Analysis of the Application of Serial Connection of the Same Type Bandpass Filters in the Sensor Signal Processing Channel in the Information Control System

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Abstract

The paper is considered the problem of constructing system components with the possibility of rearranging their characteristics depending on the operating conditions. A new approach is proposed for calculating the parameters of a new connection with a series connection of low-order components of the same type. The resulting ratio allows you to accurately calculate the frequencies of the n-th connection of the same type components. In addition, such a connection allows increasing the rate of rise and falling of the fronts of the amplitude frequency characteristic and reducing the bandwidth by 3 times with eight connected filters of the same type. Bandpass filters and notch filters are considered. Behavior of the AFC for serial connection is described.

Keywords 1

Industry 4.0-5.0, frequency-dependent components, serial connection, amplitude frequency characteristic, phase frequency characteristic, bandpass filters, notch filters.

1. Introduction

When creating an information control system for mobile and robotic systems, it is necessary to solve a number of issues of sensor signals processing in real time and with limited computing resources.

To date, the general approach to building such systems is based on the concept of Industry 4.0. Computerization and informatization of many research processes and industrial production has led to the emergence of the Industrial Internet of Things (Industrial Internet of Things, IIoT) [1].

This direction allows you to significantly automate all processes by supplying equipment with multifunctional sensors, actuators and controllers. The collected data is processed in information management system, which allows quickly make reasoned and balanced decisions to adjust the functional activity of the object. But the maximum task is to reach a level where smart systems will work without the participation of people. The role of personnel in this case is reduced to monitoring the operation of systems and responding only to emergency situations to ensure safety and reliability [1-4]. The presence of wireless networks and cloud technologies contribute to the rapid collection of data which, after primary processing, are sent to the analysis and decision center. Further development of such systems goes in the direction of humanization of decision-making and friendly contact with a person in accordance with the concept of Industry 5.0 [4].

Such tasks are encountered when using various mobile objects, as well as in systems for resolving critical situations [5–9].

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To create such systems, it is necessary to have filters in the sensor signal processing channel that are capable of restructuring their characteristics by software or hardware, depending on the operating conditions, operating modes and the presence of interference to improve the efficiency of the system as a whole. The digital channel for sensor signals processing in mobile autonomous systems includes various filters. Basically, it is created on low-order filters, because this is due to the low cost of calculating the transfer function coefficients and the number of coefficients, the ease of setting or rebuilding the filter, moderate power consumption and processing time. For example, a first-order Butterworth digital bandpass filter, which is described by a second-order transfer function

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}, \quad (1)$$

in the numerator $a_2 = -a_0$, $a_1 = 0$. For the restructuring, it is necessary to calculate one coefficient in the numerator and two in the denominator. There are three coefficients. With the fourth order of the bandpass filter, the transfer function of which is already described by the eighth order

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} + b_7 z^{-7} + b_8 z^{-8}}. \quad (2)$$

In this transfer function, the denominator coefficients must be calculated all, and in the numerator the coefficients $a_1 = a_3 = a_5 = a_7 = 0$, and $a_0 = a_8$, $a_6 = -a_2$. For such filter implementation, eight coefficients in the denominator and two coefficients in the numerator must be calculated. It is necessary to calculate 10 coefficients. The amplitude frequency characteristics (AFC) is shown in Figure.1.

