High-order network analysis for financial crash identification*

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Abstract

Network analysis is a powerful method to characterize the complexity and dynamics of socio-economic systems. However, traditional network analysis often ignores the higher-order dependencies that arise from the interactions of more than two nodes. In this paper, we propose to use high-order networks, which are generalized network structures that capture the higher-order dependencies, to study the temporal evolution of the Dow Jones Industrial Average (DJIA) index. We construct high-order networks from the DJIA time series using the visibility graph method, and we measure the topological complexity of the high-order networks using various metrics. We find that the complexity of the system changes drastically during crisis events, indicating that high-order network analysis can be used as an indicator (indicator-precursor) of financial crashes. We also show that high-order network analysis and topology can provide more insights into the nonlinear and nonstationary behavior of the DJIA index than traditional tools of financial time series analysis.

Keywords

high-order network analysis, financial crash identification, complex networks, multiplex networks, visibility graph, indicator-precursor

1. Introduction

The proliferation of extensive and finely-grained data, often with temporal resolution, has unlocked unprecedented opportunities to dissect the behaviors of complex systems spanning diverse domains such as biology, technology, finance, and economics [2, 3, 4]. These intricate systems, comprising myriad interacting units, frequently exhibit emergent properties at

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macroscopic scales due to heterogeneous interactions among their constituents [5]. Complex networks have emerged as a formidable toolset to analyze the structures and dynamics of such systems [5]. However, the tools conventionally employed in network analysis often focus on interactions between pairs of nodes, a limitation at odds with the increasing availability of empirical data illustrating group interactions within heterogeneous systems [6]. Thus, it becomes evident that interactions within systems often extend beyond dyadic connections, manifesting as collective actions involving groups of nodes [7, 8], exerting a notable influence on the interacting systems' dynamics [9, 10].

The notion of higher-order interactions finds historical roots in solid-state physics, where multiparticle potentials and quantum mechanical calculations supplanted paired interactions. Similarly, in thermodynamics and statistical physics, Tsallis introduced nonextensive interactions [11, 12]. However, in contrast to these simpler representations of higher-order interactions, complexity in complex systems demands more intricate mathematical structures like hypergraphs and simplicial complexes.

Diverse models of higher-order networks have surfaced [13], reflecting the growing importance of this domain. Here, we briefly highlight key models that have garnered attention [14, 15, 16].

Multiplex Networks: Multiplex networks, multilayer networks, and networks of networks capture interactions between various entities and have found applicability in systems with diverse interaction types [17]. However, most interactions remain dyadic and can be represented through traditional networks [18]. Their application in financial analysis is well-documented [19, 20, 21, 22, 23, 24, 25, 26, 27, 20] alongside higher-order networks [28, 29, 30, 31, 32, 33, 34, 35].

Hypergraphs and Simplicial Complexes: Algebraic topology's computational techniques, hypergraphs, and simplicial complexes encode units and hyperlinks, allowing explicit consideration of systems beyond pairwise interactions [9, 36, 7, 37].

Higher-Order Markov Models: First-order Markov models have gained traction in describing flows of information, energy, money, etc. within networks [38]. However, many flows exhibit path-dependent behaviors, necessitating higher-order Markov chain models [16].

Higher-Order Graphical Models and Markov Random Fields: Markov random fields, including the Ising model, extended to higher-order models, capture interactions between multiple objects [39, 40].

Recently, Santoro et al. [36] introduced a structure to characterize instantaneously co-fluctuating [41] signal patterns of all interaction orders. They showcased that higher-order measures discern subtleties in space-time regimes in diverse studies: brain activity, stock option prices, and epidemics. In this context, we explore the application of multiplex and higher-order network techniques to model crisis states in the stock market. Section 2 introduces a graph representation based on the visibility graph, while Section 3 presents multiplex networks' theory, including measures. Section 4 elaborates on higher-order networks and encoding methods, describing measures for both classical and high-order networks. Empirical results, including a comparative analysis of measures, are presented in Section 5. Finally, Section 6 outlines our conclusions and future directions.

2. Visibility graph

Visibility graph (VG), which was proposed by Lacasa et al. [42] is typically constructed from a univariate time series. In a visibility graph, each moment in the time series maps to a node in the network, and an edge exists between the nodes if they satisfy a "mutual visibility" condition.

