 6] for a *curvilinear coordinate parallelepiped*

$$\text{(CCP)}\,\Pi = \left\{ (\rho, \alpha, z) \in R : \rho_o < \rho < \rho_1, \alpha_0 < \alpha < \alpha_1, 0 < z < z_1 \right\}$$

where ρ, α, z are generalized cylindrical coordinates (ρ, α orthogonal curved coordinates on the plane, and z linear coordinate). At $z = 0$ and $z = z_1$ together with a temperature and electric field, gives either voltages, displacements, or a combination of them. Homogeneous boundary conditions of a special kind are set on the side surfaces ($\rho = \rho_0, \rho = \rho_1, \alpha = \alpha_0, \alpha = \alpha_1$). If a multilayer body is considered, then its layers contact along the planes $z = const$. An elastic body or layers of a multilayer body can be both transtropic and homogeneous ($z = const$ isotropy plane), and isotropic and homogeneous.

According to the above-mentioned information, the problem of elastic equilibrium of an infinite layer is generalized while simplifying the method of its solution. Simplification is achieved by: a) converting the electro-temperature problem and constructing a general solution for the class of thermo-electro elasticity problems under study; b) replacing the classical conditions set on the boundary and contact surfaces with equivalent conditions; c) using double series instead of a double integral transformation. At the end of the article, the notes provide solutions to some problems of thermo-electro elasticity.

The following can be said about the effectiveness of solutions. If, using the method of separation of variables in the domain

$$\Pi = \left\{ (\rho, \alpha, z) \in R : \rho_o < \rho < \rho_1,\ \alpha_0 < \alpha < \alpha_1,\ 0 < z < z_1 \right\},$$

it is effectively possible to construct solutions of the main boundary problems for the Laplacian equation, with zero conditions at $\rho = \rho_0, \rho = \rho_1, \alpha = \alpha_0, \alpha = \alpha_1$, then, with the same efficiency, in the same domain $\overline{\Pi}$ and by the same method, a thermo-electro elastic equilibrium can be found for the bodies under consideration.

In conclusion of the introduction, we'll indicate that the coefficients of the lamellar system ρ, α, z [7]

$$h_\rho = h_\alpha = h = \sqrt{\left(\frac{\partial x}{\partial \rho}\right)^2 + \left(\frac{\partial y}{\partial \rho}\right)^2}, \quad h_z = 1$$

and that

$$\frac{\partial x}{\partial \rho} - \frac{\partial y}{\partial \alpha} = 0, \qquad \frac{\partial x}{\partial \alpha} + \frac{\partial y}{\partial \rho} = 0,$$

$$\frac{\partial}{\partial \rho}\left(\frac{1}{h}\frac{\partial h}{\partial \rho}\right) + \frac{\partial}{\partial \alpha}\left(\frac{1}{h}\frac{\partial h}{\partial \alpha}\right) = 0,$$

where, x, y are cartesian coordinates.

2. Equations of State, Boundary Conditions, General Solution, Uniqueness of the Solution

Let the temperature field be independent of time, and the mass forces are neglected, then the system of differential equations of thermo-electro elasticity describing the state of a transtropic homogeneous body in generalized cylindrical coordinates has the following form [6, 8]:

$$
\left.\begin{aligned}
&a)\,\frac{\partial(h\sigma_\rho)}{\partial\rho}+\frac{1}{h}\frac{\partial(h^2\tau_{\rho\alpha})}{\partial\alpha}+h^2\frac{\partial\tau_{\rho z}}{\partial z}-\frac{\partial h}{\partial\rho}\sigma_\alpha=0,\\
&b)\,\frac{\partial(h\sigma_\alpha)}{\partial\alpha}+h^2\frac{\partial\tau_{\alpha z}}{\partial z}+\frac{1}{h}\frac{\partial(h^2\tau_{\alpha\rho})}{\partial\rho}-\frac{\partial h}{\partial\alpha}\sigma_\rho=0,\\
&c)\,h^2\frac{\partial\sigma_z}{\partial z}+\frac{\partial(h\tau_{\rho z})}{\partial\rho}+\frac{\partial(h\tau_{z\alpha})}{\partial\alpha}=0,
\end{aligned}\right\}\quad (1)
$$

$$
\frac{1}{h^2}\frac{\partial(h\,D_\rho)}{\partial\rho}+\frac{1}{h^2}\frac{\partial(h\,D_\alpha)}{\partial z}+\frac{\partial D_z}{\partial z}=0,\quad (2)
$$