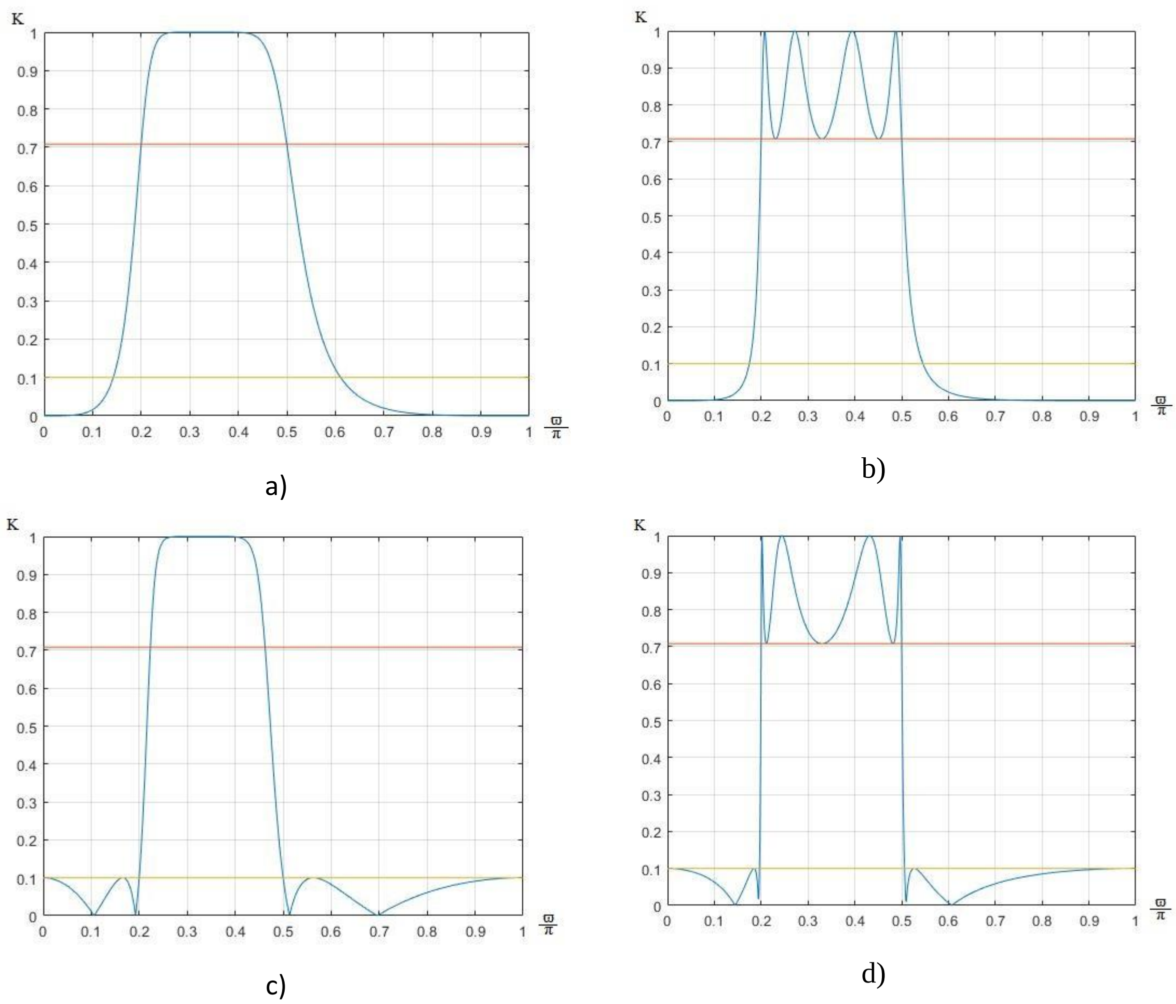


Figure 1: Amplitude frequency characteristics of digital filters of the fourth order Butterworth (a), Chebyshev of the first kind (b), Chebyshev of the second kind (c) and Elliptic (d)

As can be seen from Figure 1, fourth-order digital filters have steep edges, however, the presence of ripple in passband and stopband limits the widespread use of such filters.

In addition, when working in real time, there are restrictions on the amount and time of calculation, on the restructuring and duration of the transition process. It is known that the higher the order of the filter, the longer the transition process and the problem of filter stability arises, which is already associated with the word length of the representation of the filter coefficients and intermediate calculations.

In this regard, it is more convenient to use first-order digital filters, Figure 2. However, it should be noted that all first-order filters have the same amplitude frequency characteristic, but the steepness of the fronts of the amplitude frequency characteristic is low

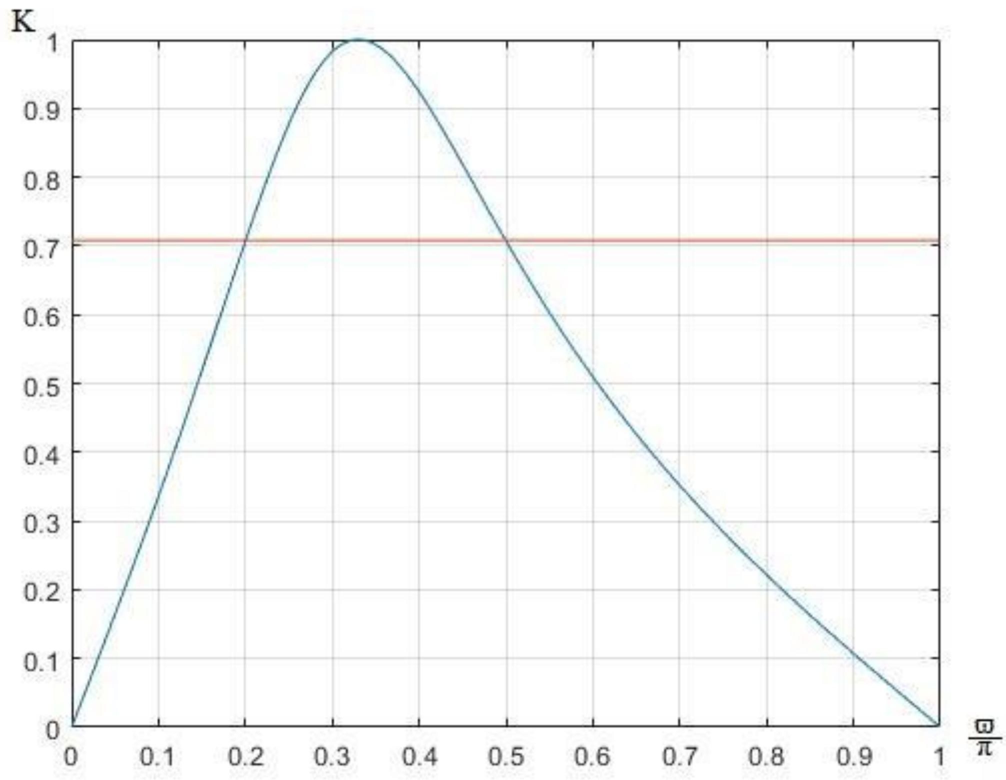


Figure 2: Amplitude frequency characteristic of a first-order digital Butterworth filter

In the sensor signal processing channel, typical tasks are - changing the cutoff frequencies and bandwidth, as well as the steepness of the fronts of the amplitude frequency characteristic.

Therefore, it is advisable to use a first-order filter (1) as a low-order filter. Formulas for tuning its cutoff frequencies and bandwidth are given [10, 11], to increase the steepness of the amplitude frequency characteristic, use a series connection of the same type of filters.

2. Series Connections of Same Type Digital Band Pass Filters and Its Effect on Frequency Characteristic

When transfer functions are connected in series, their transfer functions are multiplied

$$H(z) = \prod_{i=1}^{n} H_i(z), \tag{3}$$

where $H_i(z)$, $H(z)$ — i-th transfer functions and the transfer function of the serial connection, respectively.

Since the transfer functions are of the same type $H_o(z)$ and consist of amplitude frequency and phase frequency characteristics, can write

$$H_o(j\varpi) = H_o(\varpi) \cdot e^{j\phi_o(\varpi)}, \tag{4}$$

where $H_o(\varpi)$, $\phi_o(\varpi)$ — respectively, the amplitude frequency and phase frequency characteristics of the main filter of the same type, $\overline{\omega}$ — where is the normalized angular frequency, $\varpi = 2\pi \frac{f}{f_d}$, $\varpi \in [0, \pi]$, f, f_d, — line frequency and sample rate respectively.