"Mutual visibility" can be understood by imagining two points x_i at time t_i and x_j at time t_j as two hills of a time series, which can be understood as a landscape, and these two points are "mutually visible" if x_i has no any obstacles in the way on x_j. Formally, two points are mutually visible if, all values of x_k between t_i and t_j satisfy:

$$x_k < x_i + \frac{t_k - t_i}{t_j - t_i} \left[x_j - x_i \right], \quad \forall k : i < k < j \tag{1}$$

Horizontal visibility graph (HVG) [43] is a restriction of usual visibility graph, where two points x_i and x_j are connected if there can be drawn a *horizontal* path that does not intersect an intermediate point x_k, $i < k < j$. Equivalently, node x_i at time t_i and node x_j at time t_j are connected if the horizontal ordering criterion is fulfilled:

$$x_k < \inf(x_i, x_j), \quad \forall k : i < k < j. \tag{2}$$

Figure 1 is an approximate illustration of the construction of visibility graphs.

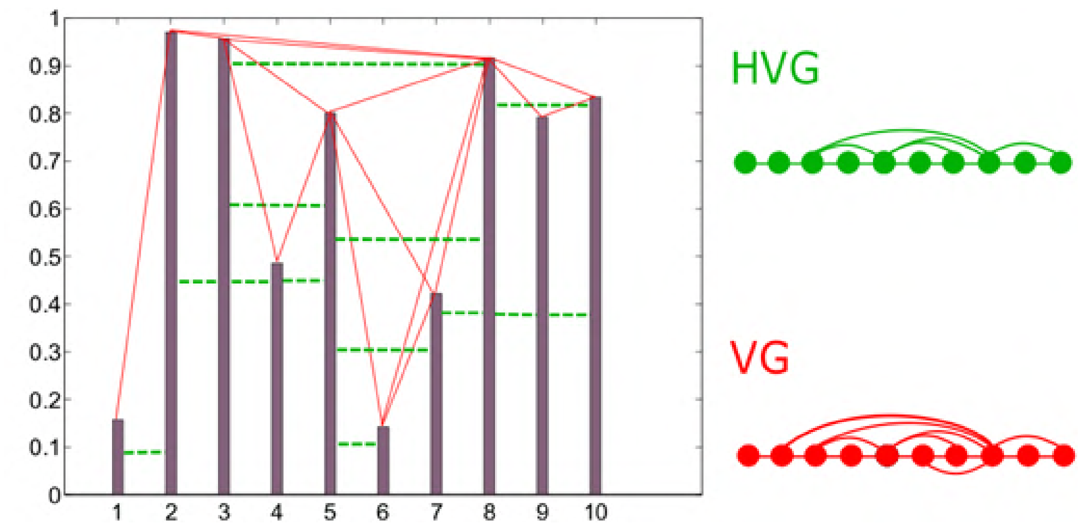

Figure 1: Schematic illustration of the VG (red lines) and the HVG (green lines). Adapted from [44].

3. Multiplex orderness and measures of complexity

Multiplex network [45] is the representation of the system which consists of the variety of different subnetworks with inter-network connections. For working with multiplex financial networks, we set two tasks:

- convert separated time series into network that represent a layer of a multiplex network. The procedure of conversion is presented in section 2;
- create intra-layer connection between each subnetwork.

Figure 2 represents an algorithm for creating a three-layered multiplex visibility graph.

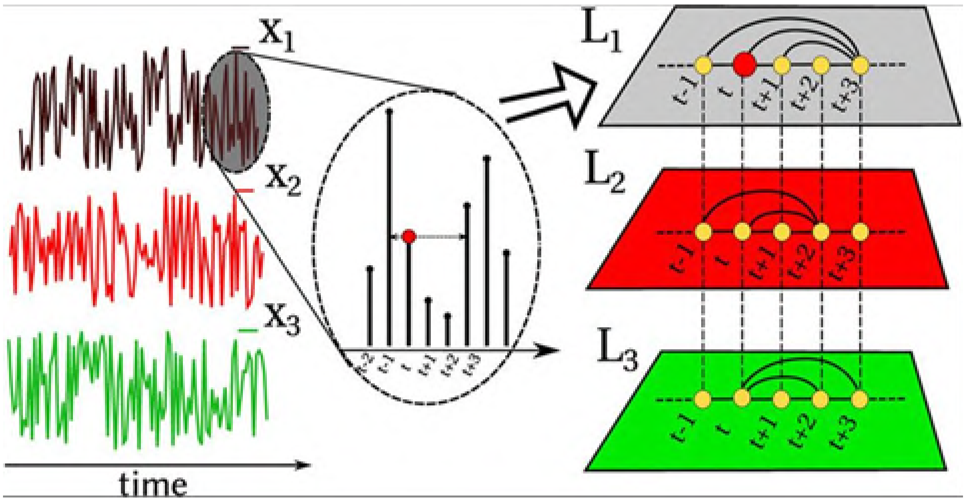

Figure 2: Illustration of the multiplex VG formation on the example of three layers. Adapted from [46].