Equations (1) are the usual elastic equilibrium equations accepted in the theory of elasticity. Equations (2) are the equations of the electric field and are called the equation of electrostatics. The following designations are used in these equations: σ_ρ, σ_α, σ_z are normal stresses; $\tau_{\rho\alpha}=\tau_{\alpha\rho},\tau_{\rho z}=\tau_{z\rho},\tau_{\alpha z}=\tau_{z\alpha}$ are tangential stresses. D_ρ, D_α, D_z are components of the electric induction vector along tangents to coordinate lines ρ,α,z.

In the case of axial polarization, the equations of state of the transtopic medium are represented by the equalities:

$$
\left.\begin{aligned}
&a)\sigma_\rho=c_1\varepsilon_{\rho\rho}+(c_1-2c_5)\varepsilon_{\alpha\alpha}+c_3\varepsilon_{zz}-e_1E_z-\beta_{10}T=\frac{c_1}{h^2}\left(\frac{\partial(hu)}{\partial\rho}+\frac{\partial(h\mathrm{v})}{\partial\alpha}\right)-\\
&-2c_5\left(\frac{1}{h}\frac{\partial\mathrm{v}}{\partial\alpha}+\frac{1}{h^2}\frac{\partial h}{\partial\rho}u\right)+c_3\frac{\partial\mathrm{w}}{\partial z}-e_1E_z-\beta_{10}T,\\
&b)\sigma_\alpha=(c_1-2c_5)\varepsilon_{\rho\rho}+c_1\varepsilon_{\alpha\alpha}+c_3\varepsilon_{zz}-e_1E_z-\beta_{10}T=\frac{c_1}{h^2}\left(\frac{\partial(hu)}{\partial\rho}+\frac{\partial(h\mathrm{v})}{\partial\alpha}\right)-\\
&-2c_5\left(\frac{1}{h}\frac{\partial u}{\partial\rho}+\frac{1}{h^2}\frac{\partial h}{\partial\alpha}\mathrm{v}\right)+c_3\frac{\partial\mathrm{w}}{\partial z}-e_1E_z-\beta_{10}T,\\
&c)\sigma_z=c_3(\varepsilon_{\rho\rho}+\varepsilon_{\alpha\alpha})+c_2\varepsilon_{zz}-e_2E_z-\beta_{20}T=\frac{c_3}{h^2}\left(\frac{\partial(hu)}{\partial\rho}+\frac{\partial(h\mathrm{v})}{\partial\alpha}\right)+\\
&+c_2\frac{\partial\mathrm{w}}{\partial z}-e_2E_z-\beta_{20}T,
\end{aligned}\right\}
$$

$$
\left.\begin{aligned}
&d)\tau_{\alpha z}=c_4\,\varepsilon_{\alpha z}-e_3E_\alpha=c_4\left(\frac{\partial\,\mathrm{v}}{\partial z}+\frac{1}{h}\frac{\partial\mathrm{w}}{\partial\alpha}\right)-e_3E_\alpha,\\
&e)\tau_{\rho z}=c_4\,\varepsilon_{\rho z}-e_3E_\rho=c_4\left(\frac{\partial\,u}{\partial z}+\frac{1}{h}\frac{\partial\mathrm{w}}{\partial\rho}\right)-e_3E_\rho,\\
&f)\tau_{\rho\alpha}=c_5\,\varepsilon_{\rho\alpha}=c_5\left(\frac{\partial}{\partial\rho}\left(\frac{\mathrm{v}}{\mathrm{h}}\right)+\frac{\partial}{\partial\alpha}\left(\frac{u}{h}\right)\right),
\end{aligned}\right\}\quad (3)
$$