Then, when similar filters are connected in series, their multiplication is converted into exponentiation

$$H(z) = \prod_{i=1}^{n} H_i(z) = [H_o(z)]^n, \tag{5}$$

and the amplitude frequency characteristic and phase frequency characteristic, respectively, are converted in this way

$$H(j\varpi) = [H_o(j\varpi)]^n = [H_o(\varpi)]^n \cdot e^{jn\phi_o(\varpi)}. \tag{6}$$

It can be seen from the last ratio that the main changes occur with the amplitude frequency characteristic.

It should be noted. that when the components are connected in parallel, their transfer functions add up.

$$H(z) = \sum_{i=1}^{n} H_i(z) = nH_o(z), \tag{7}$$

or for frequency characteristics

$$H(j\varpi) = nH_o(j\varpi) = nH_o(\varpi) \cdot e^{j\phi_o(\varpi)}. \tag{8}$$

Thus, when connecting the same type of components in parallel, the number of connected components is equivalent to multiplying by the gain factor.

In addition, when a component of the same type is included in negative feedback to the same component, the following transfer function is obtained

$$H(z) = \frac{H_o(z)}{1 + H_o^2(z)}. \tag{9}$$

With such a connection, the connection order increases, but the shape of the frequency characteristic undergoes significant changes.

As a result of the analysis of the connections of the same type of components, it is advisable to use their serial connection in the channel for processing sensor signals in the information control system.

Therefore, the aim of the work is to analyze the use of a serial connection of the same type of digital filters and improve the approach for calculating the cutoff frequencies of a new connection. Such a connection makes it possible to increase the efficiency of the information control system in autonomous mobile robotic systems by reducing the bandwidth and increasing the steepness of the amplitude frequency characteristic.

3. Increasing the Steepness of Amplitude Frequency Characteristic of Digital Bandpass Filters

With a series connection of the same type of bandpass filters, the amplitude frequency characteristic of the new connection is compressed, as it were, while the cutoff frequencies are shifted to the center frequency and the steepness of the amplitude frequency characteristic increases, Figure 3.

The transfer function of the main bandpass filter is mathematically described as follows

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}},, \tag{10}$$

where a_0, a_1, a_2, b_1, b_2 — the real coefficients of the numerator and denominator, respectively.

When substituting $z^{-1} = e^{-j\varpi}$ or by the Euler formulas $z^{-1} = \cos(\varpi) - j\sin(\varpi)$, ϖ — where is the normalized angular frequency, $\varpi = 2\pi \frac{f}{f_d}$, $\varpi \in [0, \pi], f, f_d$ — line frequency and sampling frequency respectively.

Based on this transformation, we obtain a complex transfer coefficient, and on its basis, the amplitude frequency characteristic at $a_2 = -a_0$, $a_1 = 0$ and after the transformation we get the square of the amplitude frequency characteristic in the form

$$H^2(\varpi) = \frac{\left(2a_0 sin(\varpi)\right)^2}{(1-b_2)^2+b_1^2+2b_1(1+b_2)cos(\varpi)+4b_2 cos^2(\varpi)} \tag{11}$$

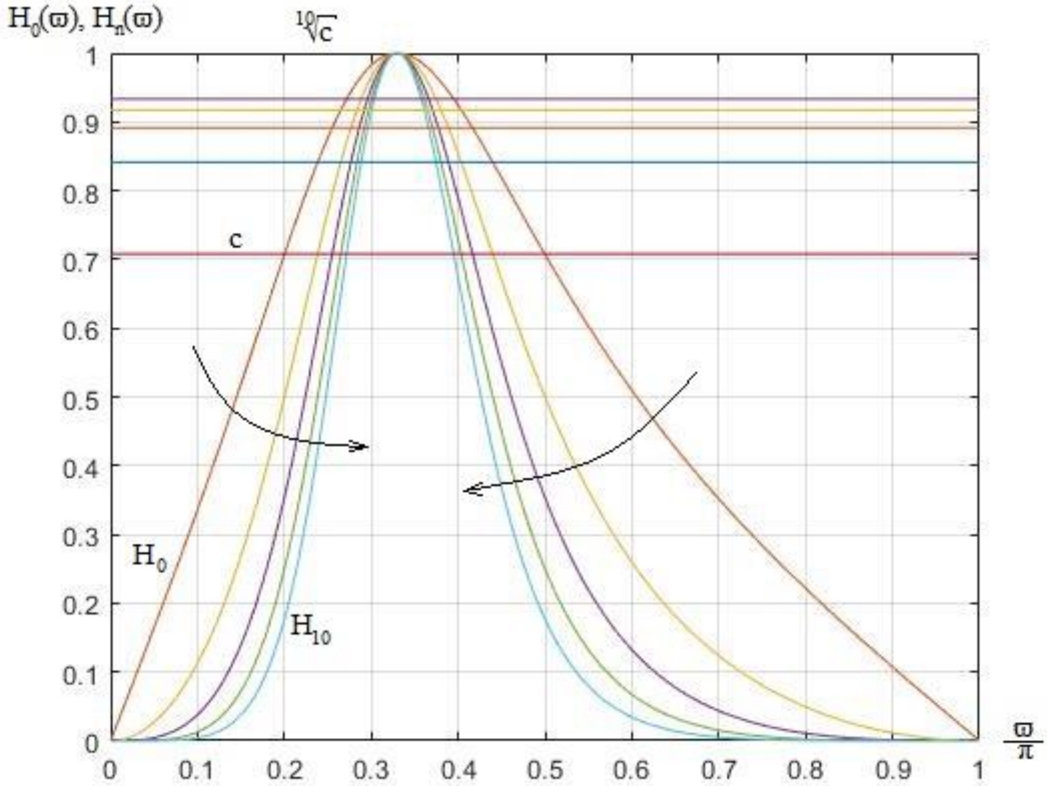


Figure 3: AFC of digital second-order Butterworth bandpass filters when they are connected in series. Where H_0 - AFC of the first order, H_{10} - AFC of the 10th order, c - the level of the cutoff frequency, $\sqrt[10]{c}$ - the level of the cutoff frequency of the 10th order