Multiplex network is the representation of a pair $M = (G, C)$, where $\{G_\alpha \,|\, \alpha \in 1, \dots, M\}$ is a set of graphs $G_\alpha = (X_\alpha, E_\alpha)$ that called layers and

$$C = \left\{E_{\alpha\beta} \subseteq X_\alpha \times X_\beta \,|\, \alpha, \beta \in 1, \dots, M, \alpha \neq \beta\right\} \tag{3}$$

is a set of intra-links in layers G_α and G_β ($\alpha \neq \beta$). E_α is intra-layer edge in M, and each $E_{\alpha\beta}$ is denoted as inter-layer edge.

A set of nodes in a layer G_α is denoted as $X_\alpha = \left\{x_1^\alpha, \dots, x_{N_\alpha}^\alpha\right\}$, and an intra-layer adjacency matrix as $A^{[\alpha]} = (a_{ij}^\alpha) \in \mathrm{Re}^{N_\alpha \times N_\alpha}$, where

$$\alpha_{ij}^\alpha = \begin{cases} 1, & (x_i^\alpha, x_j^\alpha) \in E_\alpha, \\ 0. & \end{cases} \tag{4}$$

for $1 \leq i \leq N_\alpha$, $1 \leq j \leq N_\beta$ and $1 \leq \alpha \leq M$. For an inter-layer adjacency matrix, we have $A^{[\alpha,\beta]}(a_{ij}^{\alpha\beta}) \in \mathrm{Re}^{N_\alpha \times N_\beta}$, where

$$\alpha_{ij}^{\alpha\beta} = \begin{cases} 1, & (x_i^\alpha, x_j^\beta) \in E_{\alpha\beta}, \\ 0. & \end{cases} \tag{5}$$

A multiplex network is a partial case of inter-layer networks, and it contains a fixed number of nodes connected by different types of links. Multiplex networks are characterized by correlations of different nature, which enable the introduction of additional multiplexes.

For a multiplex network, the node degree k is already a vector

$$k_i = (k_i^{[1]}, \dots, k_i^{[M]}), \tag{6}$$

with the degree $k_i^{[\alpha]}$ of the node i in the layer α, namely

$$k_i^{[\alpha]} = \sum_j a_{ij}^{[\alpha]}, \tag{7}$$

while $a_{ij}^{[\alpha]}$ is the element of the adjacency matrix of the layer α. Specificity of the node degree in vector form allows describing additional quantities. One of them is the *overlapping degree* of node i:

$$o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]}. \tag{8}$$

The next measure quantitatively describes the inter-layer information flow. For a given pair (α, β) within M layers and the degree distributions $P(k^{[\alpha]}), P(k^{[\beta]})$ of these layers, we can defined the so-called *interlayer mutual information*:

$$I_{\alpha,\beta} = \sum \sum P(k^{[\alpha]}, k^{[\beta]}) \log \frac{P(k^{[\alpha]}, k^{[\beta]})}{P(k^{[\alpha]}), P(k^{[\beta]})}, \tag{9}$$

where $P(k^{[\alpha]}, k^{[\beta]})$ is the joint probability of finding a node degree $k^{[\alpha]}$ in a layer α and a degree $k^{[\beta]}$ in a layer β. The higher the value of $I_{\alpha,\beta}$, the more correlated (or anti-correlated) is the degree distribution of the two layers and, consequently, the structure of a time series associated with them. We also find the mean value of $I_{\alpha,\beta}$ for all possible pairs of layers – the scalar $\langle I_{\alpha,\beta} \rangle$ that quantifies the information flow in the system.

The *multiplex degree entropy* is another multiplex measure which quantitatively describes the distribution of a node degree i between different layers. It can be defined as

$$S_i = -\sum_{\alpha=1}^{M} \frac{k_i^{[\alpha]}}{o_i} \log \frac{k_i^{[\alpha]}}{o_i}. \tag{10}$$

Entropy is close to zero if ith node degree is within one special layer of a multiplex network, and it has the maximum value when ith node degree is uniformly distributed between different layers.

4. High-order extension of temporal networks

4.1. Time-respecting paths

Financial networks are strongly influenced by the ordering and timing of links. In their context of their temporality, we must consider *time-respecting paths*, an extension of the concept of paths in static network topologies which additionally respects the timing and ordering of time-stamped links [47, 48, 49]. For a source node v and a target node w, a time-respecting path can be presented by any sequence of time-stamped links

$$(v_0, v_1; t_1), (v_1, v_2; t_2), ..., (v_{l-1}, v_l; t_l), \tag{11}$$

where $v_0 = v, v_l = w$ and $t_1 < t_2 < ... < t_l$. Time ordering of temporal financial networks is important since it implies causality, i.e. a node i is able to influence node j relying on two time-stamped links (i, k) and (k, j) only if edge (i, k) has occurred before edge (k, j).