$$
\left.\begin{aligned}
&a)D_\rho=e_3\,\varepsilon_{\rho z}+э_1\,E_\rho=e_3\left(\frac{\partial\,u}{\partial z}+\frac{1}{h}\frac{\partial\mathrm{w}}{\partial\rho}\right)+э_1\,E_\rho,\\
&b)D_\alpha=e_3\,\varepsilon_{\alpha z}+э_1\,E_\alpha=e_3\left(\frac{\partial\,\mathrm{v}}{\partial z}+\frac{1}{h}\frac{\partial\mathrm{w}}{\partial\alpha}\right)+э_1\,E_\alpha,\\
&c)D_z=e_1(\varepsilon_{\rho\rho}+\varepsilon_{\alpha\alpha})+e_2\varepsilon_{zz}+э_2\,E_z=e_1\left(\frac{\partial(hu)}{\partial\rho}+\frac{\partial(h\mathrm{v})}{\partial\alpha}\right)+\\
&+e_2\frac{\partial(h\mathrm{w})}{\partial z}+э_2\,E_z.
\end{aligned}\right\}\quad (4)
$$

where $u, \mathrm{v}, \mathrm{w}$ are components of the displacement vector $\vec{U}$ along tangents to coordinate lines ρ, α, z, $\varepsilon_{\rho\rho}$, $\varepsilon_{\alpha\alpha}$, ε_{zz}, $\varepsilon_{\rho\alpha} = \varepsilon_{\alpha\rho}$, $\varepsilon_{\rho z} = \varepsilon_{z\rho}$, $\varepsilon_{z\alpha} = \varepsilon_{\alpha z}$ is deformations; E_ρ, E_z, E_z, E_α is components of the electric tension vector $\vec{E}$ along tangents to coordinate lines ρ, α, z, and $\vec{E} = -grad\varphi$; φ is electrostatic potential. c_i $(i=\overline{1,5})$ is elastic modulus measured at a constant electric field; e_j $(j=\overline{1,3})$ is piezoelectric constants; $Э_1$, $Э_2$ — dielectric permittivity at constant deformations; $\beta_{10} = [\,2(c_1 - c_5)\beta_1 + c_3\beta_3]$ $\beta_{20} = [\,2c_3\ \beta_1 + c_2\beta_2]$ β_1, β_2 are coefficients of linear thermal expansion in the plane of isotropy and along z. T is the temperature of the medium obeying the equation

$$\Delta_2 T + \frac{\lambda_1}{\lambda_2}\frac{\partial^2 T}{\partial z^2} = 0$$

and the corresponding boundary conditions. λ_1, λ_2 are thermal conductivity coefficients in the isotropy plane and along z [5].

Using (3) and (4), the following system can be obtained concerning K, $\tau_{z\alpha}$, $\tau_{z\alpha}$, B, u, v, w [10, 11]

$$\left.\begin{aligned}
&a)\ \frac{\partial}{\partial z}\left(\frac{c_3}{c_1}K + \frac{c_1c_2 - c_3^2}{c_1}\frac{\partial \mathrm{w}}{\partial z}\right) + \frac{1}{\mathrm{h}^2}\left(\frac{\partial(\mathrm{h}\tau_{z\rho})}{\partial\rho} + \frac{\partial(\mathrm{h}\tau_{z\alpha})}{\partial\alpha}\right) = \frac{\partial}{\partial z}\left(\frac{c_3e_1 - c_1e_3}{c_1}\frac{\partial\phi}{\partial z}\right) + \\
&+ \frac{\partial}{\partial z}\left(\frac{c_3\beta_{20} - c_3\beta_{10}}{c_1}T\right),\\
&b)\ \frac{\partial K}{\partial\rho} - \frac{\partial B}{\partial\alpha} + \frac{\partial(\mathrm{h}\tau_{z\rho})}{\partial z} = 0\,,\\
&c)\ \frac{\partial B}{\partial\rho} + \frac{\partial K}{\partial\alpha} + \frac{\partial(\mathrm{h}\tau_{z\alpha})}{\partial z} = 0,\\
&d)\ \frac{1}{\mathrm{h}^2}\left(\frac{\partial(\mathrm{h}\tau_{z\alpha})}{\partial\rho} - \frac{\partial(\mathrm{h}\tau_{z\rho})}{\partial\alpha}\right) - c_4\frac{\partial}{\partial z}\left(\frac{1}{c_5}B\right) = 0,
\end{aligned}\right\} \qquad (6)$$