It should be noted that the peak frequency of the amplitude frequency characteristic does not change in this case and is determined by the equation, Figure 3

$$\varpi_p = arccos\left(-\frac{b_1}{1+b_2}\right). \tag{12}$$

Usually, the level at which the cutoff frequency is determined is $c = \frac{1}{\sqrt{2}} = 0{,}707$, i.e.

$$H(\varpi_c) = c, \tag{13}$$

Where ϖ_c — amplitude frequency characteristic cutoff at the level c. When multiplying the same type of amplitude frequency characteristic or raising their degree, the level remains the same, but then to determine the cutoff frequencies of the amplitude frequency characteristic of a new connection, when they are connected in series, it is necessary to extract the root of the corresponding order from the level c, i.e $\sqrt[n]{c}$, Figure 4. In Figure 1, these levels are shown by horizontal lines.

In this case, based on the amplitude frequency characteristic of the main filter cutoff frequencies can be calculated for the new compound, Figure 5.

On Figure 5 shows the correspondence between the cutoff frequencies of the main amplitude frequency characteristic of the first order at level c and the amplitude frequency characteristic when 5 (five) same-type first-order amplitude frequency characteristic are connected in series. These cutoff frequencies are determined by the main amplitude frequency characteristic, the parameters of which are known, on a new level.

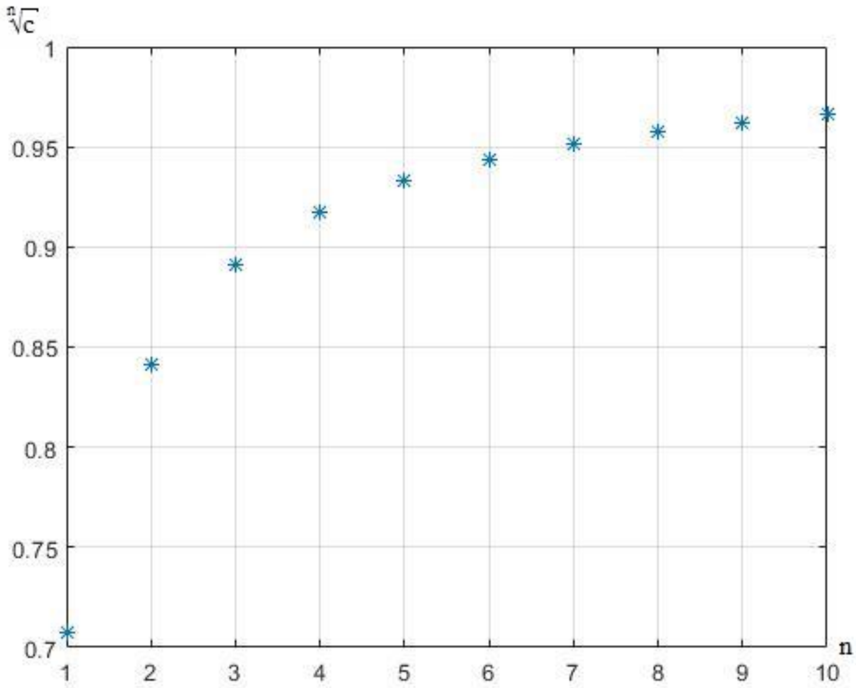


Figure 4: Graph of the cutoff level dependence on the number of connected filters of the same type

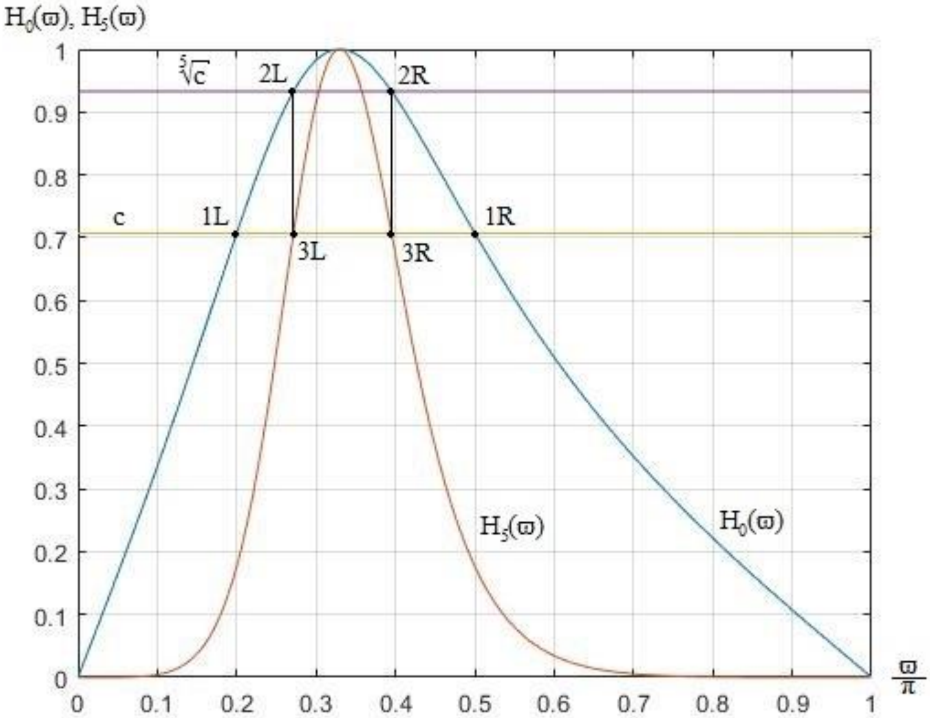


Figure. 5: AFC of the main Butterworth filter of the first order and AFC of the five same-type Butterworth filters of the first order connected into series. Where $H_0(\varpi)$ - AFC of the first order, $H_5(\varpi)$ - AFC of the 5^{th} order, c - the level of the cutoff frequency, $\sqrt[5]{c}$ - the level of the cutoff frequency of the 5^{th} order. Projection 2L and 2R points of the 1^{st} order to 3L and 3R points of the 5^{th} order respectively. 1L and 2R are base cutoff frequency of the first order.