Apart the restriction on networks to have the correct ordering, it is common to impose a maximum time difference between consecutive edges [50], i.e. there is a maximum time difference δ and, example, two time-stamped edges $(i, k; t)$ and $(k, j; t')$ that contribute to a time-respecting path if $0 \leq t' - t \leq \delta$. If $\delta = 1$, we are usually interested in paths with short time scales. For $\delta = \infty$, we impose no restrictions on time-range and consider a path definition where links can be weeks or years apart.

4.2. High-order networks

The key idea behind this abstraction is that the commonly used time-aggregated network is the simplest possible time-aggregated representation, whose weighted links capture the frequencies of time-stamped links. Considering that each time-stamped link is a time-respecting path of length one, it is easy to generalize this abstraction to higher-order time-aggregate networks in which weighted links capture the frequencies of longer time-respecting paths.

There are several variants for encoding high-order interactions [10]. The first concept of high-order links represent *hyperlink*, which can contain any number of nodes. *Hypergraph* is the generalized notion of network which is composed of nodeset V and hyper-edges E that specify which nodes from V participate in which way.

Simplex is another mathematical abstraction to accomplish high-order interaction. Formally, a *k-simplex* σ is a set of $k + 1$ fully interacting nodes $\sigma = [v_0, v_1, ..., v_k]$. Essentially, a node is 0-simplex, a link is 1-simplex, a triangle is 2-simplex, a tetrahedron is 3-simplex, etc. Since a standard graph is a collection of edges, *simplicial complexes* are collections of simplices $K = \{\sigma_0, \sigma_1, ..., \sigma_n\}$.

Figure 3 demonstrates examples of simplices and hyperlinks of orders 1, 2, and 3.

For a temporal network $G^T = (V^T, E^T)$ we thus formally define a kth order time-aggregated (or simply aggregate) network as a tuple $G^{(k)} = (V^{(k)}, E^{(k)})$ where $V^{(k)} \subseteq V^k$ is a set of node k-tuples and $E^{(k)} \subseteq V^{(k)} \times V^{(k)}$ is a set of links. For simplicity, we call each of the k-tuples $v = v_1 - v_2 - ... - v_k$ ($v \in V^{(k)}, v_i \in V$) a kth order node, while each link $e \in E^{(k)}$ is called a kth order link. Between two kth order nodes v and w exists kth order edge (v, w) if they overlap in exactly $k - 1$ elements. Resembling so-called De Bruijn graphs [51], the basic idea behind this construction is that each kth order link represents a possible time-respecting path of length k in the underlying temporal network, which connects node v_1 to node w_k via k time-stamped links

$$(v_1, v_2 = w_1; t_1), ..., (v_k = w_{k-1}, w_k; t_k). \tag{12}$$

Importantly, and different from a first-order representation, kth order aggregate networks allow to capture *non-Markovian* characteristics of temporal networks. In particular, they allow to represent temporal networks in which the kth time-stamped link $(v_k = w_{k-1}, w_k)$ on a time-respecting path depends on the $k - 1$ previous time-stamped links on this path. With this, we obtain a simple static network topology that contains information both on the presence of time-stamped links in the underlying temporal network, as well as on the ordering in which sequences of k of these time-stamped links occur.