where symbols are introduced

$$\left.\begin{aligned}
&a)\ K = c_1\frac{1}{h^2}\left[\frac{\partial(hu)}{\partial\rho} + \frac{\partial(h\mathrm{v})}{\partial\alpha}\right] + c_3\frac{\partial\mathrm{w}}{\partial z} + e_1\frac{\partial\varphi}{\partial z} - \beta_{10}T\,,\\
&b)\ B = c_5\frac{1}{h^2}\left[\frac{\partial(h\mathrm{v})}{\partial\rho} - \frac{\partial(hu)}{\partial\alpha}\right],\\
&c)\ \tau_{zr} = c_4\frac{1}{h}\left(\frac{\partial(hu)}{\partial z} + \frac{\partial\mathrm{w}}{\partial\rho}\right) + e_3\frac{1}{h}\frac{\partial\varphi}{\partial\rho}\\
&d)\ \tau_{z\alpha} = c_4\frac{1}{h}\left(\frac{\partial\mathrm{w}}{\partial\alpha} + \frac{\partial(hu)}{\partial z}\right) + e_3\frac{1}{h}\frac{\partial\varphi}{\partial\alpha}
\end{aligned}\right\}$$

Next, the thermo-electro elastic equilibrium of a *Curvilinear Coordinate Parallelepiped* (CCP) occupying a region.
$\Pi = \{\rho_0 < \rho < \rho_1, \alpha_0 < \alpha < \alpha_1,\ 0 < z < z_1,\}$ will be considered. The boundary conditions that will appear in the formulation of boundary problems have the following form:

$$\text{at } \rho = \rho_j: \qquad \left.\begin{aligned}
&a)\ \frac{\partial T}{\partial\rho} = 0, u = 0, \tau_{z\rho} = 0, B = 0, D_\rho = 0, or\\
&b)\ T = 0,\ K = 0,\ \mathrm{v} = 0,\ \ \mathrm{w} = 0,\ \ \varphi = 0.
\end{aligned}\right\} \qquad (7)$$

at $\alpha = \alpha_j$:
$$\left.\begin{array}{l} a)\ \dfrac{\partial T}{\partial \alpha} = 0,\ \mathrm{v} = 0,\ \tau_{z\alpha} = 0,\ B = 0,\ D_\alpha = 0,\ or \\ b)\ T = 0,\ K = 0,\ \mathrm{u} = 0,\ \ \mathrm{w} = 0,\ \ \varphi = 0. \end{array}\right\} \quad (8)$$

at $z = z_j$;
$$\left.\begin{array}{l} a)\ T = F_{j1}(\rho,\alpha),,\ or\ b)\ \dfrac{\partial T}{\partial \alpha} = F_{j2}(\rho,\alpha) or \\ c)\ \dfrac{\partial T}{\partial \alpha} + \ \Theta_j T = F_{j3}(\rho,\alpha). \end{array}\right\} \quad (9)$$

at $z = z_j$:
$$a)\ \varphi = F_{j4}(\rho,\alpha),,\ or\ b)\ D_z = F_{j5}(\rho,\alpha) \quad (10)$$

at $z = z_j$;
$$\left.\begin{array}{l} a)\ \sigma_Z = F_{j6}(\rho,\alpha),\ h\tau_{z\rho} = F_{j7}(\rho,\alpha),\quad h\tau_{z\alpha} = F_{j8}(\rho,\alpha),\ or \\ \mathrm{b})\ \mathrm{w}\ \ = f_{\mathrm{j}1}(\rho,\alpha),\ \ hu = f_{j2}(\rho,\alpha),\qquad h\mathrm{v} = f_{\mathrm{j}3}(\rho,\alpha),\ or \\ c)\ \ \mathrm{w} = f_{\mathrm{j}1}(\rho,\alpha),\quad h\tau_{z\rho} = F_{j7}(\rho,\alpha),\ \ h\tau_{z\alpha} = F_{\mathrm{j}8}(\rho,\alpha),\ \ or \\ d)\ \sigma_z = F_{j6}(\rho,\alpha),\ \ hu = f_{j2}(\rho,\alpha),\qquad h\mathrm{v} = f_{\mathrm{j}3}(\rho,\alpha). \end{array}\right\} \quad (11)$$

where j = 0,1 at that $z_0 = 0$ is specified constants. The conditions imposed on the functions f_{jk} $\left(k = \overline{1,3}\right)$ F_{jl} $\left(l = \overline{1,8}\right)$ will be discussed below, we will only indicate that these functions are such that the coordination conditions are met on the edges of the CCP. Now we give a technical interpretation of the boundary conditions:

$(8a)$, $(9a)$, $(11c)$ at $f_{\mathrm{j}1}(\rho,\alpha)$ = 0 and $F_{j6}(\rho,\alpha)$ = 0 condition III_0.