To determine the cutoff frequencies of the new AFC after connecting n filters of the same type according to the main AFC, it is necessary to solve the equation [12]

$$H^2(\varpi) = \frac{(2a_0 sin(\varpi_{1n}))^2}{(1-b_2)^2+b_1^2+2b_1(1+b_2)cos(\varpi_{1n})+4b_2cos^2(\varpi_{1n})} = \sqrt[n]{c^2} = c^{\frac{2}{n}} \tag{14}$$

where ϖ_{1n} — cutoff frequency at a new level $\sqrt[n]{c}$ according to the main AFC.

However, when replacing a_0 with another expression from [10]

$$a_0 = \frac{1-b_2}{2} \tag{15}$$

and instead of $sin(\varpi_{1n})^2 = 1 - cos^2(\varpi_{1n})$, solving this equation, find formulas for determining the cutoff frequencies for the n-th connection of the same type filters. To simplify the representation of the result, introduce new notation

$$A = 4b_2c^{\frac{2}{n}} + (1-b_2)^2; \tag{16}$$

$$B = -b_1(1+b_2)c^{\frac{2}{n}}; \tag{17}$$

$$C = (1-b_2)\sqrt{\left(1-c^{\frac{2}{n}}\right)\left[(4b_2-b_1^2)c^{\frac{2}{n}} + (1-b_2)^2\right]}; \tag{18}$$

As a result, obtain the cutoff frequencies of the amplitude frequency characteristic with the n-th connection of the same type of filters, Figure. 6

$$\varpi_{cn1} = arccos(\frac{B+C}{A}); \tag{19}$$

$$\varpi_{cn2} = arccos(\frac{B-C}{A}). \tag{20}$$

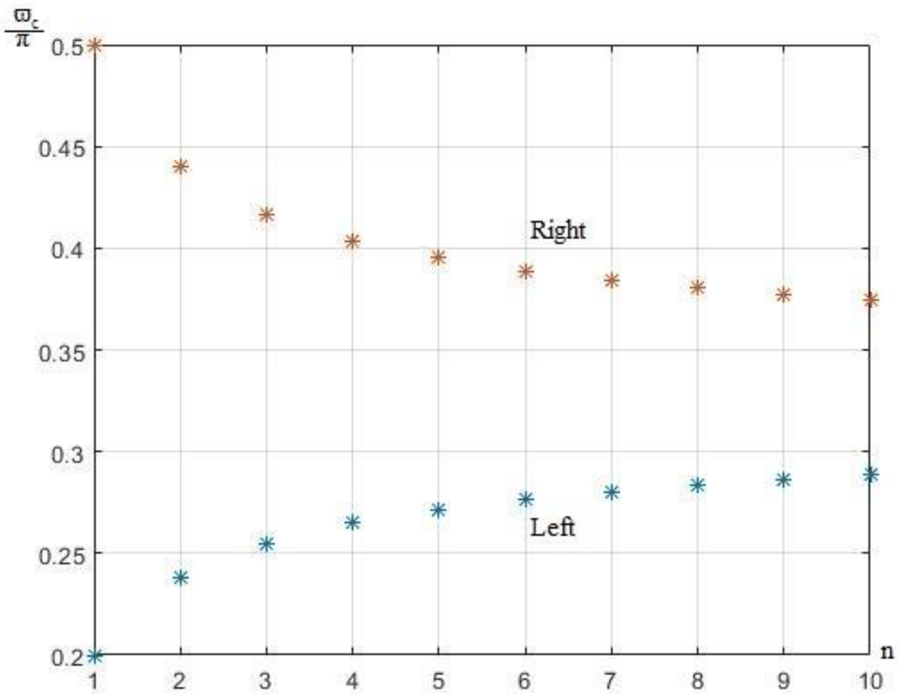


Figure 6: Graph of the dependence of the cutoff frequencies when the same type filters are connected in series on the number of connections

In accordance with the formulas obtained, it is possible to determine the bandwidth of such a compound as $\varpi_{BP} = |\varpi_{cn1} - \varpi_{cn2}|$, Figure 7.

As can be seen from Figure 7, the bandwidth decreases exponentially. In this case, it is possible to show how many times the bandwidth will decrease with a serial connection, Figure 8.

For example, when connecting four components of the same type, the bandwidth is reduced by more than two times, and with eight - more than three times, and ten - the bandwidth is reduced by 3.5 times.