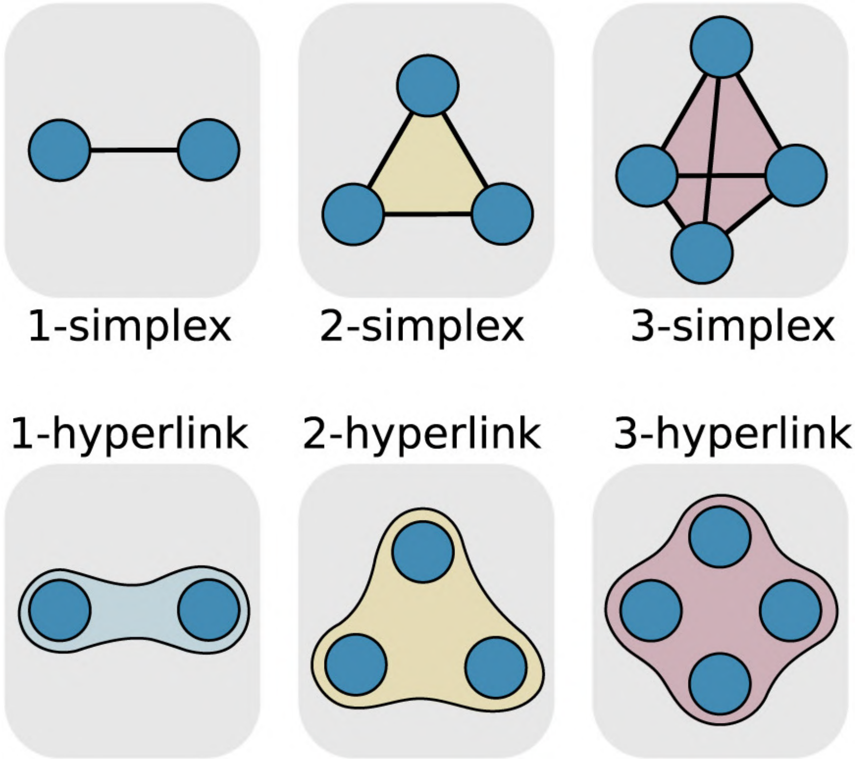

Figure 3: High-order connections in terms of simplices and hyperlinks. Adapted from [9].

4.3. Degree centrality

Network centralities are node-related measures that quantify how "central" a node is in a network. There are many ways in which a node can be considered so: for example, it can be central if it is connected to many other nodes (degree centrality), or relatively to its connectivity to the rest of the network (path based centralities, eigenvector centrality). One of the simplest centrality measure is the *degree of a node*, which counts the number of edges incident to an ith node.

For any adjacency matrix the degree of a node i can be defined as

$$D_i = \sum_j A_{ij}. \tag{13}$$

High-order degree centrality counts the number of kth-order edges incident to the kth-order node i. To get a scalar value which will serve as an indicator of high-order dynamics, we obtain mean degree D_{mean}:

$$D_{mean} = \frac{1}{N} \sum_{i=1}^{N} D_i. \tag{14}$$

Except this measure, we can calculate nth moment of the degree distribution, which can be defined as

$$\langle k^n \rangle = \sum_{k_{\min}}^{\infty} k^n p_k \approx \int_{k_{\min}}^{\infty} k^n p_k dk. \tag{15}$$

In this study we will present the dynamics of the first moment, which is the mean weighted degree of a network, and its high-order behavior.

4.4. Assortativity coefficient

Assortativity is a property of network nodes that characterizes the degree of connectivity between them. Many networks demonstrate "assortative mixing" on their nodes, when high-degree nodes tend to be connected to other high-degree nodes. Other networks demonstrate disassortative mixing when their high-degree nodes tend to be connected to low-degree nodes. Assortativity of a network can be defined via the Pearson correlation coefficient of the degrees at either ends of an edge. For an observed network, we can write it as

$$r = \frac{M^{-1} \sum_i j_i k_i - \left[M^{-1} \sum_i \frac{1}{2} (j_i + k_i) \right]^2}{M^{-1} \sum_i \frac{1}{2} \left(j_i^2 + k_i^2 \right) - \left[M^{-1} \sum_i \frac{1}{2} (j_i + k_i) \right]^2}, \tag{16}$$

where $-1 \leq r \leq 1$; j_i, k_i are the degrees of the nodes at the ends of the ith edge, with $i = 1, ..., M$, where M is the number of edges of a network.

This correlation function is zero for no assortative mixing. If $r = 1$, then we have perfect assortative mixing pattern. For $r = -1$, we can observe perfect disassortativity.

Studying financial networks, with time-respecting paths, we can consider four type of assortativity: $r(in, in), r(in, out), r(out, in), r(out, out)$, which will correspond to tendencies to have similar in and out degrees. We can denote one of the studied in/out pairs as (α, β). Suppose, for a given ith edge, we have got the source (i.e. tail) node of the edge and target (i.e. head) node of the edge. We can denote them as α-degree of the source (j_i^{α}) and β-degree of the target (k_i^{β}). Assortativity coefficient for degrees of a specific type can be defined as

$$r(\alpha, \beta) = \frac{\sum_i \left(j_i^{\alpha} - \overline{j^{\alpha}} \right) \left(k_i^{\beta} - \overline{k^{\beta}} \right)}{\sqrt{\sum_i \left(j_i^{\alpha} - \overline{j^{\alpha}} \right)^2} \sqrt{\sum_i \left(k_i^{\beta} - \overline{k^{\beta}} \right)^2}}, \tag{17}$$

where $\overline{j^{\alpha}}$ and $\overline{k^{\beta}}$ are the average α-degree of sources and β-degree of targets.