$(8b)$, $(9b)$, $(11d)$ at $f_{\mathrm{j}1}(\rho,\alpha)$ = 0 and $F_{j6}(\rho,\alpha)$ = 0- condition IV_0.

In the case of conditions III_0, we assume that the cylindrical or flat boundary S of the CCP is connected, respectively, with a smooth cylindrical or plane boundary surface S of a rigid body, which is a thermo-electric insulator.

Due to the absolute rigidity of the body, the component of the displacement vector normal to S vanishes, and due to the absolute smoothness of S $B = 0, \tau_{z\rho} = 0$, either $B = 0, \tau_{z\alpha} = 0$ or $\tau_{z\rho} = 0, \tau_{z\alpha} = 0$.

In the case of condition VI_0, we will assume that an absolutely flexible, but inextensible and incompressible thin plate is glued to the cylindrical or flat boundary surface S of the CCP (naturally, the plate takes the form of a surface S).

Due to the absolute inextensibility and incompressibility of the plate $\mathrm{v} = 0, \mathrm{w} = 0$ or $\mathrm{u} = 0, \mathrm{w} = 0$ or $\mathrm{u} = 0, \mathrm{v} = 0$, and due to absolute flexibility $\mathrm{K} = 0$ (the conditions $\mathrm{T} = 0, \mathrm{D} = 0$ at $\rho = \rho_j$ and $\alpha = \alpha_j$ are achieved by other technical means).

Note. The smaller the curvature of the boundary cylindrical surface $\rho = \rho_j$, the less the conditions differ $(7a)$ and, $(7b)$ accordingly, from the conditions

$$\left.\begin{array}{l} a)\ \dfrac{\partial T}{\partial \rho} = 0,\ u = 0,\ \tau_{z\rho} = 0,\ \tau_{\alpha\rho} = 0,\ D_\rho = 0,\ u \\ b)\ T = 0,\ \sigma_r = 0,\ \ \mathrm{v} = 0,\ \ \mathrm{w} = 0,\ \ \varphi = 0. \end{array}\right\} \quad (12)$$

at $\rho = \rho_j$. The conditions $(7a)$ and $(7b)$ are equivalent to the conditions $(12a)$ and $(12b)$ when $\rho = \rho_j$ is a plane. Everything is the same for surface $\alpha = \alpha_j$ and conditions (8).

According to operation [10] in a thermally homogeneous medium, when $\lambda_1 = const$ and, $\lambda_2 = const$ the thermal conductivity equation (5) takes the form

$$\Delta_2 T + \lambda_0 \frac{\partial^2 T}{\partial \mathrm{z}^2} = 0 \quad (13)$$

where $\lambda_0 = \lambda_1 / \lambda_2$ In this case, using the method of separating variables, the function T in the domain $\Pi = \{\rho_0 < \rho < \rho_1, \alpha_0 < \alpha < \alpha_1,\ 0 < z < \mathrm{z}_1,\}$ can be represented as follows:

$$T = t_0 + t_1 z + \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(A_{Tmn}e^{-p_r z} + B_{Tmn}e^{p_r(z-z_1)}\right)\psi_{mn}(\rho,\alpha), \tag{14}$$

where $t_0, t_1, p_r = \lambda_0^{-0,5} p(m,n) \geq 0$, A_{Tmn}, B_{Tmn} is permanent; $\psi_{mn}(\rho,\alpha)$ is a nontrivial solution to the following Sturm-Liouville problem [9].

$$\Delta_2\psi_{mn} + p^2\psi_{mn} = 0, \tag{15}$$

$$\text{at } \rho = \rho_j:\ a)\ \psi_{mn}(\rho,\alpha) = 0, or\ \ b)\ \frac{\partial\psi_{mn}}{\partial\rho} = 0; \tag{16}$$

$$\text{at } \alpha = \alpha_j:\ a)\ \psi_{mn}(\rho,\alpha) = 0, or\ \ b)\ \frac{\partial\psi_{mn}}{\partial\alpha} = 0; \tag{17}$$

Conditions (16) and (17) follow from conditions (7) and (8). Note that in a Cartesian coordinate system, x, y, z the function ψ_{mn} is the product of a trigonometric function; in the case of circular cylindrical coordinates r, α, z, the function ψ_{mn} is the product of a trigonometric function and a Bessel function; for a cylindrical-elliptic system ψ_{mn}, the product of the Mathieu function, and for a cylindrical-parabolic coordinate system, the product of the Weber function.