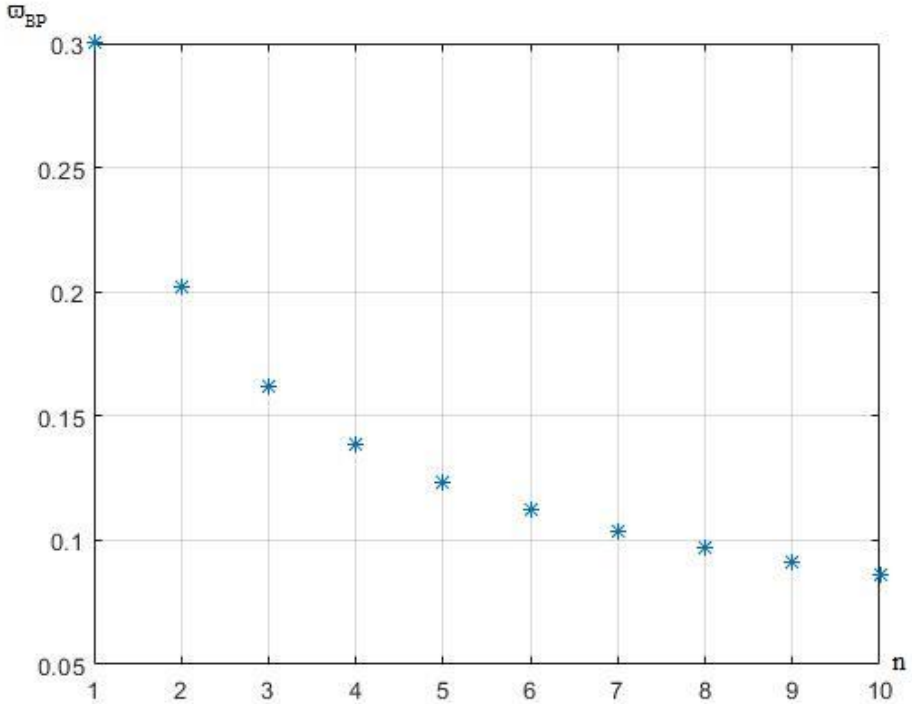


Figure 7: Plot of bandwidth ϖ_{BP} versus connection number

4. Approach Implementation

The implementation of this approach can be both hardware and software.

The hardware implementation is based on serial connection of n components of the same type. The main component is calculated based on the data flow and work mode. For example, cutoff frequencies, band pass, gain are determined based on data flow. Block diagram of possible implementation is shown on Figure 9.

The scheme proposes a serial connection of several $\mathrm{H_{oi}}$ components of the same type at the same time and registers $\mathrm{Rg_i}$ at the outputs of these components. The MX switch commutes the outputs of the registers to the output of a device. This allows to reduce the time for re-commutation of the components and the time of the transient process, since the necessary data is already in the registers. Such an approach can be implemented by an FPGA.

The software implementation was applied for an ultrasonic obstacle sensor data processing in mobile ICS based on the same type of first-order components.

For this, a generalized algorithm of the n-th order of the form

$$y_n = a_0^i x_n + i a_0^{i-1} a_1 x_{n-1} + \frac{i!}{2^{i-2}} a_0^{i-2} a_1^2 x_{n-2} + i a_0^{i-3} a_1^3 x_{n-3} + a_1^i x_{n-4} - b_1\ y_{n-1} - \\ - \frac{i!}{2^{i-2}} b_1^2 y_{n-2} - i b_1^3 y_{n-3} - b_1^4 y_{n-4}, \tag{21}$$

where a_i and b_1 – coefficients of the numerator and denominator of the first order respectively, i – count of the connected first-order elements, $i = 1, 2, 3, 4$.

Such algorithm was implemented, but turned out to be complex for implementation and operations. It requires additional computation, although some components of this algorithm were pre-calculated and stored in memory.