5. Empirical results

To build indicators (indicators-precursors) based on multiplex and high-order networks, the following is done:

- databases of 6 most influential stock market indices for the period from 02.01.2004 to 18.10.2022 were selected for multiplex analysis (see figure 4). The data were extracted using Yahoo! Finance API based on Python programming language [52];
- the indicators described in the previous sections were calculated using the sliding window procedure [12, 53, 54, 55, 56, 57, 58]. The essence of this procedure is that: (1) a fragment (window) of a series of a certain length w was selected; (2) a network measure was calculated for it; (3) the measure values were stored in a pre-declared array; (4) the window was shifted by a predefined time step h, and the procedure was repeated until the series was completely exhausted; (5) further, the calculated values of the network measure were compared with the dynamics of the stock index. Subsequently, conclusions were drawn regarding the further dynamics of the market. In our case, window length $w = 500$ days and time step $h = 10$ day. The choice of step was limited by the counting time for high-order networks;
- multiplex and high-order indicators are compared with the Dow Jones Industrial Average (DJIA) index.

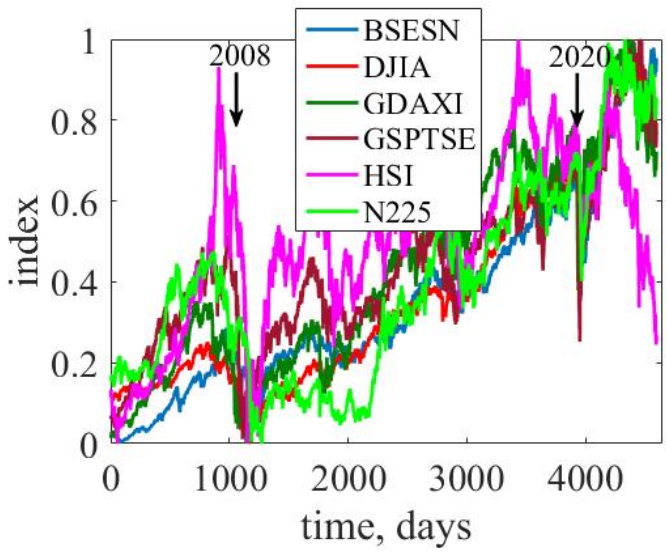

Figure 4: The dynamics of stock market indices for studying multiplex characteristics.

In figure 5 presented the dynamics of inter-layer mutual information (I) and multiplex degree entropy (S) along with the DJIA index.

From figure 5 we can see that multiplex mutual information increases before the crisis of 2008. Also, it noticeably becomes higher before COVID-19 crash. For the last months, it demonstrates decreasing pattern, which indicates that the economies of different countries may be experiencing different evolutions now. Nevertheless, it can be seen that, as a rule, this indicator is characterized by growth, indicating an increase in the interconnection of the economies of different countries. In a crisis, this indicator usually declines, demonstrating different resistance to the collapse events of the stock markets of countries and the difference in the actions that they take. Entropy indicator shows asymmetric behavior.

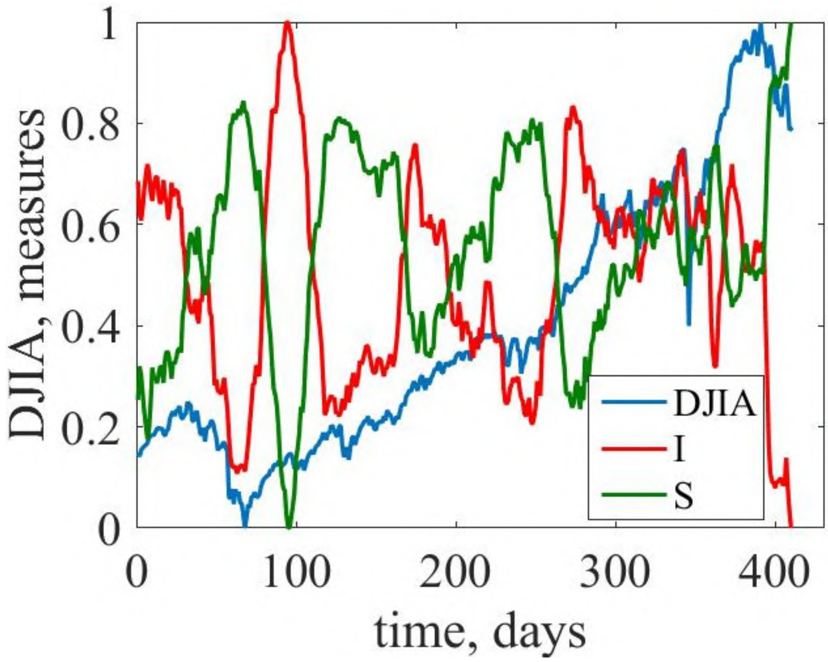

Figure 5: The dynamics of inter-layer mutual information (I) and multiplex degree entropy (S) along with the DJIA index.