Further, in a thermally homogeneous medium, we will assume

$$T = \frac{\partial^2\tilde{T}}{\partial z^2}, \tag{18}$$

where

$$\tilde{T} = \tilde{T}_0 + \tilde{T}_1 = \frac{t_0}{2}\left(z^2 - \frac{\lambda_0}{2}r^2\right) + \frac{t_1}{6}\left(z^3 - \frac{3\lambda_0}{2}zr^2\right) + \\ + \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{p_T^2}\left(A_{Tmn}e^{-p_T z} + B_{Tmn}e^{p_T(z-z_1)}\right)\psi_{mn}(\rho,\alpha). \tag{19}$$

The function $\tilde{T}$ satisfies the same equation *T*. In the expression for $\tilde{T}$, $\tilde{T}_0$ is the polynomial part $\tilde{T}$ (terms with coefficients t_0, t_1), and $\tilde{T}_1$ the remaining part $\tilde{T}$. For the convenience of constructing boundary value problems, conditions (11) are replaced, respectively, by the following conditions

$$\text{at } z = z_j \quad \left.\begin{array}{ll} a)\ \sigma_z = F_{i1}(r,\alpha),\ \Gamma_1(h\tau_{zr}, h\tau_{z\alpha}) = \tilde{F}_{i2}(r,\alpha), & \\ \Gamma_2(h\tau_{zr}, h\tau_{z\alpha}) = \tilde{F}_{i3}(r,\alpha), & or \\ b)\ \mathrm{w} = f_{i1}(r,\alpha),\ \Gamma_1(h\mathrm{u}, h\mathrm{v}) = \tilde{f}_{i2}(r,\alpha), & \\ \Gamma_2(h\mathrm{u}, h\mathrm{v}) = \tilde{f}_{i3}(r,\alpha), & or \\ c)\ \mathrm{w} = f_{i1}(r,\alpha),\ \Gamma_1(h\tau_{zr}, h\tau_{z\alpha}) = \tilde{F}_{i2}(r,\alpha), & \\ \Gamma_2(h\tau_{zr}, h\tau_{z\alpha}) = \tilde{F}_{i3}(r,\alpha), & or \\ d)\ \sigma_z = F_{i1}(r,\alpha),\ \Gamma_1(h\mathrm{u}, h\mathrm{v}) = \tilde{f}_{i2}(r,\alpha), & \\ \Gamma_2(h\mathrm{u}, h\mathrm{v}) = \tilde{f}_{i3}(r,\alpha), & \end{array}\right\} \tag{20}$$

where

$$\Gamma_1(g_1, g_2) = \frac{1}{h^2}\left(\frac{\partial g_1}{\partial\rho} + \frac{\partial g_2}{\partial\alpha}\right), \Gamma_2(g_2, g_1) = \frac{1}{h^2}\left(\frac{\partial g_2}{\partial\rho} - \frac{\partial g_1}{\partial\alpha}\right),$$

at that $g_1 = h\tau_{z\rho}$ or

where

$$\Gamma_1(g_1,g_2)=\frac{1}{h^2}\left(\frac{\partial g_1}{\partial\rho}+\frac{\partial g_2}{\partial\alpha}\right),\Gamma_2(g_2,g_1)=\frac{1}{h^2}\left(\frac{\partial g_2}{\partial\rho}-\frac{\partial g_1}{\partial\alpha}\right),$$

at that $g_1 = h\tau_{z\rho}$ or $g_1 = hu$, $g_2 = h\tau_{z\alpha}$ or $g_2 = h\text{v}$. We assume that the functions $\tilde{F}_{i2}(r,\alpha)$ and $\tilde{F}_{i3}(r,\alpha)$ itself function $F_{i1}(r,\alpha)$ together with their first and second derivatives, they decompose the problems (15–17).