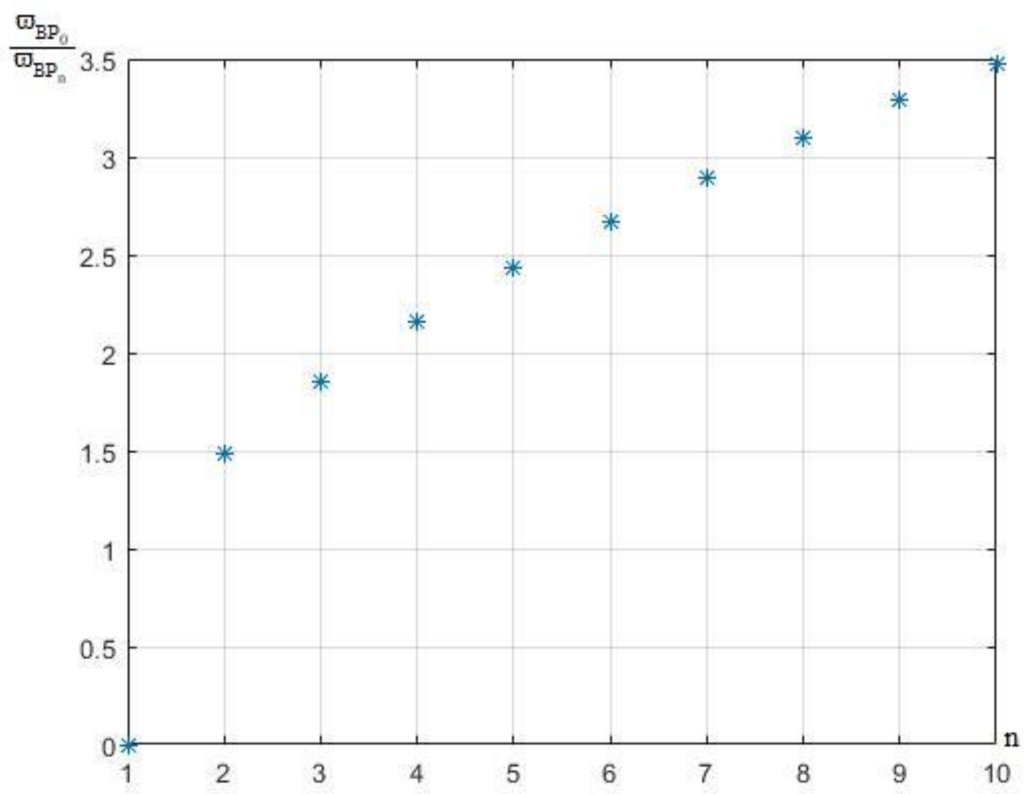


Figure 8. Bandwidth reduction ϖ_{BP} versus number of connections

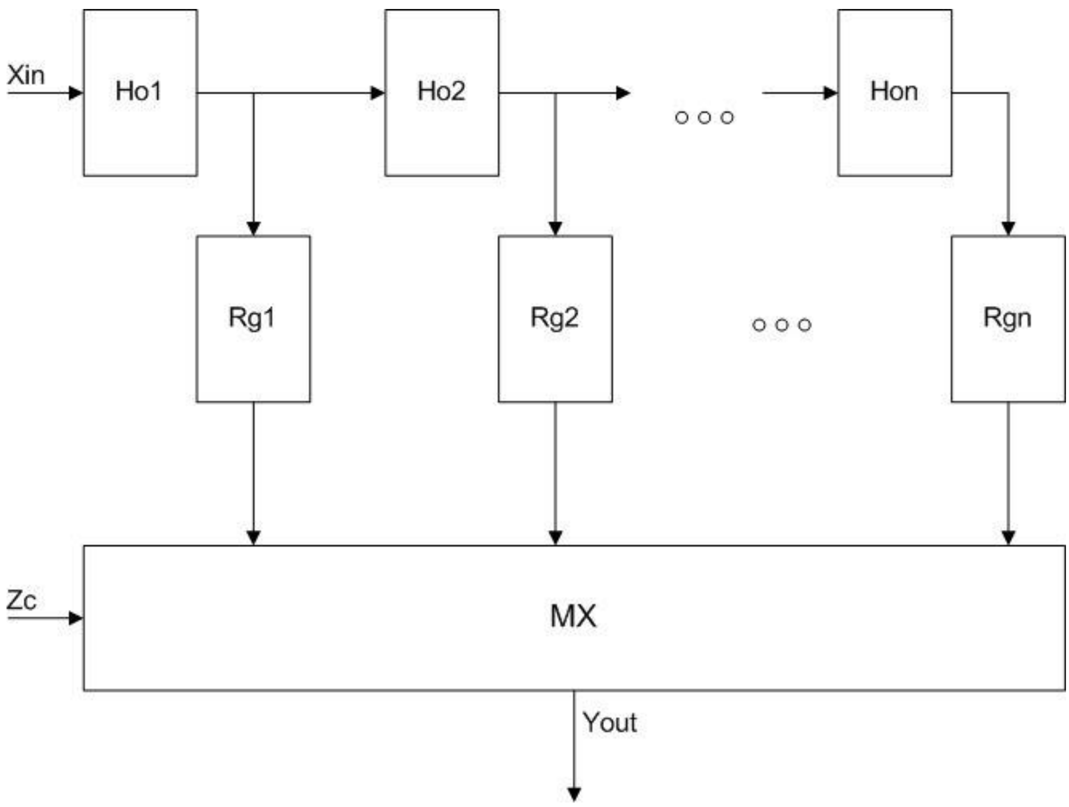


Figure 9: Block diagram of the serial connection of the same type elements

The algorithm was developed and a signal processing program was written according to a different principle for the ATMEL MEGA128 microcontroller. This made it possible to reduce the hardware of the system, since all sensors are connected to the ADC, which is located in the microcontroller, and also to reduce the processing time of data from the ADC because it is on the same data bus with the processor.

The signal graph of a first-order bandpass filter is shown in Figure 10.

Based on the signal graph a system of equations of the node states ordered signal graph, according which the calculation algorithm was compiled:

$$\begin{cases} x_1[i] = x_n(i); \\ x_2[i] = x_3[i-1]; \\ x_3[i] = x_4[i-1]; \\ x_4[i] = kx_i[i] - b_1x_3[i] - b_2x_2[i]; \\ x_5[i] = a_0 * \{x_4[i] + x_2[i]\} + 2x_3[i]; \\ y_n(i) = x_5[i]; \end{cases} \quad (22)$$

where $x_n, x_i[i], y_n$ – respectively input sequence, state of the graph node, output sequence.

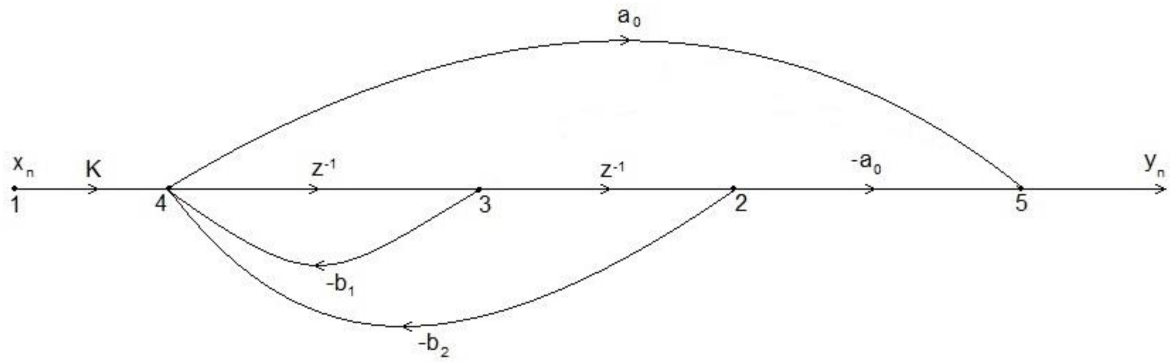


Figure 10: Signal graph of the first-order digital bandpass filter

This solution is more convenient for implementation and operation and allowed to reduce the computation time, since some constants were calculated in advance and stored in memory cells.In addition, subroutines were written that were connected if it was necessary to increase the order of the filter and the steepness of the AFC.

Modeling fragment of the filter is shown at the Figure 11.

5. Application of This Approach to Notch Filters

Band-pass filters include notch filters, which do not pass the necessary frequencies, but cut them out. By analogy with band-pass filters, in this case there will be compression of the amplitude frequency characteristic. But when connected in series, the amplitude frequency characteristic of notch filters does not narrow, but expands in accordance with the transfer function of this filter.