Next, we compare one of the multiplex measure, overlapping degree (o), with the mean degree of a network (D_{mean}). Figure 6 represents this result.

In figure 6 we can see that both D_{mean} and o are characterized by similar dynamics. These indicators increase near the crash, which indicates an increase in the concentration of connections for some network nodes, and further, based on the indicators during the crisis, there is a decline in concentration both in the dynamics of the DJIA and the inter-layer connectedness of stock indices. We may see that the multiplex approach does not significantly change the dynamics of the concentration degree indicator in comparison with the indicator based on the classical univariate graph.

Figure 7 demonstrates the dynamics of mean weighted degree (equation (15)) for order 1 and 2 along with the DJIA index.

In figure 7 we can see that the second-order D_{mean} is slightly different from the first-order one. The second-order D_{mean} starts to increase a slightly earlier before the crisis of 2008. We can see that before crisis of 2020 second-order D_{mean} declines more noticeably comparing to the first-order one. However, this difference between the first and second order is still insignificant, what can we say about the fact that the classical visibility graph can reflect all the information that the series under study can represent.

Next, let us present high-order dynamics of the assortativity coefficient for the DJIA index (see figure 8).

Figure 8 presents the assortativity coefficient for first, second, and third orders. Assortativity declines before crashes and increases during them. We see that high-orderness does not change

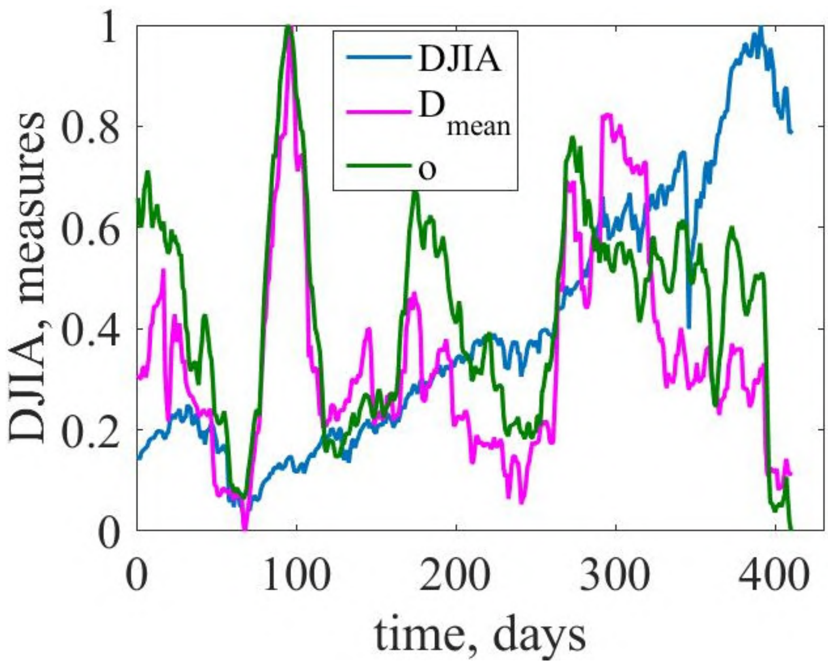

Figure 6: The dynamics of the mean degree (D_{mean}) and overlapping degree (o) along with the DJIA index.

radically change the dynamics of this indicator. Third-order assortativity responds better for the crash of 2008, but worse for the COVID-19 crisis, comparing to first- and second-order assortativity.

6. Conclusions

In this article, we have introduced methods to measure and model systems with causal, multiplex, and high-order interactions. We have shown that these methods can capture the long-range spatio-temporal correlations that characterize non-Markovian, non-stationary, non-linear systems, which are better described by the high-order paradigm. We have used hypergraphs [59, 60, 61] and simplicial complexes [62, 63, 64] as richer types of links that allow us to go beyond typical nodes and encode higher-order clusters and temporal dependencies.