The decomposition by functions ψ_{mn} can be considered valid, at least formally, and in the case when in equation (15) the variables are not separated, for example, cylindrical-bipolar coordinates. Now taking into account the matching conditions on the edges of the CCP, it can be argued that the boundary conditions (11) and (20) will be equivalent if in the domain $\Pi=\{\rho_0<\rho<\rho_1,\alpha_0<\alpha<\alpha_1\}$ the following boundary terms have only a trivial (zero) solution

$$\frac{\partial g_1}{\partial\rho}+\frac{\partial g_2}{\partial\alpha}=0\,,\ \frac{\partial g_2}{\partial\rho}-\frac{\partial g_1}{\partial\alpha}=0, \qquad (21)$$

$$\text{at } \rho=\rho_j:\ a)\ g_2=0,\ or\ \frac{\partial g_1}{\partial\rho}=0\ or\ b)\ g_1=0,\ or\ \frac{\partial g_2}{\partial\rho}=0; \qquad (22)$$

$$\text{at } \alpha=\alpha_j:\ a)\ g_1=0,\ or\ \frac{\partial g_2}{\partial\rho}=0\ or\ b)\ g_2=0,\ or\ \frac{\partial g_1}{\partial\rho}=0. \qquad (23)$$

According to the Keldysh-Sedov theorem [9], the boundary value problem (21–23), except problems (21, 22a, 23b) and (21), (22b, 23a), has a solution

$$g_1=0,\ \ g_2=0$$

The boundary problem (21), (22a), (23b) has a solution

$$g_1=g_{10}=const,\ \ g_2=0, \qquad (24)$$

a boundary problem (21), (22b), (23a) solution

$$g_1=0,\quad g_2=g_{20}=const. \qquad (25)$$

As we can see, boundary problems (21, 22a, 23b) and (21, 22b, 23a) have non-zero solutions. To overcome the problem that has arisen, to solve the boundary value problems (5, 6, 7a, 8b, 9, 10, 20) a solution is being added [13–16]

$$hu=0,\quad \text{w}=0, h\text{v}=b_1+b_2l_z. \qquad (26)$$

to boundary problems (5, 6, 7b, 8a, 9, 10, 20) there is a solution:

$$h\text{v}=0,\quad \text{w}=0, h\text{v}=b_3+b_4l_z.$$

where $l_z=c_4^{-1}z$ b_1,b_2,b_3,b_4 is permanent.

Let us now use equations (6) and (6'). From (6 b, c, d) follows:

$$\Delta_2 B+\frac{c_4}{c_5}\frac{\partial^2 B}{\partial z^2}=0 \qquad (27)$$

Before going further, it is necessary to make the following remark of material significance regarding the boundary conditions (7–8). From these conditions, it follows that on the side surfaces $\rho=\rho_j$ and $\alpha=\alpha_j$ t CCP, the function B itself or its normal derivative equals to zero. As for surfaces, it follows from (10)

$$\Gamma_2(h\tau_{zr},h\,\tau_{z\alpha})=\frac{c_4}{c}\frac{\partial B}{\partial z},$$

$$\Gamma_1(h\text{v},h\,u)=\frac{1}{c_5}B$$

Thus, to determine the function B, we get the classical problem of mathematical physics. It is necessary to determine the function B from equation (27), when either on boundary surfaces are given the function B itself or its normal derivative, or the function itself is specified on the partial boundary surfaces and its normal derivative is specified on the remaining part [17–20].

Using the method of separating variables, we present the functions B as

$$B=b_{10}+b_{12}l_z+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}B_{mn}(z)\psi_{mn}(\rho,\alpha). \qquad (28)$$

where b_{10},b_{12} constants $B_{mn}(z)$ are solving the equation

$$\frac{c_4}{c_5}\frac{d^2B_{mn}}{dz^2}-p_1^2B_{mn}=0,$$

where $p_1=p_1(m,n)$. $\psi_{mn}(\rho,\alpha)$—solving the problem (15–17). From the condition

$$\int_{\rho_0}^{\rho_1}\int_{\alpha_0}^{\alpha_1} Bh^2 d\rho d\alpha = 0$$

it follows that $b_{10} = 0, b_{12} = 0$ and final for the considered class of boundary problems

$$B = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn}(z)\psi_{mn}(\rho,\alpha). \qquad (29)$$