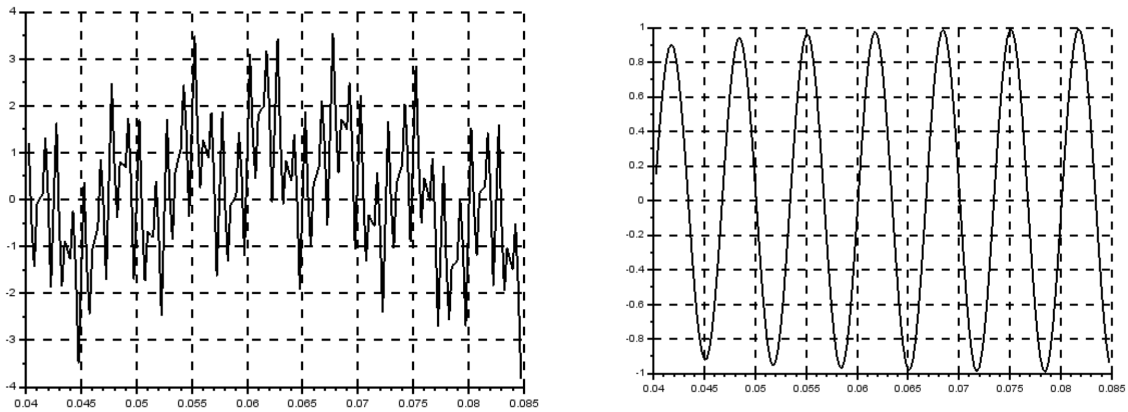
$$H(z) = \frac{a_0 + a_1z^{-1} + a_0z^{-2}}{1 + b_1z^{-1} + b_2z^{-2}}. \quad (23)$$

The mathematical description of the amplitude frequency characteristic of the notch filter looks as this.

$$H(\overline{\omega}) = \sqrt{\frac{(a_1 + 2a_0 sin\,(\overline{\omega}))^2}{(1-b_2)^2 + b_1{}^2 + 2b_1(1+b_2)\,cos(\overline{\omega}) + 4b_2 cos\,(\overline{\omega})^2}}. \quad (24)$$

With the expansion of the amplitude frequency characteristic, its steepness also increases with the expansion of the frequency band cut out from the signal spectrum, Figure 12.

This is not convenient in some issues, since with an increase the order it is necessary to narrow the cut-off band with an increase in the steepness of the amplitude frequency characteristic.



a) b)

Figure 11: Fragment of the input signal (a) and output signal (b) of the digital bandpass filter

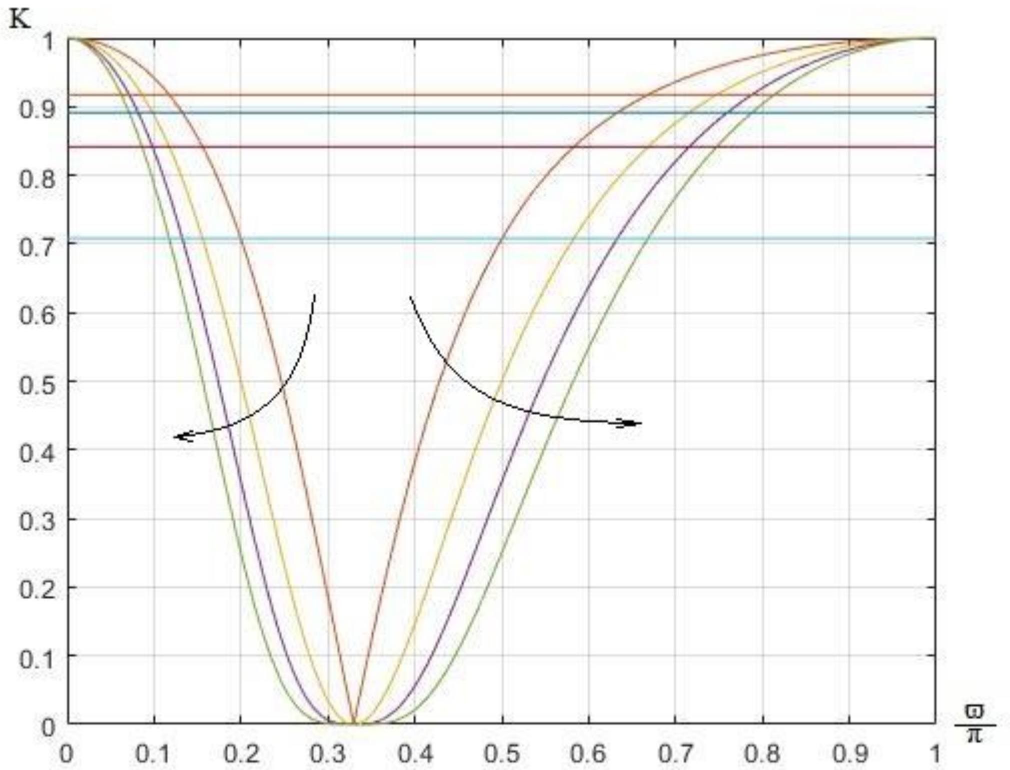


Figure 12: Amplitude frequency characteristic of first-order Butterworth digital notch filters when they are connected in series

6. Conclusion

Serial connection of frequency-dependent components of low order of the same type leads to exponentiation of the transfer function and AFC. This leads to compression of the AFC with an increase in the steepness of the AFC. The paper shows, using the example of a band-pass filter, that with such a connection, the center frequency does not change. It remains in its place, the cutoff frequencies (left and right) are shifted to the center. In addition, a new approach is obtained for calculating the exact values of the cutoff frequencies and bandwidth.

The advantage of such connection is simple increase in the front of the amplitude frequency characteristic and decrease in the bandwidth of the connection by three times. In the example shown in Figure 8, the bandwidth reduction is - 3.5 times with ten connected filters of the same type. The disadvantages of such connection include not so fast rise and fall of the AFC.

The possibility of implementing this approach on a mobile ICS is shown. This allows on board such systems to calculate the necessary "compression" of the amplitude frequency characteristic, and with limited computing capabilities on board, use a preliminary calculation in the form of a table of values that are stored in memory.

This approach allows you to automatically increase the operational security of data processing in the ICS in the presence of interference.

In addition, the analysis showed that a similar connection of notch filters does not allow obtaining a similar result for processing sensor signals in order to cut out unwanted frequencies in a narrow frequency band. As shown, this results not in a narrowing of the cutting band, but in widening the band, which is not desirable in such systems.

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