We have presented indicators (indicators-precursors) based on classic visibility graphs, multiplex networks, and high-order networks. We have applied these indicators to the time series of the Dow Jones Industrial Average (DJIA) index and a database of six stock indices from different countries and sectors. We have used the sliding window algorithm to calculate various network measures, such as the mean degree of a node (D_{mean}), the first-moment degree of a network, the assortativity coefficient, the inter-layer mutual information (I), the multiplex degree entropy (S), and the mean overlapping degree of a network (o). We have found that multiplex and high-order networks do not differ significantly from the traditional pairwise visibility model in terms of their dynamics. This may suggest that the classical visibility graph reflects all possible

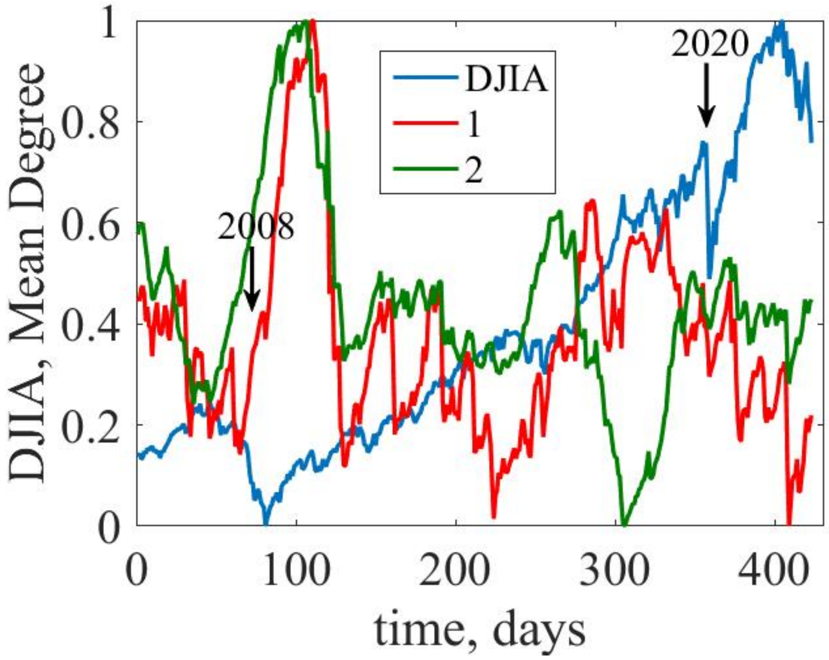

Figure 7: The dynamics of first- and second-order mean (weighted) degree along with the DJIA index.

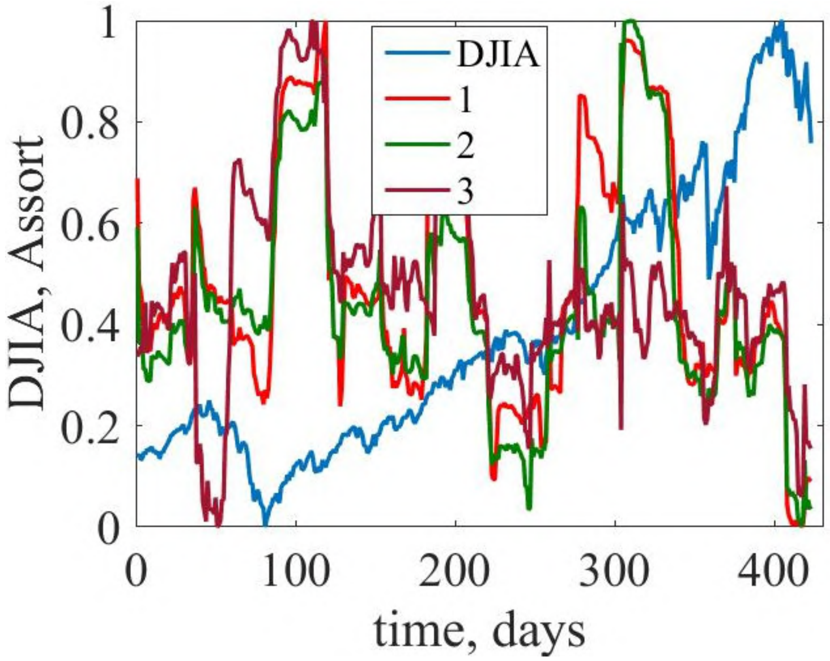

Figure 8: The dynamics of first-, second-, and third-order assortativity along with the DJIA index.

short-term and long-term dependencies in the DJIA index. We have also found that all the presented measures work similarly as indicators (indicators-precursors) of critical financial events, increasing or decreasing before and during them. However, multiplex and high-order network indicators still need further development and improvement for studying complex financial time series. A possible solution may be to combine Markov chains of multiple, higher orders into a multi-layer graphical model that captures temporal correlations in pathways at multiple length scales simultaneously [65]. Another perspective may be to use neuro-fuzzy forecasting and clustering methods of complex financial systems [66, 67, 68, 69, 70, 71, 72].

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