Without limiting the generality, we present the function B in the following form:

$$B = \frac{c_4}{c_5}\frac{\partial^2 \Psi_0}{\partial z^2} \qquad (30)$$

where

$$\Delta_2\psi_0 + \frac{c_4}{c_5}\frac{\partial^2\psi_0}{\partial z^2} = 0 \qquad (31)$$

and taking into account (29)

$$\psi_0 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\psi_{0mn}(z)\psi_{mn}(\rho,\alpha), \qquad (32)$$

where ψ_{0mn} is solving the equation

$$\frac{c_4}{c_5}\frac{d^2\psi_{0mn}}{dz^2} - p_1^2\psi_{0mn} = 0.$$

Theorem. For the considered class of boundary problems of thermo-electro elasticity, the general solution in the class of regular functions is represented a

$$\begin{aligned}
\mathrm{w} &= -\frac{\partial}{\partial z}\left(\Psi_2 + \frac{1}{2c_4}\Psi_1\right) + \frac{1}{c_4}\frac{\partial\Psi_1}{\partial z} - \frac{e_3}{c_4}\varphi,\\
h\mathrm{v} &= \frac{\partial}{\partial\alpha}\left(\Psi_2 + \frac{1}{2c_4}\Psi_1\right) - \frac{1}{c_5}\frac{\partial\Psi_0}{\partial\rho},\\
hu &= \frac{\partial}{\partial\rho}\left(\Psi_2 + \frac{1}{2c_4}\Psi_1\right) + \frac{1}{c}\frac{\partial\Psi_0}{\partial\alpha}.
\end{aligned} \qquad (33)$$

Here

$$\left.\begin{aligned}
&a)\Delta_2\Psi_0 + \frac{c_4}{c_5}\frac{\partial^2\Psi_0}{\partial z^2} = 0\\
&b)\Delta_2\Psi_1 + G_1\frac{\partial^2\Psi_1}{\partial z^2} - G_2\frac{\partial^2\Psi_2}{\partial z^2} - G_3\frac{\partial\varphi}{\partial z} - G_4T = 0\\
&\mathrm{c})\Delta_2\Psi_1 + G_5\frac{\partial^2\Psi_1}{\partial z_2} + G_6\Delta_2\Psi_2 - \frac{2c_3c_4}{c_1}\frac{\partial^2\Psi_2}{\partial z^2} - G_7\frac{\partial\varphi}{\partial z} - G_8T = 0\\
&d)\frac{\text{Э}_1\ c_4 + e_3}{c_4}\Delta_2\varphi + \frac{\text{Э}_1\ c_4 + e_2e_3}{c_4}\frac{\partial^2\varphi_2}{\partial z^2} = e_1\Delta_2\frac{\partial\Psi_2}{\partial z} - e_2\frac{\partial^3\Psi_2}{\partial z^3} +\\
&+\frac{e_1 + 2e_3}{c_4}\Delta_2\frac{\partial\Psi_1}{\partial z} + \frac{e_2}{c_4}\frac{\partial^3\Psi_1}{\partial z^3}
\end{aligned}\right\} \qquad (34)$$

where

$$G_1 = \frac{c_1c_2 - c_3^2 - 2c_3c_4}{2c_1c_4}, \mathrm{S}\, G_2 = \frac{c_1c_2 - c_3^2}{2c_1c_4},\ G_3 = \frac{\left(c_1c_2 - c_3^2\right)_3 + \left(c_3e_1 - c_1e_2\right)c_4}{c_1c_4}$$

$$G_4 = \frac{c_1\beta_{20} - c_3\beta_{10}}{c_1}, G_5 = \frac{c_3 + 2c_4}{2c_1c_4}, G_6 = 2c_4,\ G_7 = \frac{2\left(c_3e_3 - c_4e_1\right)}{c_1},\ G_8 = \frac{2c_4\beta_{10}}{c_1}.$$

Note. We do not give a proof of the theorem. The obtained general solutions will be used to solve boundary-contact problems for multilayer bodies.

3. Conclusions

Was shown that the condition of the constituent elements of telecommunications and radio-electronic complexes is largely determined by the trends of changes in the parameters of electro-radio components. During the monitoring and control of the specified parameters of radio-electronic equipment, one of the indicators that must be taken into account is thermoelectric elasticity. Information on models of thermo-electro elasticity of multi-layered structures, which form various basic elements of telecommunication and radio-electronic complexes, is necessary for determining the dynamics of changes in the technical condition of equipment under certain operating conditions. Thus, this paper presents a new very effective solution to boundary problems of thermo-electro elasticity in the generalized cylindrical coordinates, which can be used for determining the electromagnetic parameters of radio-electronic equipment of modern telecommunication systems [21–27].

4. References

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